



## PRESSURE AND VELOCITY DISTRIBUTION IN SLIDE JOURNAL BEARING LUBRICATED MICROPOLAR OIL

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### Abstract

Present paper shows the results of numerical solution Reynolds equation for laminar, steady oil flow in slide bearing gap. Lubrication oil is fluid with micropolar structure. Materials engineering and tribology development helps to introduce oils with the compound structure (together with micropolar structure) as a lubricating factors. Properties of oil lubrication as of liquid with micropolar structure in comparison with Newtonian liquid, characterized are in respect of dynamic viscosity additionally dynamic couple viscosity and three dynamic rotation viscosity. Under regard of build structural element of liquid characterized is additionally microinertia coefficient. In modeling properties and structures of micropolar liquid one introduced dimensionless parameter with in terminal chance conversion micropolar liquid to Newtonian liquid. The results shown on diagrams of hydrodynamic pressure, velocity and velocity of microrotation distribution in dimensionless form in dependence on coupling number  $N^2$  and characteristic dimensionless length of micropolar fluid  $\Lambda_1$ . Differences were showed on graphs in the schedule of the circumferential velocity oils after the height of the gap in the flow of the micropolar and Newtonian liquid. In presented flow, the influence of lubricating fluid inertia force and the external elementary body force field were omit. Presented calculations are limited to isothermal models of bearing with infinite length.

**Keywords:** micropolar lubrication, journal bearing, hydrodynamic pressure, velocity, velocity of microrotation

### 1. Introduction

Presented article take into consideration the laminar, steady flow in the crosswise cylindrical slide bearing gap. Non-Newtonian fluid with the micropolar structure is a lubricating factor. Exploitation requirements incline designers to use special oil refining additives, to change viscosity properties. As a experimental studies shows, most of the refining lubricating fluids, can be included as fluids of non-Newtonian properties with microstructure [3],[4],[6]. They belong to a class of fluids with symmetric stress tensor that we shall call polar fluids, and include, as a special case, the well known Navier-Stokes model. Presented work dynamic viscosity of isotropic micropolar fluid is characterized by five viscosities: shearing viscosity  $\eta$  ( known at the Newtonian fluids ), micropolar coupling viscosity  $\kappa$  and by three rotational viscosities. This kind of micropolar fluid viscosity characteristic is a result of essential compounds discussed in works [3],[4]. Regarding of limited article capacity please read above works. In presented flow, the influence of lubricating fluid inertia force and the external elementary body force field were omit [4],[5],[9].

## 2. Reynolds equations, hydrodynamic pressure

Basic equation set defining isotropic micropolar fluid flow are describe following equations [2],[4],[5]: momentum equation, moment of momentum equation, energy equation, equation of flow continuity. Incompressible fluid flow is taken into consideration with constant density skipping the body force. Further equation analysis were taken in rolling co-ordinate system, where the wrapping coordinate  $\varphi$  describes the wrapping angle of the bearing, the coordinate  $r$  describes radial direction from the journal to the bearing, the coordinate  $z$  describes longitudinal direction of crosswise bearing. In order to make the analysis of basic equations in dimensionless form [7], we input dimensionless quantities characterizing individual physical quantities. Oil velocity vector components are:

$$V_\varphi = UV_1 \quad V_r = \psi UV_2 \quad V_z = \frac{U}{L_1} V_3 \quad (1)$$

Reference pressure  $p_0$  caused by journal rotation with the angular velocity  $\omega$  was assumed in (7) taking into consideration dynamic viscosity of shearing  $\eta$  and the lubricating gap height  $h_1$  at the wrapping angle  $\varphi$  was taken in relative eccentricity function  $\lambda$  :

$$p_0 = \frac{\omega\eta}{\psi^2} \quad ; \quad h_1(\varphi, \lambda) = 1 + \lambda \cos\varphi \quad (2)$$

The constant viscosity of micropolar oil, independent from thermal and pressure condition in the bearing. Quantity of viscosity coefficient depend on shearing dynamic viscosity  $\eta$ , which is decisive viscosity in case of Newtonian fluids. Reference pressure  $p_0$  is also described with this viscosity, in order to compare micropolar oils results with Newtonian oil results. In micropolar oils decisive impact has quantity of dynamic coupling viscosity  $\kappa$  [1],[4]. In some works concerning bearing lubrication with micropolar oil, it's possible to find the sum of the viscosities as a micropolar dynamic viscosity efficiency. In presented article coupling viscosity was characterized with coupling number  $N^2$ , which is equal to zero for Newtonian oil:

$$N = \sqrt{\frac{\kappa}{\eta + \kappa}} \quad 0 \leq N < 1 \quad (3)$$

Quantity  $N^2$  in case of micropolar fluid, define a dynamic viscosity of coupling share in the oil dynamic viscosity efficiency.

From the dynamic rotational viscosities at the laminar lubrication, individual viscosities are compared to viscosity  $\gamma$ , which is known as the most important and it ratio to shearing viscosity  $\eta$  is bounded to characteristic flow length  $\Lambda$ , which in case of Newtonian flow assume the zero quantity. Dimensionless quantity of micropolar length  $\Lambda_1$  and micropolar length  $\Lambda$  are defined:

$$\Lambda = \sqrt{\frac{\gamma}{\eta}} \quad ; \quad \Lambda\Lambda_1 = \varepsilon \quad (4)$$

Dimensionless micropolar length  $\Lambda_1$  in case of Newtonian oil approach infinity.

Reynolds equation for stationary flow of laminar micropolar fluid in the crosswise cylindrical slide bearing gap can be present [1],[2],[3],[8] in dimensionless form:

$$\frac{\partial}{\partial \varphi} \left( \Phi_l(\Lambda_l, N, h_l) \frac{\partial p_l}{\partial \varphi} \right) + \frac{I}{L_l^2} \frac{\partial}{\partial z_l} \left( \Phi_l(\Lambda_l, N, h_l) \frac{\partial p_l}{\partial z_l} \right) = 6 \frac{dh_l}{d\varphi} \quad (5)$$

for  $0 \leq \varphi \leq \varphi_k; 0 \leq r_l \leq h_l; -1 \leq z_l \leq 1$

$$\text{where: } \Phi_l = h_l^3 + 12 \frac{h_l}{\Lambda_l^2} - 6 \frac{Nh_l^2}{\Lambda_l} \coth\left(\frac{h_l N \Lambda_l}{2}\right) \quad (6)$$

Below solutions (5) for infinity length bearing is presented. In this solution the Reynolds boundary conditions, applying to zeroing of pressure at the beginning ( $\varphi=0$ ) and at the end ( $\varphi=\varphi_k$ ) of the oil film and zeroing of the pressure derivative on the wrapping angle at the end of the film where fulfill. The pressure distribution function in case of the micropolar lubrication has a form:

$$p_l(\varphi) = 6 \int_0^\varphi \frac{h_l - h_{lk}}{\Phi_l(\Lambda_l, N, h_l)} d\varphi; \quad p_{lN}(\varphi) = 6 \int_0^\varphi \frac{h_l - h_{lk}}{h_l^3} d\varphi \quad (7)$$

where:  $h_{lk} = h_l(\varphi_k)$  lubricating gap height at the end of the oil film.

In the boundary case of lubricating Newtonian fluid, pressure distribution function is a pressure  $p_{lN}(\varphi)$ . Example numerical calculation were made for the infinity length bearing with the relative eccentricity  $\lambda=0,6$ .

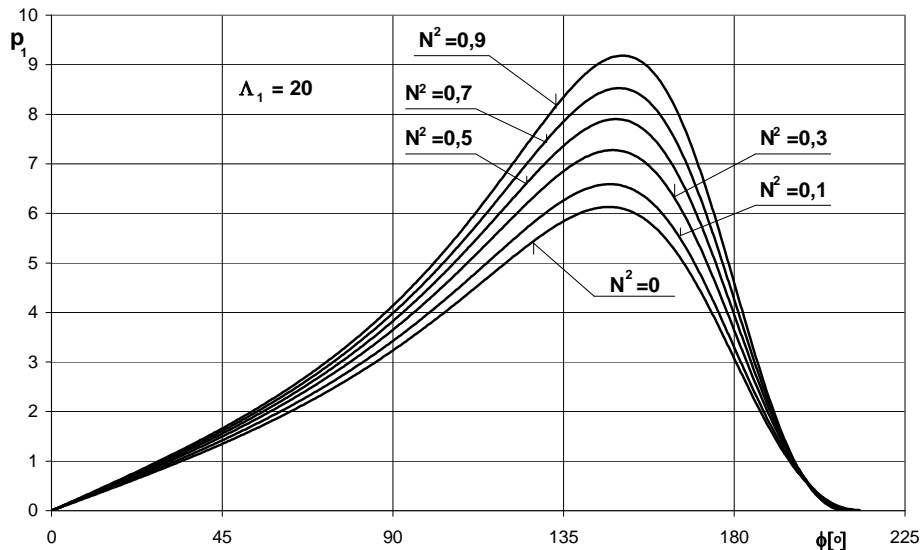


Fig.1 The dimensionless pressure distributions  $p_l$  in direction  $\varphi$  in dependence on coupling number  $N^2$  by micropolar ( $N^2 > 0$ ) and Newtonian ( $N^2 = 0$ ) lubrication for dimensionless eccentricity ratio  $\lambda=0,6$  and characteristic dimensionless length of micropolar fluid  $\Lambda_1=20$

Analyzing the influence of coupling number  $N^2$  and the influence of dimensionless micropolar length  $\Lambda_1$  on hydrodynamic pressure distribution in the bearing liner circuital direction. At the Fig.1 pressure distribution for individual coupling numbers at constant micropolar length  $\Lambda_1 = 20$ . The pressure increase effect is caused by oil dynamic viscosity efficiency increase as a result of coupling viscosity  $\kappa$ . At  $N^2 = 0,5$ , coupling viscosity is equal to shearing viscosity. Pressure graph in the Fig.1 for micropolar oil lubrication ( $N^2 > 0$ ) find themselves above the pressure graph at the Newtonian oil lubrication ( $N^2 = 0$ ). Pressure distribution is higher for higher coupling number. It is caused by oil viscosity dynamic efficiency. In the Fig.2 the course of dimensionless pressure  $p_l$  for few micropolar length quantity  $\Lambda_1$  is shown: Decrease of this parameter determine the increase of

micropolar oil rotational dynamic viscosity. Pressure distribution are presented at the constant coupling number  $N^2=0,4$ . Newtonian oil pressure in the course 1. Rotational viscosity increase determine the pressure distribution increase and is caused, because both the oil flow and velocities of microrotation are coupled. Quantities of coupling number  $N^2$  and dimensionless micropolar

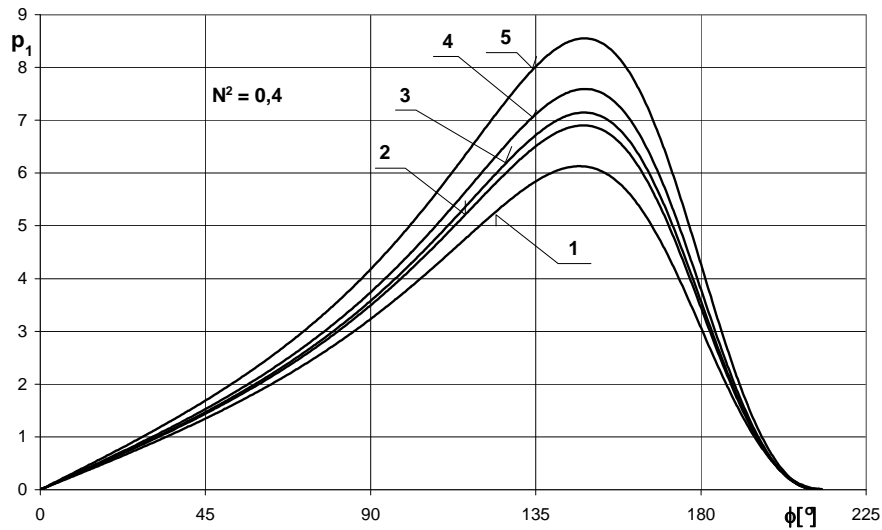


Fig.2 The dimensionless pressure distributions  $p_1$  in direction  $\phi$  in dependence on characteristic dimensionless length of micropolar fluid  $\Lambda_1$ : 1) Newtonian oil, 2)  $\Lambda_1=40$ , 3)  $\Lambda_1=30$ , 4)  $\Lambda_1=20$ , 5)  $\Lambda_1=10$ , for dimensionless eccentricity ratio  $\lambda=0,6$  and coupling number  $N^2=0,4$

length where taken from works [1],[2]. Based on given hydrodynamic pressure distribution  $p_1$  on wrapping angle of the bearing  $\phi$ , the numerical quantities of maximal pressure  $p_{1m}$  and the angular

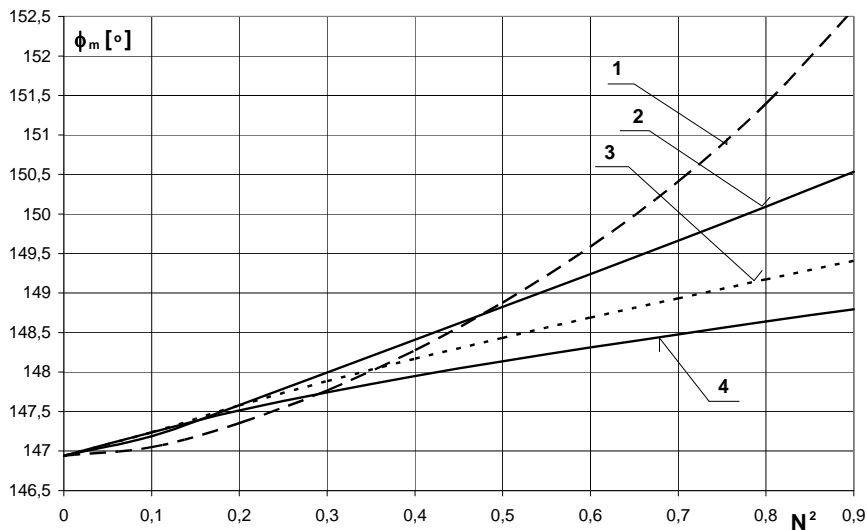


Fig.6 Angle  $\phi_m$  situated maximal pressure  $p_{1m}$  in dependence on coupling number  $N^2$  for characteristic dimensionless length of micropolar fluid  $\Lambda_1$ : 1)  $\Lambda_1=10$ , 2)  $\Lambda_1=20$ , 3)  $\Lambda_1=30$ , 4)  $\Lambda_1=40$

coordinate  $\phi_m$  (at the maximal position) were obtain. Quantities  $p_{1m}$  are presented in the Fig.3 in the coupling Number  $N^2$  function for chosen micropolar length  $\Lambda_1$ . All lines are coming out from the maximal pressure point in case of Newtonian fluid flow. We observe maximal pressure increase when the coupling number  $N^2$  increases.

### 3. Velocity and velocity of microrotation distribution

The field equation of micropolar fluid with general lubrication theory assumptions is simplified into two systems of coupled ordinary differential equation. He interests us in the case of the bearing of the infinite breadth the arrangement the coupling the velocity  $V_\phi$  and the microrotation velocity  $\Omega_z$  according [8] and introduced in dimensionless form:

$$\begin{aligned} (1-N^2) \frac{\partial p_1}{\partial \phi} &= \frac{\partial^2 V_1}{\partial r_1^2} + N^2 \frac{\partial \Omega_3}{\partial r_1} \\ \frac{1-N^2}{N^2 A_1^2} \frac{\partial^2 \Omega_3}{\partial r_1^2} - \frac{\partial V_1}{\partial r_1} - 2\Omega_3 &= 0 \end{aligned} \quad (8)$$

Dimensionless velocity of microrotation  $\Omega_3$  are in formula:

$$\Omega_z = \Omega_3 \frac{U}{\varepsilon} \quad (9)$$

The profiles of the schedule of velocity and microrotation velocity after the height of the gap (coordinate  $r$ ) comply following boundary conditions  $V_1(s_1)$  and  $\Omega_3(s_1)$  [8] on the surface of the bearing  $s_1 = 0$  and on the slide  $s_1 = 1$ :

$$\begin{cases} V_1(0) = 1 \\ V_1(1) = 0 \end{cases} ; \quad \begin{cases} \Omega_3(0) = 0 \\ \Omega_3(1) = 0 \end{cases} ; \quad s_1 = \frac{r_1}{h_1} \quad (10)$$

Thus, the expressions for velocity  $V_\phi$  as a results of the solutions of the abowe equations with boundary conditions (10) are [8] in dimensionless form  $V_1$ :

$$V_1 = \frac{1}{2} s_1^2 h_1^2 \frac{\partial p_1}{\partial \phi} + \left( s_1 h_1 - \frac{N}{A_1 h_1} \sinh s_1 N A_1 h_1 \right) A_1 + (1 - \cosh s_1 N A_1 h_1) \frac{2N}{A_1} \frac{A_3}{A_2} + 1 \quad (11)$$

where:

$$\begin{aligned} A_1 &= \frac{-\sinh N A_1 h_1}{\left[ \sinh N A_1 h_1 - \frac{2N}{A_1 h_1} (\cosh N A_1 h_1 - 1) \right]} - \frac{h_1^2}{2} \frac{\partial p_1}{\partial \phi} \\ A_2 &= h_1 \left[ \sinh N A_1 h_1 - \frac{2N}{A_1 h_1} (\cosh N A_1 h_1 - 1) \right] \\ A_3 &= \frac{1}{2} (\cosh N A_1 h_1 - 1) + \frac{h_1^2}{2} \frac{\partial p_1}{\partial \phi} \left[ \frac{1}{2} (\cosh N A_1 h_1 - 1) + 1 - \frac{2N}{A_1 h_1} \sinh N A_1 h_1 \right] \end{aligned} \quad (12)$$

The profile velocity  $V_1$  was introduced on Fig.4 along the height of the gap in the point where maximum hydrodynamic pressure steps out for three parameters of the micropolar oil. Nonlinearity visible is insignificant in the relation to the linear graph in the case of lubrication from newtonian oil. Differences among the schedule of the velocity of micropolar and newtonian oil it was introduced on Fig. 5. in points of the maximum pressure. The difference of the speed was marked from dependence:

$$\Delta V_1 = V_{1p} - V_{1N} \quad (13)$$

where:

$V_{1p}$  - dimensionless velocity for micropolar oil flow marked from (12),

$V_{IN}$  - dimensionless velocity for newtonian oil flow marked from (14)

$$V_{IN} = 1 - s_1 + \frac{h_1}{2} (s_1^2 - s_1) \frac{\partial p_1}{\partial \varphi} \quad (14)$$

Change velocities  $\Delta V_1$  in point of the maximal pressure is asymmetric in relation to the centre of the height of the gap. For  $0 < s_1 < 0,5$  the velocity of the micropolar flow is larger from the

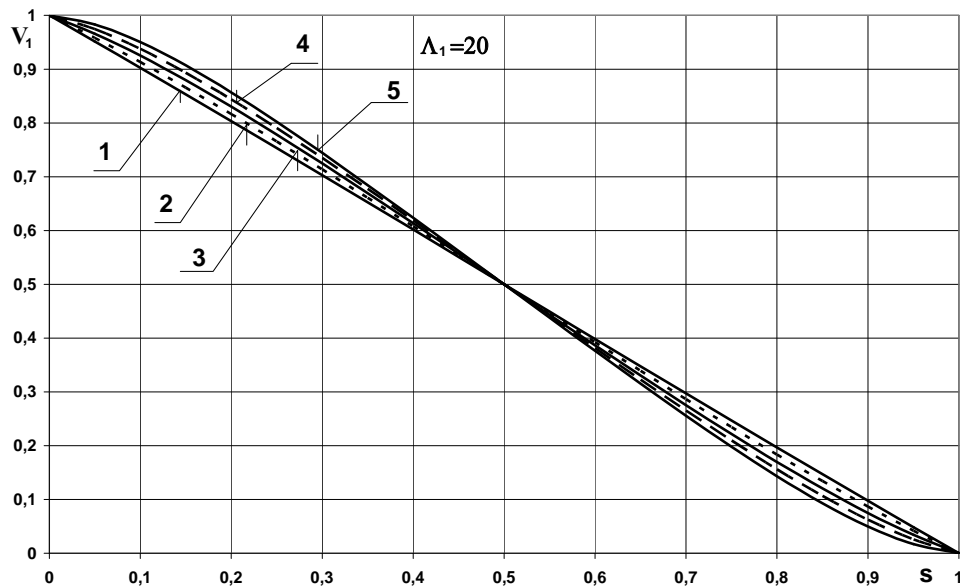


Fig.4 Velocities  $V_1$  in points of the maximal pressure of micropolar fluid  $\Lambda_1=20$  : 1)  $N^2=0,1$ ; 2)  $N^2=0,3$ ; 3)  $N^2=0,5$ ; 4)  $N^2=0,7$ ; 5)  $N^2=0,9$

velocity of the flow of the Newtonian oil and is for  $0,5 < s_1 < 1$  smaller. The growth of the coupling viscosity (coupling number  $N$ ) causes the growth of the nonlinearity of velocity  $V_1$ .

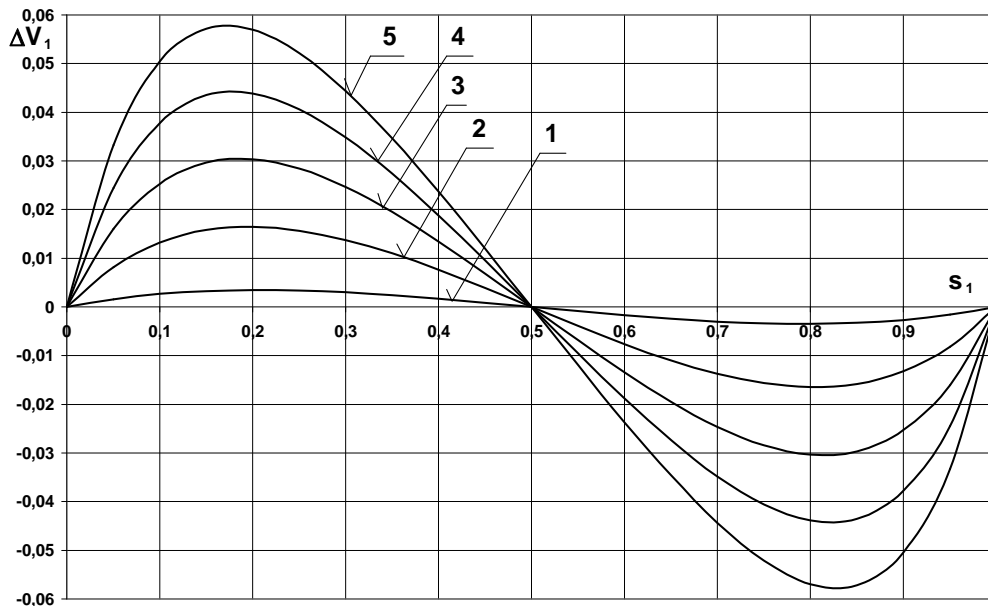


Fig.5. Change velocities  $\Delta V_1$  in points of the maximal pressure of micropolar fluid  $\Lambda_1=20$  : 1)  $N^2=0,1$ ; 2)  $N^2=0,3$ ; 3)  $N^2=0,5$ ; 4)  $N^2=0,7$ ; 5)  $N^2=0,9$

The expressions for velocity of microrotation  $\Omega_z$  as a results of the solutions of the above equations with boundary conditions (10) are [8] in dimensionless form  $\Omega_3$  :

$$\Omega_3 = -s_1 \frac{h_1}{2} \frac{\partial p_1}{\partial \varphi} + (\cosh s_1 N \Lambda_1 h_1 - 1) \frac{A_1}{2h_1} + \frac{A_3}{A_2} \sinh s_1 N \Lambda_1 h_1 \quad (15)$$

In the Fig.6 are presented profile velocity of microrotation  $\Omega_3$  in points of the maximum pressure in the coupling Number  $N^2$  function for chosen micropolar length  $\Lambda_1 = 20$ .

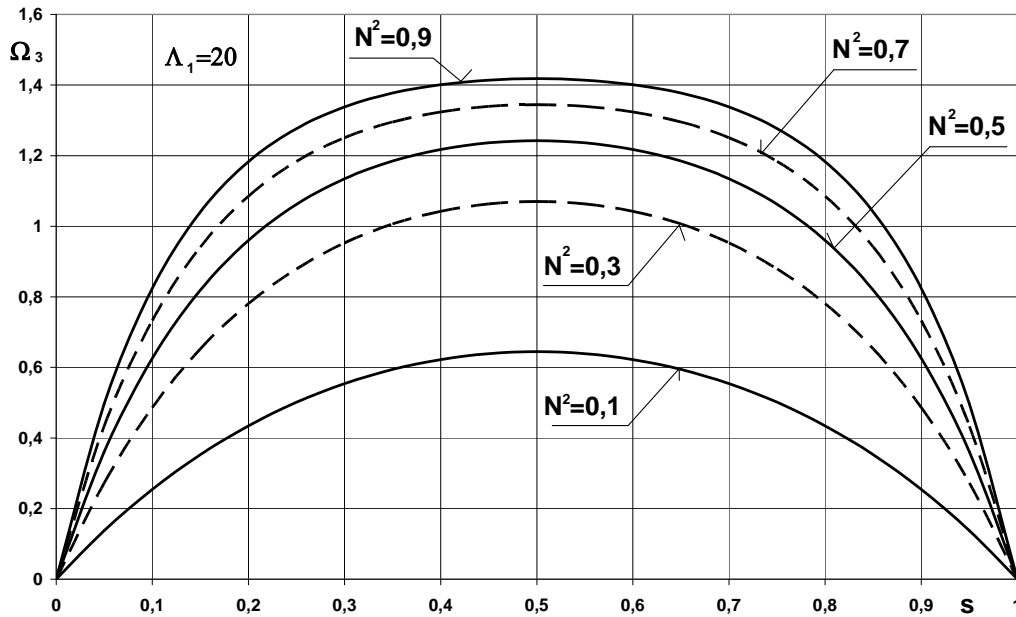


Fig.6. Velocities of microrotation  $\Omega_3$  in points of the maximal pressure of micropolar fluid  $\Lambda_1=20$  :1)  $N^2=0,1$  ;2)  $N^2=0,3$  ; 3)  $N^2=0,5$  ; 4)  $N^2=0,7$  ;5)  $N^2=0,9$

The profile of the velocity of microrotation  $\Omega_3$  is symmetrical in relation to the centre of gap. He is more even (flat) after the height of the gap in the case of the growth of the coupling number  $N$ .

#### 4. Conclusions

Presented example of the Reynolds equation solutions for steady laminar non-Newtonian lubricating oil flow with micropolar structure, enable the hydrodynamic pressure distribution introductory estimation as a basic exploitation parameter of slide bearing. Comparing Newtonian oil to oils with micropolar structure, can be used in order to increase hydrodynamic pressure and also to increase capacity load of bearing friction centre. Micropolar fluid usage has two sources of pressure increase in view of viscosity properties: increase of fluid efficient viscosity (coupling viscosity increase) and the rotational viscosity increase (characteristic length parameter  $\Lambda$ ). Author realize that he made few simplified assumptions in the above bearing centre model and in the constant parameter characterizing oil viscosity properties. Despite this calculation example apply to bearing with infinity length, received results can be usable in estimation of pressure distribution and of capacity force at laminar, steady lubrication of cylindrical slide bearing with infinity length. Presented results can be usable as a comparison quantities in case of numerical model laminar, unsteady flow Non-Newtonian fluids in the lubricating gaps of crosswise cylindrical slide bearings.

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## Notation

- $L_1$  dimensionless bearing length  $L_1=b/R$   
 $N$  coupling number  
 $R$  radius of the journal (m)  
 $U$  peripheral journal velocity (m/s)  $U = \omega R$   
 $V_i$  components of oil velocity in co-ordinate  $i = \varphi, r, z$  (m/s)  
 $V_i$   $i=1,2,3$  dimensionless components of oil velocity in co-ordinate  $\varphi, r, z$   
 $\Lambda$  characteristic length of micropolar fluid (m)  
 $\Lambda_1$  dimensionless characteristic length of micropolar fluid  
 $\Omega_i$  components of oil microrotation velocity in co-ordinate  $i = \varphi, r, z$  (1/s)  
 $\Omega_i$   $i=1,2,3$  dimensionless components of oil microrotation velocity in co-ordinate  $\varphi, r, z$   
 $b$  length of the journal (m)  
 $h$  gap height (m)  
 $h_1$  dimensionless gap height  $h = \varepsilon h_1$   
 $p$  hydrodynamic pressure (Pa)  
 $p_0$  characteristic value of pressure (Pa)  
 $p_1$  dimensionless hydrodynamic pressure  $p_1 = p/p_0$   
 $r$  co-ordinate in radial of the journal (m)  
 $z$  co-ordinate in length of the journal (m)  
 $z_1$  dimensionless co-ordinate in length of the journal  $z_1 = z/b$   
 $\alpha, \beta, \gamma$  micropolar rotational viscosities in co-ordinate  $\varphi, r, z$  ( $\text{Pa s m}^2$ )  
 $\varepsilon$  radial clearance (m)  
 $\eta$  dynamic oil viscosity (Pa s)  
 $\kappa$  micropolar coupling viscosity (Pa s)  
 $\lambda$  dimensionless eccentricity ratio  
 $\rho$  oil density ( $\text{kg/m}^3$ )  
 $\varphi$  the angular co-ordinate  
 $\varphi_e$  the angular co-ordinate for the film end  
 $\psi$  dimensionless radial clearance ( $10^{-4} \leq \psi \leq 10^{-3}$ )  $\psi = \varepsilon/R$   
 $\omega$  angular journal velocity (1/s)