CHARACTERISTICS OF COUNTERCURRENT FLOW HEAT EXCHANGERS

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Abstract

The paper includes equations described heat exchange in countercurrent flow heat exchangers. Taking the energy balance formula as a basis and dividing the heat exchanger into sections, the thermal balances of the cooled fluid, plate and heated fluid are prepared and pertinent system of three differential equations is derived.

The ε -NTU method is used in the analysis. Exemplary temperature profiles in steady state conditions are presented in a graphic form. Transfer functions and dynamic characteristic are determined. Response on step disturbance of inlet temperature is found.

Responses on step or on sin disturbance of inlet temperature in steam condensation heat exchangers are found. The results of calculations are presented in a graphic form.

Keywords: *heat exchanger, thermal balance, temperature profile, and transfer function*

1. Introduction

A heat exchanger is a device in which energy is transferred from one fluid to another across a solid surface. Exchanger analysis and design therefore involve both convection and conduction. Radiative transfer between the exchanger and the environment can usually be neglected unless the exchanger is uninsulated and its external surfaces are very hot. Exemplary countercurrent flow heat exchangers are presented in Figures 1 and 2.

The base for elaboration of control systems is knowledge of static and dynamic characteristics of heat exchangers. Two important problems in heat exchanger analysis are: rating existing heat exchangers and sizing heat exchangers for a particular application.

Rating involves determination of the rate of heat transfer, the change in temperature of the two fluids, and the pressure drop across the heat exchanger. Sizing involves selection of a specific heat exchanger from those currently available or determining the dimensions for the design of a new heat exchanger, given the required rate of heat transfer and allowable pressure drop. The *LMTD* method can be readily used when the inlet and outlet temperatures of both the hot and cold fluids are known. When the outlet temperatures are not known, the *LMTD* can only be used in an iterative scheme. In this paper the *ε-NTU* method is used to simplify the analysis.

2. Energy Considerations

The first Law of Thermodynamics, in rate form, applied to a control volume (CV) between crosses 1 – 1 and 2 – 2, can be expressed as:

energy inflow = energy outflow + exchanged heat + accumulated energy

or $$Q_{inf} = Q_{out} + Q_{exch} + Q_{acc} \,. \tag{1}$$

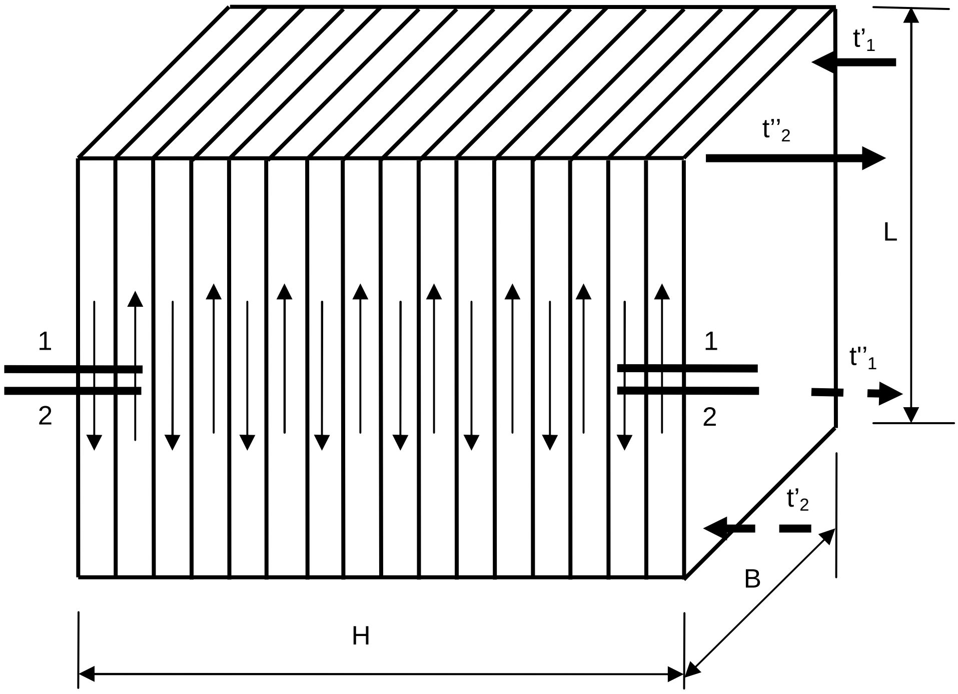

Fig. 1. Schema of plate type heat exchanger

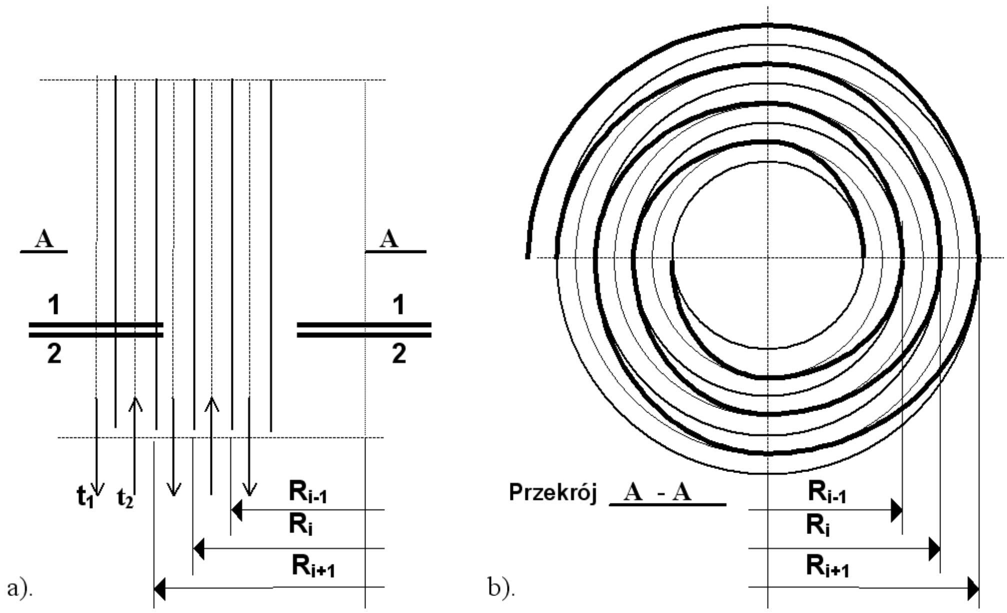

Fig. 2. Longitudinal flow heat spiral exchanger, a) longitudinal section, b) cross- section

This simplified form of the First Law assumes no work- producing processes, no energy generation inside the CV, and negligible kinetic and potential energy in the fluid streams entering and leaving the CV. In steady state operation the energy residing in the CV is constant, meaning that $Q_{acc} = 0$.

If, furthermore, the boundary of the CV is adiabatic (i.e., perfectly insulated), then $Q_{exch} = 0$.

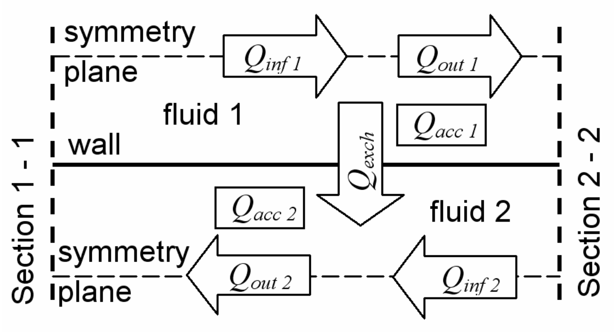

Fig. 3. Control volumes of cooled fluid (marked: fluid 1) and of heated fluid (marked: fluid 2) between sections 1-1 and 2-2 in the countercurrent flow heat exchangers

3. The thermal balances of a single stream of the cooled fluid, of plate and of the heated fluid

The following simplifying assumptions are introduced in the analysis:

- heat conductivity along the plate is disregarded,
- densities and heat capacities are constant in considered temperature range,
- overall heat transfer coefficient in constant in all points of heating surface,
- heat losses to environment are disregarded.

The ingredients of the energy balance for a single stream of the cooled fluid are as follows:

$$Q_{inf} = G'_1 c_1 \vartheta_1 dt, \tag{2}$$

$$Q_{out} = G'_1 c_1 (\vartheta_1 + \frac{\partial \vartheta_1}{\vartheta l}\partial l)\, dt, \tag{3}$$

$$Q_{exch} = \alpha_1 S_1 (\vartheta_1 - \vartheta_{sc})\, dl\, dt, \tag{4}$$

$$Q_{acc} = A_1 \rho_1 c_1 \frac{\partial \vartheta_1}{\partial t}\, dl\, dt, \tag{5}$$

By substitution equations (2 ÷ 5) into (1) we get

$$A_1 \rho_1 c_1 \frac{\partial \vartheta_1}{\partial t} dl + G'_1\, c_1 \frac{\partial \vartheta_1}{\partial l} = \alpha_1 S_1 (\vartheta_{sc} - \vartheta_1) dl\,, \tag{6}$$

thus

$$T_1 \frac{\partial \vartheta_1}{\partial t} + v_1 T_1 \frac{\partial \vartheta_1}{\partial l} = \vartheta_{sc} - \vartheta_1\,, \tag{7}$$

where:

$$T_1 = \frac{A_1 \cdot \rho_1 \cdot c_1}{\alpha_1 \cdot S_1} = \frac{D_{h1} \cdot \rho_1 \cdot c_1}{4 \cdot \alpha_1}\,, \quad D_{h1} = \frac{4 \cdot A_1}{S_1}\,, \quad G'_1 = \frac{G_1}{i} = \frac{A_1 \cdot i \cdot \rho_1 \cdot v_1}{i} = A_1 \cdot \rho \cdot v_1 = \frac{W'_1}{c_1}.$$

and

α – heat transfer coefficient [W m^{-2} K^{-1}]
A – area of duct cross – sectional [m^2]
B, L, H – width of heat exchanger, length of duct, length of exchanger, respectively [m]
c – specific heat [J kg^{-1} K^{-1}]
dl – element of length [m]
D_h – hydraulic diameter [m]

ρ – density [kg m^{-3}]
S – wetted periphery [m]
ϑ – temperature [K]
t – time [s]
T – time constant [s]
$G'c = W'$ – water equivalent for one duct [W K^{-1}]
v – velocity [m.s^{-1}]
1, 2, sc – cooled, heated fluid, plate, respectively,

The thermal balance of plate takes the form:

$$Sg\rho_{sc}c_{sc}\frac{\partial\vartheta_{sc}}{\partial t}dl = \alpha_1 S(\vartheta_1 - \vartheta_{sc})dl - \alpha_2 S(\vartheta_{sc} - \vartheta_2)dl\,, \tag{8}$$

and

$$\frac{\partial\vartheta_{sc}}{\partial t} = \frac{1}{T_{1s}}(\vartheta_1 - \vartheta_{sc}) - \frac{1}{T_{2s}}(\vartheta_{sc} - \vartheta_2), \tag{9}$$

where:

g – wall thickness [m],

and

$$T_{1s} = \frac{(S\cdot g\cdot\rho_{sc})\cdot c_{sc}}{\alpha_1\cdot S}, \quad T_{2s} = \frac{(S\cdot g\cdot\rho_{sc})\cdot c_{sc}}{\alpha_2\cdot S}$$

The thermal balance of the heated fluid takes the form:

$$A_2\rho_2 c_2\frac{\partial\vartheta_2}{\partial t}dl - G'_2 c_2\frac{\partial\vartheta_2}{\partial l}dl = \alpha_2 S_2(\vartheta_{sc} - \vartheta_2)dl\,, \tag{10}$$

thus

$$T_2\frac{\partial\vartheta_2}{\partial t} - v_2 T_2\frac{\partial\vartheta_2}{\partial l} = \vartheta_{sc} - \vartheta_2\,, \tag{11}$$

where:

$$T_2 = \frac{A_2\cdot\rho_2\cdot c_2}{\alpha_2 S_2} = \frac{D_{h2}\cdot\rho_2\cdot c_2}{4\cdot\alpha_2}, \quad D_{h2} = \frac{4\cdot A_2}{S_2}, \quad G'_2 = \frac{G_2}{i+1} = \frac{A_2\cdot i\cdot\rho_2\cdot v_2}{i+1} = A_2\cdot\rho_2\cdot v_2 = \frac{W'_2}{c_2}.$$

4. Static characteristics of plate heat exchangers

If we disregard time derivatives in the balance equations (6, 8, 10), we obtain differential equations for steady state conditions:

$$W'_1\frac{\partial\vartheta_1}{\partial l} = \alpha_1 S_1(\vartheta_{sc} - \vartheta_1), \tag{12}$$

$$0 = \alpha_1(\vartheta_1 - \vartheta_{sc}) - \alpha_2(\vartheta_{sc} - \vartheta_2), \tag{13}$$

$$-W'_2\frac{\partial\vartheta_2}{\partial l} = a_2 S_2(\vartheta_{sc} - \vartheta_2), \tag{14}$$

Boundary conditions are as follows:

$$\text{for } l=0 \quad \vartheta_1 = t'_1, \tag{15}$$

$$\text{for } l=L \quad \vartheta_2 = t'_2. \tag{16}$$

where

t' – inlet temperature [K]

If we solve equations (12, 13, 14) taking into account boundary conditions (15 ÷ 16), we obtain of formulae describing temperatures for the steady state conditions:

$$\vartheta_1(l) = t'_1 - (t'_1 - t'_2)\frac{1-\exp((W'_1/W'_2 - 1)\cdot k\cdot F(l)/W'_1)}{1-(W'_1/W'_2)\exp((W'_1/W'_2-1)\cdot k\cdot F(L)/W'_1)}, \tag{17}$$

$$\vartheta_2(l) = t'_2 + (t'_1 - t'_2)\frac{W_1}{W_2}\frac{\exp((W'_1/W'_2-1)\cdot k\cdot F(l)/W'_1) - \exp((W'_1/W'_2-1)\cdot k\cdot F(L)/W'_1)}{1-(W'_1/W'_2)\exp((W'_1/W'_2-1)\cdot k\cdot F(L)/W'_1)}, \tag{18}$$

$$\vartheta_{sc}(l) = \frac{\alpha_1\cdot\vartheta_1(l) + \alpha_2\cdot\vartheta_2(l)}{\alpha_1+\alpha_2}, \tag{19}$$

where:

$$\frac{1}{k} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{g}{\lambda} + R_Z, \; F(l) = 2\cdot B\cdot l, \; \frac{F(l)}{W'_1} = \frac{2\cdot B\cdot l}{A_1\cdot v_1\cdot\rho_1\cdot c_1} = \frac{4}{D_{h1}\cdot v_1\cdot\rho_1\cdot c_1}\cdot l, \tag{20}$$

and

F – heat transfer area [m^2]
k – overall heat transfer coefficient [$W\ m^{-2}\ K^{-1}$]
λ – thermal conductivity coefficient [$W\ m^{-1}\ K^{-1}$]
R_Z – additional thermal resistance of calcium precipitations [$W^{-1}\ m^2\ K$]

In balanced heat exchangers $\frac{W'_1}{W'_2} = 1$. Hence from equations (17, 18) we obtain:

$$\vartheta_1(l) = t'_1 - (t'_1 - t'_2)\cdot\frac{k\cdot F(L)/W'_1}{1+k\cdot F(L)/W'_1}\cdot\frac{l}{L}, \tag{17a}$$

$$\vartheta_2(l) = t'_2 - (t'_1 - t'_2)\cdot\frac{k\cdot F(L)/W'_1}{1+k\cdot F(L)/W'_1}\cdot\left(1-\frac{l}{L}\right), \tag{18a}$$

In steam heat exchangers, $W_2 < W_1 = \infty$ and from equations (17, 18) we obtain:

$$\vartheta_1(l) = t'_1, \tag{17b}$$

$$\vartheta_2(l) = t'_2 + (t'_1 - t'_2)\cdot\left(\exp\left(\frac{k\,(F(L)-F(l))}{W_2}\right)-1\right), \tag{18b}$$

Taking into account that:

$$t''_1 = \vartheta_1(L), \tag{21}$$

$$t''_2 = \vartheta_2(0), \tag{22}$$

the outlet temperatures *t”₁*, *t”₂* can be calculated from system of equation (17, 18), in balanced heat exchangers from equations (17a, 18a) and in steam heat exchangers from equations (17b, 18b).

5. Dynamic characteristic of plate heat exchangers. Response to disturbance of temperature

Plate heat exchanger is considered as element with two input and two output quantities:

$$\mathbf{Y_2} = \mathbf{G_{2\,x\,2}}\,\mathbf{X_2}, \tag{23}$$

$\mathbf{X_2} = \mathbf{X}\,[t'_1, t'_2]$ – input signal, two – dimensional vector
$\mathbf{Y_2} = \mathbf{Y}\,[t''_1, t''_2]$ – output signal, two – dimensional vector

G – transmittance matrix: $\mathbf{G} = \mathbf{G}_{2x2} = \begin{Vmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{Vmatrix} = \begin{Vmatrix} \dfrac{t''_1(s)}{t'_1(s)} & \dfrac{t''_1(s)}{t'_2(s)} \\ \dfrac{t''_2(s)}{t'_1(s)} & \dfrac{t''_2(s)}{t'_2(s)} \end{Vmatrix}.$

If we disregard heat accumulation in plates of exchanger (9) in the thermal balance equations (7, 9, 11), we should obtain system of two differential equations:

$$T_1\frac{\partial\vartheta_1}{\partial t} + v_1 T_1\frac{\partial\vartheta_1}{\partial l} = \vartheta_2 - \vartheta_1, \tag{24}$$

$$T_2\frac{\partial\vartheta_2}{\partial t} - v_2 T_2\frac{\partial\vartheta_2}{\partial l} = \vartheta_1 - \vartheta_2, \tag{25}$$

where T_1 and T_2 are calculated for overall heat transfer coefficient.

Let’s consider small changes of temperatures from state of equilibrium:

$$\vartheta_{1,2} = \overline{\vartheta_{1,2}} + \Delta\vartheta_{1,2},$$

where $\bar{\vartheta}$ – temperature in state of equilibrium, $\Delta\vartheta = \Theta$ – change of temperature ϑ.

Let’s define:

$$L^*_1 = \frac{L}{v_1 T_1},\quad L^*_2 = \frac{L}{v_2 T_2},\quad v^* = \frac{v_1}{v_2},\quad t^* = \frac{v_1\cdot t}{L},\quad l^* = \frac{l}{L},\ \text{i. e.: } dt^* = \frac{v_1}{L}dt,\quad dl^* = \frac{dl}{L}.$$

Then equations (24, 25) can be written in dimensionless form [1]:

$$\frac{\partial\Theta_1}{\partial t^*} + \frac{\partial\Theta_1}{\partial l^*} = L^*_1(\Theta_2 - \Theta_1), \tag{26}$$

$$v^*\frac{\partial\Theta_2}{\partial t^*} - \frac{\partial\Theta_2}{\partial l^*} = L^*_2(\Theta_1 - \Theta_2), \tag{27}$$

We solve system of equations (26, 27) by the Laplace – transform method. Boundary conditions (15, 16) are transformed too and included into solution. Then, the transfer functions can be written as:

$$G_{11} = \frac{2q\exp(b+q)}{q-p+(p+q)\exp(2q)}, \tag{28}$$

$$G_{12} = L*_1 \frac{1-\exp(-2q)}{p+q-(p-q)\exp(-2q)}, \tag{29}$$

$$G_{21} = L*_2 \frac{1-\exp(-2q)}{p+q-(p-q)\exp(-2q)}, \tag{30}$$

$$G_{22} = \frac{2q\exp(-b+q)}{-p+q-(p+q)\exp(2q)}, \tag{31}$$

where:

$$b = -\left(\frac{1-r}{2}s + \frac{b_1-b_2}{2}\right),\; p = \left(\frac{1+r}{2}s + \frac{b_1+b_2}{2}\right),\; q = \sqrt{p^2 - b_1 b_2}\;.$$

6. Simulations example

Change of outlet temperature of cooled fluid $\Delta\vartheta''(t)$, which is response to step change of inlet temperature of heated fluid $\Delta\vartheta'(t)$ could be calculated from equation.

$$\Delta\vartheta''(t) = \frac{M(0)}{N(0)} + \sum_{k=1}^{n} \frac{M(s_k)}{s_k\, N'(s_k)} e^{s_k t}, \tag{32}$$

where $M(s_k)$ is numerator, $N(s_k)$ denominator of the taken into consideration transfer function, s_k are roots of an equation $N(s_k) = 0$ and $N'(s_k) = [dN(s)/ds]_{s=sk}$.

First summand in right part of equation (32) represents steady-state conditions, following – transient components.

The transfer functions G_{12}, G_{21} are interesting particularly. In case steam condensation heat exchangers $b_1 b_2 = 0$, hence $p = q$ and the transfer functions G_{21} takes the form:

$$G_{21} = L*_2 \frac{1-\exp(-b_2)\exp((1+r)s)}{(1+r)s+b_2}, \tag{33}$$

Supplementary schema of heat exchanger under equation (33) with sine-response function is showed in Figure 4.

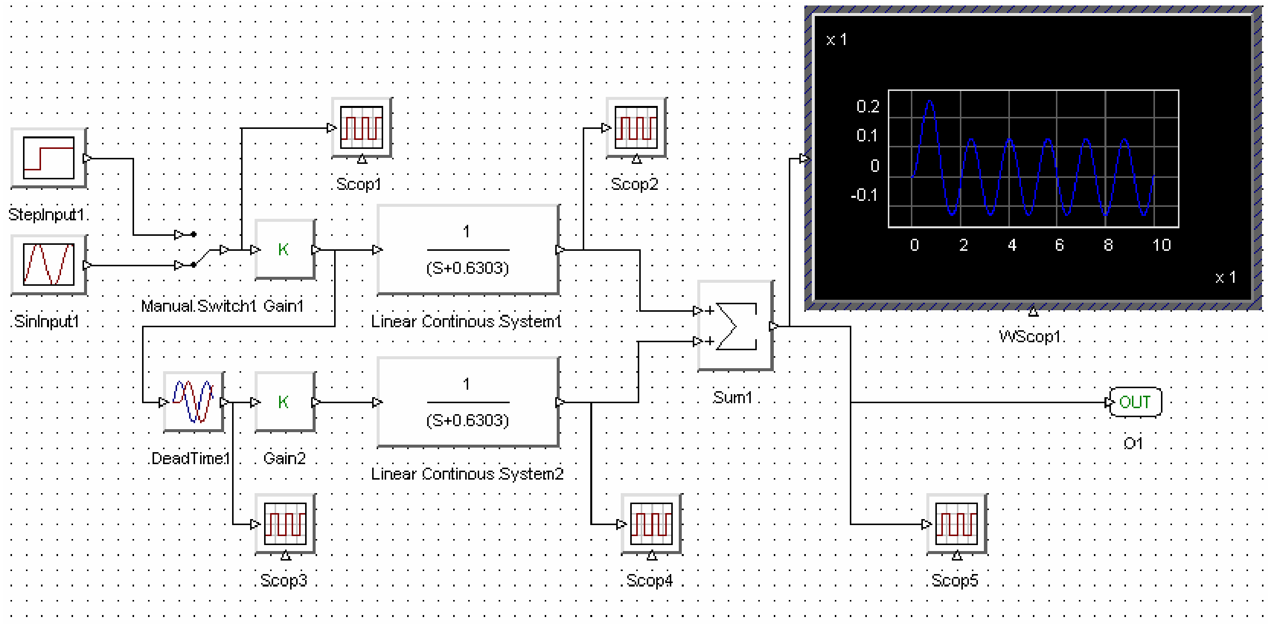

Fig. 4. Supplementary schema of heat exchanger under equation (33) with sine-response function

7. Conclusions

Values of temperatures in individual sections countercurrent flow exchangers could be computed using presented method of calculations.

Important element of this computations is determination of heat transfer coefficients α_1, α_2 and overall heat transfer coefficient k.

Better knowledge of specificities enables farther optimization heat exchangers and exchanger control systems.

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References

[1] Adamski, M., *Theoretical characteristics of plate heat exchangers*, VII Internationales Symposium: Wärmeaustausch und Erneuerbare Energiequellen, Polnische Akademie der Wissenschaften, Technische Universität Szczecin, Szczecin 1998, pp. 9 -16.

[2] Chmielnicki, W., Zawada, B., *Właściwości dynamiczne przepływowych wymienników ciepła*, Nowa technika w Inż. San., Ogrzewnictwo i Wentylacja Nr 14, Arkady, Warszawa 1982.

[3] Żelazny, M., *Podstawy automatyki*, PWN, Warszawa 1976.