SIMULATION OF VIBRATIONS OF MACHINE ELEMENTS ON THE EXAMPLE OF VIBRATIONS OF MARINE DIESEL CYLINDER LINERS

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Abstract

The article presents new methods for studying mechanisms of vibrations by means of their simulation in the electronic environment “Electronics Workbench”. 4CH 8,5/11 marine diesel cylinder liner was represented as a mechanical circuit of two-terminals – elements with mass, internal friction (dissipation), and stiffness. The use of electro-mechanical analogy of vibration systems has allowed the transition from mechanical chain to a chain of electrical circuits in the electronic environment «Electronics Workbench». The obtained values of the parameters of 4CH 8,5/11 diesel liner vibrations in the electronic environment (frequency, amplitude and vibration velocity) were in good reliance with the results of natural experiment.

Key words: Marine Diesel, cylinder, vibrations, simulation, modelling

1. Introduction

Corrosive and erosive destruction of the cooling surfaces of marine diesel engines, and in particular the surfaces of the cooling cylinder liners, significantly reduces their resource indicators. The research indicates that the resource of cylinder liners exposed to the erosion - corrosion attack, reduces up to 50% of its theoretical value. It is commonly assumed that these processes are generated by vibrations of liners which are caused by piston impulses coupled with transposition of connecting rod at the moment of overtravel. Impulses generated by cylinder liner vibrations are responsible for creating in the cooling water favorable environmental conditions for cavitation. For more active influence on the negative effects of vibration cavitation in the engines the benchmark trials are required as well as development of methods for calculating the durability of parts under the influence of cavitation erosion.

This problem can be solved by obtaining a reliable picture of the nature of cylinder liner vibrations at any alighting in a cylinder block considering influence of all force factors (the value of piston impulse, pulse of operating gas pressure in a cylinder). A visual model, one that reflects real conditions of liner vibrations or with some assumptions close to them, can be achieved by modelling of the process in computer virtual environment. The obtained parameters of vibration (frequency, amplitude and vibration acceleration) will simulate the characteristics of the damping device (stiffness, mass and dissemination property).

2. Assumptions to the mathematical model
This was done by introducing the mechanical system in the form of a combination of separate elements with different properties selecting active elements that can deliver energy into a mechanical system and stimulate its movement [1]. The mentioned mechanical chain will reproduce dynamic properties of the original mechanical system with an acceptable accuracy.

In the experiment two-terminals (elements with one input and one output) were used as components of the mechanical chain.

A mechanical chain with separate units of cylinder parts with the same wall thickness can be used as a model of liner with flanges and alighting zones.

In this way the entire liner is divided into separate rings and thin-walled cylinder with a constant wall thickness that corresponds to the middle of the liner between the alighting zones (Fig.1). Each unit in this mechanical chain is characterized by its mass, resilience, viscous friction, and consists of the following basic two-terminals: resilient element (stiffness), dissipative element (damper) and inertia element (mass).

An equivalent scheme of a two-terminal connection into a mechanical chain of ten units representing a cylinder liner of the 4CH 8,5/11 type is shown in Fig.2. During the set up of this scheme the following rules were adopted:

- Elements of mass, stiffness and dampers are connected to a fixed bearing by one fastener. Thus, the displacement of one of the poles of each component in relation to the fixed bearing is zero, which simulates the installation conditions of liner edges inside the cylinder block;
- If all elements of the mechanical system move with equal speed, their two-terminals are connected in parallel, and if the same force is transmitted through all elements, two-terminals are connected sequentially.

Fig.2 shows two-terminals which characterize mass of the rings denoted by symbols \( M_1, M_2, \ldots, M_9 \), two-terminals characterizing ring stiffness are denoted by the \( k_1, k_2, \ldots, k_9 \) symbols; symbols \( B_1, B_2, \ldots, B_9 \) - two-terminals, which characterize the dissipative properties of the rings, \( F \) - an active two-terminal of force determined by value of the force activating liner vibrations.

![Fig. 1. An example of the 4CH 8,5/11 diesel engine liner division into units of mechanical chain](image)
Thus, the task of determination of liner vibration frequency is brought to the determination of parameters of each element of the equivalent mechanical chain and solution of Kirchhoff’s equations, performed for each node and contour of the chain.

Fig. 2. Model of a mechanical chain of engine 448.5/11 cylinder liner (sleeve)

According to the decomposition of a simulated liner into separate sections (see Fig. 1), each of the units, except the third one, is a model of a ring and the third unit is a model of a smooth thin-walled cylindrical shell.

3. The mathematical model of cylinder shell vibrations

Each of the rings will generate asymmetric bending vibrations, during which the radial displacement (normal to median surface) of deformable elements of the ring is accompanied by a circumferential shift (tangent to the contour of cross-section). Radial pressure of elastic forces of the ring, resulting from deformation and referred to the center line, is expressed by the following equation [3]:

\[ q = \frac{1}{r^2} \left( M + \frac{d^2 M}{d\varphi^2} \right), \]  

where:

- \( r \) – radius of curvature of the centerline of the ring,
- \( M \) – bending moment at any section of the ring,
- \( \varphi \) – angular coordinate of the considered section.

In its turn, bending moment is determined by dependence:

\[ M = \frac{EJ}{r^2} \left( U + \frac{d^2 U}{d\varphi^2} \right), \]  

(2)
where:

$E$ – modulus of elasticity of the ring material,
$J$ – centroidal moment of inertia of the ring section,
$U$ – radial displacement of the points of the center line within the deformation of the ring.

Then, the formula for the elastic force pressure will take the following form:

$$q = \frac{EJ}{r^4} \left( w + 2 \frac{d^2w}{d\varphi^2} + \frac{d^4w}{d\varphi^4} \right).$$

The frequency of characteristic vibrations of the ring can be determined by the formula of S.P. Timoshenko [4]:

$$\omega^2 = \frac{EJ}{r^4 \rho \delta} \frac{n^2(n^2 - 1)^2}{(n^2 + 1)},$$

where:

$\rho$ – density of the ring material,
$\delta$ – the cross-sectional area of the ring,
$n$ - number of radial half waves in the cross section of the ring.

Note that the formula for the free vibration frequencies of the ring (4) can be obtained directly from the differential equations of the ring motion at a known formula for the elastic forces (3) and radial displacements given in the form:

$$w(t) = w_0 \cos n\varphi \sin \omega t.$$  

An analysis of the obtained equations (3) and (4) suggests that the ring stiffness element as a link of the equivalent scheme, can be expressed as follows:

$$k_i = \frac{EJ_i}{r_i^4} \frac{n^2(n^2 - 1)^2}{(n^2 + 1)},$$

and mass element -

$$m_i = \rho \delta_i.$$  

For the third unit of the equivalent mechanical chain modeling the smooth cylindrical shell with a wall thickness $\delta$, the expression for the free vibration frequency is given by the formula:

$$\omega^2 = \frac{D_{ww}}{R^4 \rho \delta} \frac{n^2(n^2 - 1)^2}{(n^2 + 1)}.$$  

Where the cylindrical stiffness of the shell is:
\[ D_u = \frac{E \delta^3}{12(1 - \mu^2)}, \quad R - \text{radius of the cylinder.} \]

Comparing the natural frequencies of the ring and the cylinder (4) and (8) it is easy to see that they are the same, if we replace the bending stiffness \( D_u \) by the normal stiffness \( EJ \) in formula (8), assuming \( \delta \) cross-sectional area of the ring.

If \( R/L \geq 0,1 \), then the finite length of the cylinder must be taken into account and the following formula should be used:

\[
\omega^2 = \frac{1}{\rho \delta} \frac{E \delta \left( \frac{\pi R}{L} \right)^4 + \frac{D_u}{R^2} \left( n^2 - 1 \right)^2 n^4}{R^2 \left( \frac{\pi R}{L} \right)^2 + \left( n^2 + 1 \right)n^2}. \tag{9}
\]

Then, similarly to formulas (6) and (7) the value of stiffness and mass of the third unit of the chain for the case \( R/L < 0,1 \) would be:

\[
k = \frac{D_u}{r^4} \frac{n^2 \left( n^2 - 1 \right)^2}{\left( n^2 + 1 \right)}, \tag{10}
\]

and for the case \( R/L \geq 0,1 \)

\[
k = \frac{E \delta \left( \frac{\pi R}{L} \right)^4 + \frac{D_u}{R^2} \left( n^2 - 1 \right)^2 n^4}{R^2 \left( \frac{\pi R}{L} \right)^2 + \left( n^2 + 1 \right)n^2}, \tag{11}
\]

\[ m = \rho \delta. \tag{12} \]

### 4. Identification of model parameters

Parameters of damper elements of the mechanical chain can be defined on the basis of reference data on the logarithmic decrement for the liner material. Simultaneously it should be kept in mind that determination of the damping characteristics of the material in a methodological sense is a much more complicated problem than the experimental determination of any other mechanical characteristics of the material, since the definition of the dissipation energy of the material at its cyclic deformation within elastic limit requires sophisticated methods of experiment, that border with physical ones. Therefore, the question of methods for determining the damping characteristics of the material is extremely important and the reliability and validity of experimental data depend on how well the method has been chosen.

Lack of attention to the specific characteristics of methods for the determination of the dissipation energy in the materials cyclically deformed in the range of Hooke’s law, has led to extensive evidence about a damping ability of the same materials, often contradicting each other.
As a result, it is necessary to select references on the damping ability of materials very carefully, taking into account a high sensitivity of damping rates to the experimental methods of their obtaining.

Certain reference data on damping properties of various brands of cast iron were obtained for the frequencies of cyclic deformation of samples in the range of 10-30 Hz. However the frequency of cyclic deformation plays an important role in determining the damping properties of a material only in ultrasonic range – 20000 Hz and more. Natural frequencies of cylinder liner vibrations are in the range of 1000-2000 Hz, so a cyclic deformation within this range would not be significant.

Within the heterogeneous states of stress observed in the vibrations of cylinder liner inflexibly fixed at the top collar, the damping decrement is an integral characteristic of damping properties of the entire material and is usually associated with the amplitude of maximum tension. Therefore, when determining damping properties of components of the chain, it is important to chose the value of the decrement corresponding to the maximum amplitude of liner vibration stresses.

The damper resistance coefficient $b_i$ is related to the logarithmic decrement $\theta$ by the following equation:

$$b_i = \frac{1}{\pi} \theta \sqrt{m_i k_i},$$

(13)

where:

$m_i, k_i$ – parameters of the mass of the elements and the stiffness of a chain link.

Basing on these assumptions calculations of mechanical chain parameters of the simulation model of the 4Ч8,5/11 diesel liner have been made.

Parameters of mass elements of all links of the chain which model the liner of the 4Ч8,5/11 diesel engine were determined from formulas (7), (10). Mass elements in this case will have the dimension of linear density. Density of the liner material $\rho = 7840$ kg/m$^3$.

<table>
<thead>
<tr>
<th>№ chain link</th>
<th>$\delta_i$, m</th>
<th>$m_i = \rho \delta_i$, kg/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring1</td>
<td>$2.985 \times 10^{-3}$</td>
<td>23,399</td>
</tr>
<tr>
<td>Ring 2</td>
<td>$1.414 \times 10^{-3}$</td>
<td>11,084</td>
</tr>
<tr>
<td>Cylinder 3</td>
<td>$4.000 \times 10^{-3}$</td>
<td>31,36</td>
</tr>
<tr>
<td>Ring 4</td>
<td>$1.414 \times 10^{-3}$</td>
<td>11,084</td>
</tr>
<tr>
<td>Ring 5</td>
<td>$6.872 \times 10^{-4}$</td>
<td>5,388</td>
</tr>
<tr>
<td>Ring 6</td>
<td>$1.414 \times 10^{-3}$</td>
<td>11,084</td>
</tr>
<tr>
<td>Ring 7</td>
<td>$6.872 \times 10^{-4}$</td>
<td>5,388</td>
</tr>
<tr>
<td>Ring 8</td>
<td>$1.414 \times 10^{-3}$</td>
<td>11,084</td>
</tr>
<tr>
<td>Ring 9</td>
<td>$1.118 \times 10^{-3}$</td>
<td>8,768</td>
</tr>
<tr>
<td>Ring 10</td>
<td>$8.388 \times 10^{-4}$</td>
<td>6,576</td>
</tr>
</tbody>
</table>

Stiffness parameters of the elements of the mechanical chain of the 4CH 8,5/11 diesel liner were determined from formula (6) for all units, except the third – simulating a smooth cylinder where formula (11) was used. According to these formulas stiffness per length unit is determined, so the dimension of the stiffness parameters is N/m$^2$. 
Parameters of damping elements were calculated from formula (13) and have in this case the
dimension of mechanical resistance per unit of length.

\[ b_i = \frac{1}{\pi} \rho \sqrt{m_i k_i}, \quad \frac{kg}{s \cdot m} \]

<table>
<thead>
<tr>
<th>№ chain unit</th>
<th>( k_i ), N/m²</th>
<th>( b_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring 1</td>
<td>6,741 ( 10^{11} )</td>
<td>3,641 ( 10^{04} )</td>
</tr>
<tr>
<td>Ring 2</td>
<td>3,529 ( 10^{11} )</td>
<td>1,813 ( 10^{04} )</td>
</tr>
<tr>
<td>Cylinder 3</td>
<td>1,462 ( 10^{09} )</td>
<td>6,013 ( 10^{03} )</td>
</tr>
<tr>
<td>Ring 4</td>
<td>3,529 ( 10^{11} )</td>
<td>1,813 ( 10^{04} )</td>
</tr>
<tr>
<td>Ring 5</td>
<td>1,811 ( 10^{11} )</td>
<td>9,055 ( 10^{03} )</td>
</tr>
<tr>
<td>Ring 6</td>
<td>3,529 ( 10^{11} )</td>
<td>1,813 ( 10^{04} )</td>
</tr>
<tr>
<td>Ring 7</td>
<td>1,811 ( 10^{11} )</td>
<td>9,055 ( 10^{03} )</td>
</tr>
<tr>
<td>Ring 8</td>
<td>3,529 ( 10^{11} )</td>
<td>1,813 ( 10^{04} )</td>
</tr>
<tr>
<td>Ring 9</td>
<td>2,852 ( 10^{11} )</td>
<td>1,450 ( 10^{04} )</td>
</tr>
<tr>
<td>Ring 10</td>
<td>2,137 ( 10^{11} )</td>
<td>1,087 ( 10^{04} )</td>
</tr>
</tbody>
</table>

5. Methodology of problem solution

As presented above, a further analysis of the mechanical chain is connected with the solution
of the system of Kirchhoff’s equations, which must be performed for each node and contour of the
chain. There are other ways of treating a chain consisting of individual units. In particular,
the amplitude-frequency and phase-frequency characteristics of the system are determined by the
inverse Laplace transform transfer function of the system, which in turn is the multiplication or the
sum of the transfer functions of the individual units, depending on how they are connected with
each other – in a sequential or parallel way.

Since the procedure of describing of the characteristics of individual units and their
interconnections through the transfer functions is sufficiently formalized and there are plenty of
studies on circuit calculations based on this technique, determination of the amplitude-
frequency characteristics of the cylinder liner simulator is not complicated. However, a more
vivid and interesting from the research point of view, is the method of virtual experiment. By
changing the parameters of various elements of the chain in this experiment, it is possible,
together with multivariable analysis of the mechanical system, to get its response to a variety
of effects.

For the virtual experiment, the software for automotive design Electronics Workbench (EWB)
[2], has been used.

As this software was developed for designing electrical circuits it was necessary to perform a
transfer from a mechanical chain model to an electrical circuit one to work in the Electronics
Workbench (EWB) environment. Such a transfer is made on the basis of physical analogies
between mechanical and electrical vibration processes. Vibrations in different systems: vibrations
of load under the force of elastic spring, electrical vibrations in the contours with inductivity and
capacity, spread of sound waves have common mathematical formulation and expressed by the
same differential equations. Thus the movement of load on an elastic spring in a viscous medium
under the driving force is described by the equation

\[ M \frac{d^2x}{dt^2} + B \frac{dx}{dt} + k \cdot x = Q(t), \]  \( (14) \)
and a differential equation of electrical circuit with one pair of nodes would be:

\[ C \frac{d^2u}{dt^2} + \frac{1}{R} \frac{du}{dt} + \frac{1}{L} \cdot u = \frac{di}{dt}. \]  

(15)

Comparing equations (14) and (15), we can establish the similarity between mechanical and electrical values. Dynamic equations for an \( n \) – contour electrical circuit consisting of a current source \( i_j \), resistors \( R_{jk} \), capacitors \( C_{jk} \) and inductors \( L_{jk} \), are Lagrange equations of the second kind:

\[ \frac{d}{dt} \left( \frac{dT_e}{dq_j} \right) + \frac{dU_e}{dq_j} + \frac{dF_e}{dq_j} = \frac{dj}{dt}, \quad (j = 1, 2,...,n), \]  

(16)

where the energy of the electric field

\[ T_e = \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} C_{jk} \cdot \dot{u}_j \cdot \dot{u}_k, \]  

(17)

Energy of magnetic field

\[ U_e = \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \frac{1}{L_{jk}} \cdot u_j \cdot u_k, \]  

(18)

dissipation function

\[ F_e = \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \frac{1}{R_{jk}} \cdot \dot{u}_j \cdot \dot{u}_k. \]  

(19)

Equations (14), (15) and (16) result from the fact that kinetic energy of a mechanical system corresponds to the energy of electric field, potential energy – energy of the magnetic field, generalized forces – rate of current. Then, Lagrange equations of the second kind for an electrical system by the adopted analogy of “force – current” express the first law of Kirchhoff: algebraic sum of currents in the node is equal to zero [1].

The obtained under these rules equivalent electrical scheme for the solution of a system of equations describing the vibrations of a mechanical system (liner) was designed in the dialog box of Electronics Workbench (EWB) and is presented in Figure 3.
6. Programme and results of experiments

According to input analogies between mechanical and electrical values the parameters of elements of the electrical circuit were defined: mass elements of units \( m_i \), expressed in \( \text{kg/m} \), are compared to capacitance \( C_i \), expressed in nanofarads; elements of stiffness \( k_i \), expressed in pascals, compared to the inductance in H so that \( 1/L_i = k_i \), and coefficient of damper resistance \( b_i \), expressed in \( \text{kg/m·s} \),- electrical resistance \( R_i \), defined as \( R_i = 1/b_i \) and expressed in ohms.

The obtained parameters of the equivalent electrical circuit for the 4CH8,5/11 diesel liner are presented in Table 3.

<table>
<thead>
<tr>
<th>№ chain unit</th>
<th>Inductance ( L ) ( \text{mH} )</th>
<th>Capacitance ( C ) ( \text{nF} )</th>
<th>Resistance ( R ) ( \text{Om} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring 1</td>
<td>1,484</td>
<td>23,399</td>
<td>2,308</td>
</tr>
<tr>
<td>Ring 2</td>
<td>2,833</td>
<td>11,084</td>
<td>4,635</td>
</tr>
<tr>
<td>Cylinder 3</td>
<td>72,87</td>
<td>31,36</td>
<td>13,97</td>
</tr>
<tr>
<td>Ring 4</td>
<td>2,833</td>
<td>11,084</td>
<td>4,635</td>
</tr>
<tr>
<td>Ring 5</td>
<td>5,522</td>
<td>5,388</td>
<td>9,281</td>
</tr>
<tr>
<td>Ring 6</td>
<td>2,833</td>
<td>11,084</td>
<td>4,635</td>
</tr>
<tr>
<td>Ring 7</td>
<td>5,522</td>
<td>5,388</td>
<td>9,281</td>
</tr>
<tr>
<td>Ring 8</td>
<td>2,833</td>
<td>11,084</td>
<td>4,635</td>
</tr>
<tr>
<td>Ring 9</td>
<td>3,506</td>
<td>8,768</td>
<td>5,797</td>
</tr>
<tr>
<td>Ring 10</td>
<td>4,679</td>
<td>6,576</td>
<td>7,733</td>
</tr>
</tbody>
</table>

Generation of unit vibrations in the simulating circuit was carried out by an impulse generator available at Electronics Workbench (EWB) library. An oscilloscope is included in the equivalent electrical scheme as a controller to register the frequency of vibrations.

The virtual experiment with an electrical scheme – simulating a model of the 4CH8,5/1 diesel liner 1 in the EWB environment was held under the following parameters of the output impulse generator signal:
- signal waveform – rectangular,
- frequency of signal 200 Hz,
- impulse duration 1%, which is 0,05 ms.

Another operation mode with the signal frequency of 200 Hz and impulse duration of 2% was also applied in the experiment. Such frequency and duration of an impulse corresponds to the shock of the piston within a degree of turn of the crankshaft with rotating frequency equal to 1500 rpm. The resulting vibration oscillograms are shown in Figures 4 and 5.
7. Final remarks

The nature of the obtained vibration oscillograms shows that impulse influence on the system generates vibrations of several harmonics at once, and in the time of approx. 3.5 ms, the higher harmonic is practically damped and vibrations occur with the frequency of the first harmonic. Table 4 shows the measurement results of first harmonic frequency.

<table>
<thead>
<tr>
<th>Signal frequency, Hz</th>
<th>Impulse duration, %</th>
<th>Vibration period ms</th>
<th>Vibration frequency, f, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>1</td>
<td>522.959</td>
<td>1912.2</td>
</tr>
<tr>
<td>200</td>
<td>2</td>
<td>581.224</td>
<td>1720.5</td>
</tr>
</tbody>
</table>

Tab. 4. The frequencies of the first harmonic vibrations of the liner
Simultaneously parameters of $448.5/11$ diesel cylinder liner vibrations were measured on a mechanical test bench where the frequency of the natural diesel liner vibration was $f = 1727 \pm 198$ Hz. In this way, the results obtained in the virtual experiment match well with the results of the natural vibrations of the liner.

References
