



## THE MODELLING OF ACCUMULATION AND DISSIPATION OF ENERGY IN MECHANICAL DRIVE SYSTEM

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### **Abstract**

*This paper presents a modelling of accumulation and dissipation energy in simple drive system. The simple drive system is presented by laboratory stand designed for calculate fatigue life of material for bending and rotation tests pieces. The laboratory stand was prepared to diagnosis of fatigue life of material by use energetic methods. This idea was submitted as utility model to Polish Patent Office. This laboratory stand gives ability to research a influence of change of fatigue life for energetic symptoms (like: temperature of material, vibration, Acoustic Emission, rotary speed, electrical energy). This paper presents influence of change of fatigue life for rotary speed. The rotary speed is value describing effect of accumulation and dissipation energy in drive system (in material). A difference between energy taken from power supply and consumed for giving rotary speed is used for fatigue of material. This simple theory is presented and modelling in this paper. The next paper (in this book) describes results of research that could verify this theory.*

**Keywords:** *modelling, accumulation and dissipation of energy, mechanical drive systems*

### **1. Introduction**

The necessary of diagnostic of machines is obvious consequence which is result of their maintenance. This concerns especially important and expensive machines. For many engineers and scientists a very difficult problem is how to calculate or predict the future life of mechanical elements that works by high cyclic load. We know that fatigue life is very complicated problem and it is difficult to precise calculation. We know too that the fatigue life is effect of accumulation of energy in material. The difficult problem is how to calculate this energy. We don't have to do this (at work of machines) but we can observe symptoms that are effect of accumulation energy. For example a growth of temperature is a value defining a increase of internal energy in material. We know that the growth of temperature depends on kind of material (elastic-plastic or plastic-brittle) and this is symptom describing the future fatigue life for this mechanical element.

The fatigue of material at work of mechanical elements could be described by many symptoms and parameters. Their are effect of change of properties of material. The cyclic load makes hardening or weakening of material that influences on rigidity. This makes that the bearings could be less loaded (the lesser deflection of shaft in place of support) and less energy is wasted there. The change of rigidity influences on change of vibration of mechanical drive system. For example a stable work after hardening of material (after 20 hours of work) makes stability of vibration

amplitude. This symptom is useful to diagnostic of technical state all machines or engines.

The change of properties of material influences on rotary speed. In used laboratory state the change of rotary speed is visible diagnostic symptom describing accumulation of energy in material. This accumulation of energy describes fatigue life of steel test piece. The observation of multi symptoms by use temperature, deflection, vibration, electrical energy: current and voltage, Acoustic Emission and rotary speed gives information about effects of accumulation energy in material. This same allows to estimate a fatigue life.

The multi symptoms research allow to make physical and mathematical model. It is necessary to verification results and could allow to predict fatigue life.

## 2. The purpose of test stand building

Every mechanical drive system owns a efficiency. The main stream of energy is passed to any working equipments. The part of energy is lost in mechanical elements (bearings, shafts, gear wheels, oil) which pass energy from source (electrical motor or engine) to working equipments. This small part of lost energy is used to change properties of material. The bearing and tooth of gear wheels are cyclic deformed. A oil is plastic deformed under cyclic loading of shaft neck. These all cyclic changes make growth of temperature and change material's properties. These results are effect of dissipation energy in material and dissipation energy in mechanical system (growth of vibrations).

Any delivered energy makes a change of situation of body (substance). This principle concerns to every situation: huge machines that are described like closed system and every elements (inside of substances). When it is considered to separate elements the dissipation of energy makes change of material's properties. Overflow of limit of this dissipation energy in material makes complete wear or crack of material. Present research should show how can describe technical state of working elements by use theory about dissipation energy in material. This problem is difficult to solve but it will give good information about calculate of reliability of mechanical structures.

The used laboratory stand is prepared to research fatigue life of bending and rotary elements working by cyclic loading. This laboratory stand is patterned on well-known fatigue machines - Schenck [4] but now it is used to calculate and diagnose of material by use energetic methods. The improvement of laboratory stand and new patterns was described in utility model [the number of application W.121416].

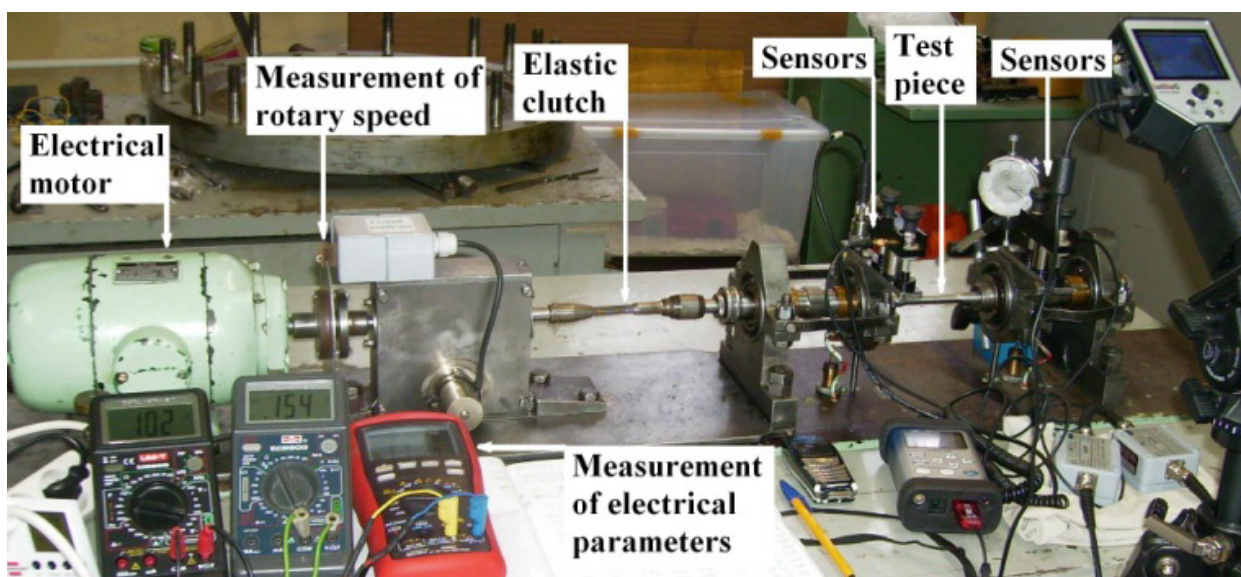


Fig. 1. The laboratory stand

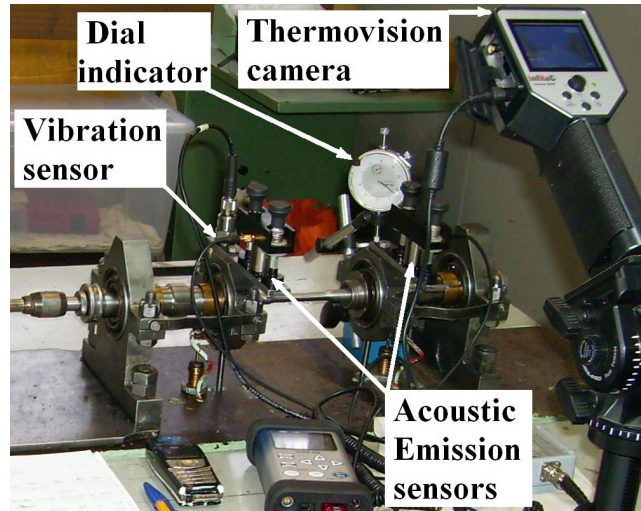


Fig. 2. The laboratory stand – place of sensor's installation

The principle of laboratory stand is possibility of observation of energetic symptoms for bending of rotary test piece. The main principle of laboratory stand is cause that all stream of energy is used in mechanical rotary system. The energy had been taken from electrically source could be used for two causes: giving kinetic energy to rotary elements or accumulation energy in material (what makes his fatigue). This allows to know better symptoms describing change of properties of material at cyclic loaded work. The main formula describing behaviour of drive model is relationship:

$$J \frac{d\omega}{dt} = M_d - M_{res} \quad (1)$$

where:

$J$  – moment of inertia [ $\text{kg m}^2$ ],

$\omega$  – angular velocity [ $\text{rad/s}$ ],

$t$  – time [ $\text{s}$ ],

$M_d$  – drive moment [ $\text{Nm}$ ],

$M_{res}$  – resistance moment [ $\text{Nm}$ ].

The principle 1 describes a behaviour of drive system as change between drive moment  $M_d$  and resistance moment  $M_{res}$ . The drive moment is value of energy delivered to system. The resistance moment describes wasting of energy in mechanical system: accumulation energy in material (increase of internal energy) and dissipation energy in mechanical system (growth of kinetic energy – increase of rotary speed). A difference between  $M_d$  and  $M_{res}$  is used to give rotary speed. So a value and a change of the rotary speed can describe a part of energy that is not used to change material's properties. The resistance moment  $M_{res}$  can be described generally as sum of two moments:

$$M_{res} = M_f + M_{str} \quad (2)$$

where:

$M_f$  – friction moment (in bearings only – the simple model) [ $\text{Nm}$ ],

$M_{str}$  – moment of change of material's structure [ $\text{Nm}$ ].

The  $M_f$  is simple function of one physical quantity - loading (stresses in material) because it makes more loading of bearings and more friction and lost energy. The principle between  $M_f$  and loading is linear function and can be simple to modeling. In laboratory stand is not more important elements where might be lost of energy.

The moment of change of material's structure  $M_{str}$  is difficult problem. This is nonlinear function of many physical quantities:

- temperature;
- deflection;
- rotary speed;
- parameters of Acoustic Emission (energy, amplitude, number of events, duration of signal, RMS);
- vibrations.

The  $M_{str}$  is following function:

$$M_{str} = f(T, D, \omega, AE, V) \quad (3)$$

where:

$M_f$  – temperature [K],  
 $D$  – deflection [mm],  
 $\omega$  – angular velocity [rad/s],  
 $AE$  – parameters of Acoustic Emission,  
 $V$  – RMS of vibrations.

### 3. The physical model of transformation of energy in mechanical drive system

Three values are necessary to modeling of behavior of drive system: input, output and disruptions. The input energy is measured by using electrical parameters:

- current (measured on rotor);
- voltage (measured on rotor);
- electrical input energy (energy delivered from power supply);

The drive moment  $M_d$  is calculated as electromagnetic moment [5]. This principle does not contain a efficiency of rotor's system (decrease a voltage in carbon brush and in rotor's winding) but this efficiency is high and might have solid number what helps by modeling. The main principle is shown below:

$$M_d = \frac{U \cdot I}{\omega} \quad (4)$$

where:

$U$  – voltage (on electrical terminal of rotor) [V],  
 $I$  – current (on electrical terminal of rotor) [A],  
 $\omega$  – angular velocity [rad/s].

The output values was specified before  $M_{str} = f(T, D, \omega, AE, V)$  and present it is not necessary precise model describing  $M_{str}$  (moment of change of material's structure). The future research can help to make this precise model. In this paper I can show a influence of fatigue life on change of the rotary speed  $\omega$ , the temperature  $T$  and the drive moment  $M_d$ . The main view of all mechanical drive system (laboratory stand) is shown below [3]:

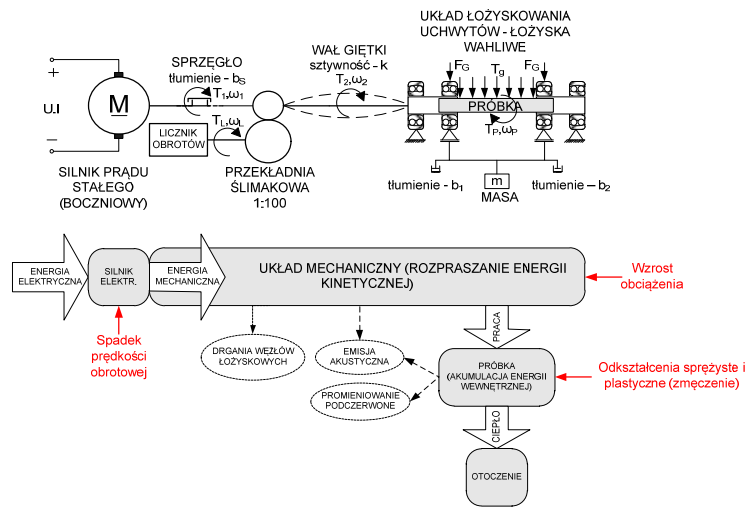


Fig. 3. The model of laboratory stand (simple mechanical drive system) [3]

The model on Fig. 3. shows a flow of energy in mechanical drive system. A increase of load makes change of diagnostic (energetic) parameters. A increase of load makes decrease of rotary speed and increase of temperature. The physical model is shown on Fig. 4. This presents relationships between mechanical elements (bearings, absorbers, test piece, electrical motor) and physical values that describe accumulation and dissipation energy in mechanical drive system. The main principle of physical model is relationship 1. The other relationships are less taken into consideration but that are going to model in relationship 3. The disruptions value is the load. The load is made by weight hanging under laboratory stand. This makes constant bending stresses on all length of test piece. The electrical motor makes that the bending stresses has changeable progress (sinusoidal progress). At normal work of machines a change of load is value which makes change of behaviour of system (for example as load is increasing as rotary speed is decreasing). The description of laboratory stand was presented in earlier articles of Author [1, 2].

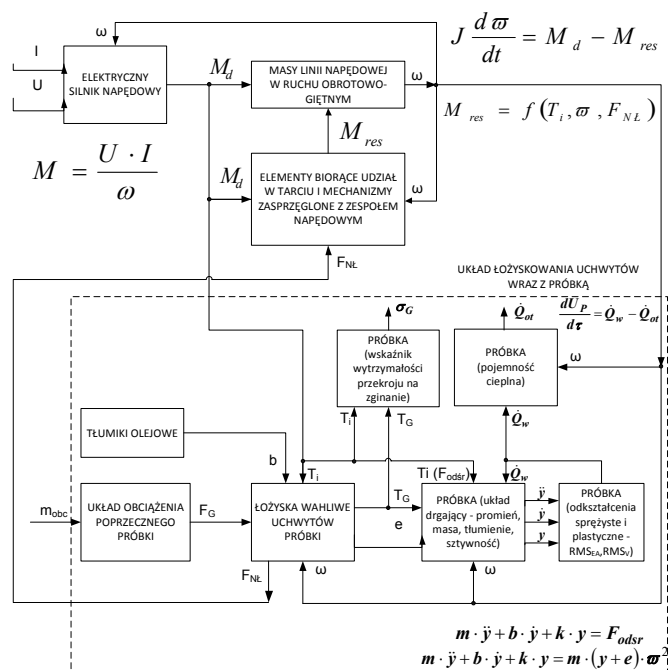


Fig. 4. Physical model of simple mechanical drive system and relationships between physical values [3]

#### 4. The example of test's results

The experiment was made for five test pieces. The conditions of work were constant. The material of test pieces used in this experiment is designed for mechanical drive elements (shafts or gear wheels). The test pieces were made from one round bar that makes smaller spread of results what is obvious for fatigue wear. The parameters of starting work and kind of material are shown in Table 1:

Tab. 1. Parameters of test

number of test piece	outside temperature	loading	starting rotary speed	material
	[K]	[MPa]	[r/min]	C45+C
1,2,3,4,5	296	297 ≈0,35Rm	4000	Rm = 830 MPa (round bar)

The test's results corroborate a simple theory described in point 3. The progress and change of energy and rotary speed is depended on fatigue life. Results of five test pieces are not representative population for one test but are a part of bigger research. This test was planned by use earlier test (15 test pieces) [2]. A experiment and the laboratory stand were improved (the measured values and period of measurement). The behaviour of material under changeable loading is known but in present test are more values describing accumulation and dissipation of energy.

The results of test show a change of behaviour of drive system depending on fatigue life (number of cycles until crack). The results confirm the simple theory described in point 3: as a fatigue life is higher as rotary speed  $R(N)$  is higher and energy from power supply  $P(N)$ , drive moment  $M_d(N)$  and temperature  $T(N)$  are lesser.

The behaviour of mechanical drive system and accumulation of energy can describe follow: energy delivered to drive system is not waste in material but makes higher rotary speed – the energy is not accumulated in material and does not make changes in material (elastic-plastic deformations and increase of temperature). The temperature is a value describing the internal energy. For cyclic loading the internal energy is describing as a range of hysteresis lop between stresses and deformations. This statement was described by Szala and Boroński [6] and Kaleta [7]. In this test the internal energy is value describing fatigue life and reliability of mechanical drive systems.

The behaviour of mechanical system and principle 1 is shown on Fig. 5. The Fig 5 makes some questions:

- why does the rotary speed increase when conditions of work are this same?
- why does the drive moment  $M_d$  decrease?
- why does the resistance moment  $M_{res}$  decrease?

$$J \frac{d\omega}{dt} = M_d - M_{res}$$

Fig. 5. Behaviour of mechanical drive system

The answers might be contained in progress of measured values. These characteristics show change of rotary speed, power supply, drive moment, temperature and current of rotor as function of fatigue life. These all characteristics are parameters describing a change of stream of energy at work of mechanical drive system. The main results of test are shown in table 2:

*Tab. 2. Results of test*

Test piece	number of cycles until crack	temperature at moment of crack	rotary speed (ending, stable)	power supply (ending, stable)	drive moment (ending, stable)
	[-]	[K]	[r/min]	[W]	[Nm]
1	162x10 <sup>3</sup>	313,3	3740	324	0,247
2	140x10 <sup>3</sup>	315,0	3400	345	0,258
3	263x10 <sup>3</sup>	308,8	4030	315	0,243
4	274x10 <sup>3</sup>	309,4	4700	300	0,230
5	218x10 <sup>3</sup>	312,1	3850	325	0,270

The current of rotor has a direct relationship with drive moment. For electrical motor (permanent current) the drive moment has follow relationship [5]:

$$M_d = c \cdot \phi \cdot I \quad (5)$$

where:

c – constant factor depending on construction of electrical motor [-],

$\phi$  – magnetic flux [Wb],

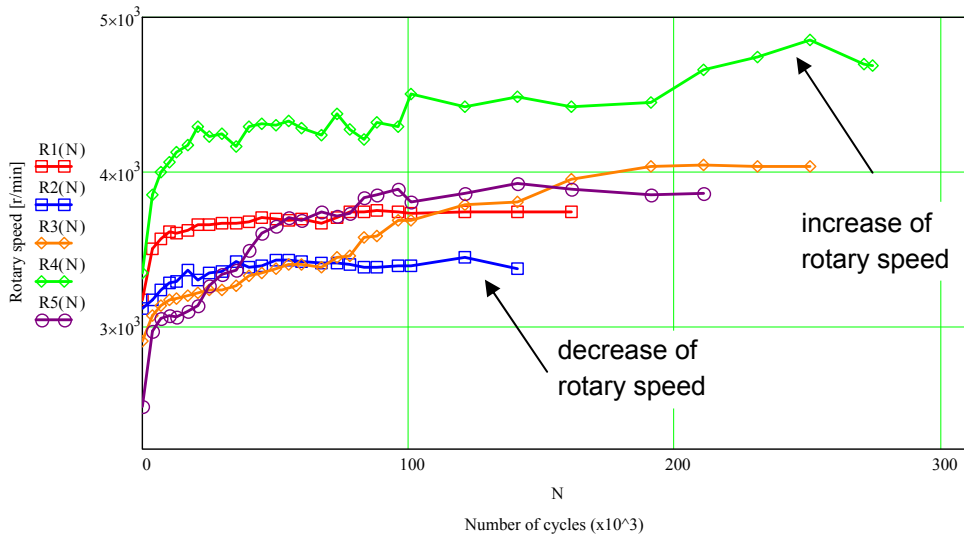
I – current (on electrical terminal of rotor) [A].

This relationship shows that drive moment (indirectly energy delivered to drive system) can be described by using the current of rotor only (magnetic flux is depended on induction current and is permanent). A calculation of drive moment  $M_d$  by using principle 5 is difficult but a trend of characteristic is kept.

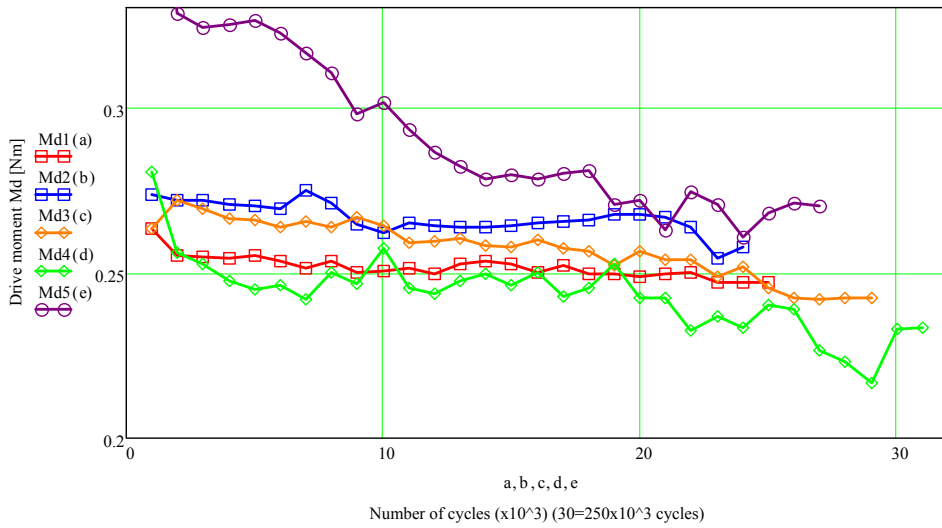
## 5. Conclusion

The results of test confirm theory of accumulation energy in mechanical drive system. The energy which is not lost in material is used to make increase of kinetic energy (increase of rotary speed). This shows that rotary speed is value which can be used to describing fatigue life and reliability of drive systems.

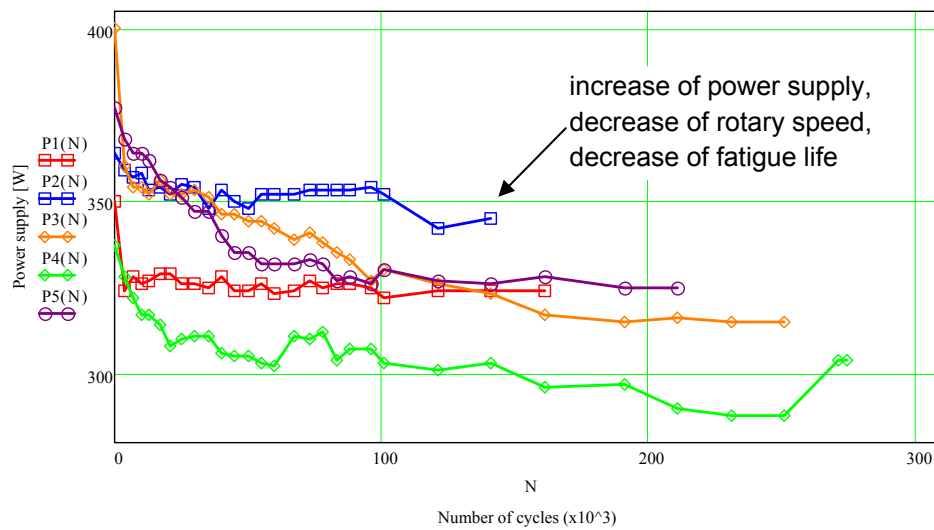
The figure 7 shows interesting progress of power supply. The energetic theory of fatigue wear could be described as a energy delivered to material. The endings of progress of power supply show Wöhler's characteristic.



a)

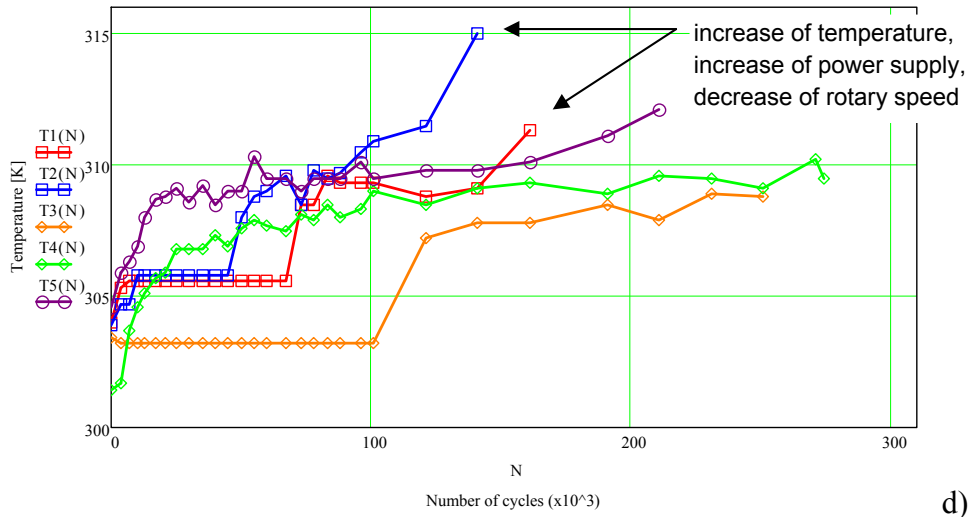


b)

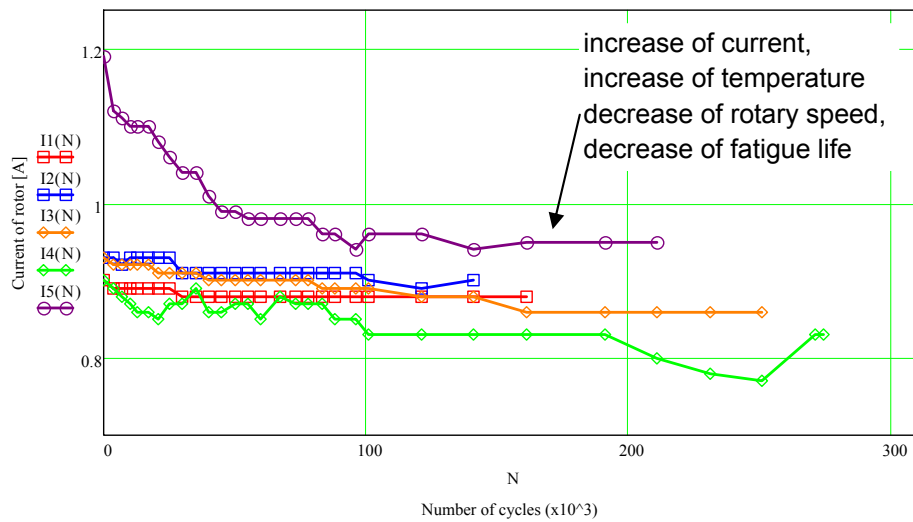


c)





d)



e)

Fig. 6. Change of physical values in depending on fatigue life (number of cycles  $N$ ): a) rotary speed  $R(N)$ , b) drive moment  $M_d$ , c) power supply  $P(N)$ , d) temperature  $T(N)$ , e) current of rotor  $I(N)$

The results show following conclusions:

- the change of rotary speed is depending on fatigue life,
- the temperature is a value describing internal energy accumulated in material and is depending on fatigue life,
- the drive moment and the power supply are value describing energy delivered to mechanical drive system and are depending on fatigue life.

At this test were used other diagnostic parameters: vibrations and Acoustic Emission (AE). These parameters confirm energetic methods using by diagnostics of mechanical elements and fatigue wear.

This test allows to predict a behaviour of diagnostic parameters in the next part of experiment. Thanks to this test is making now a method of research of the future experiment and a statistical analysis.

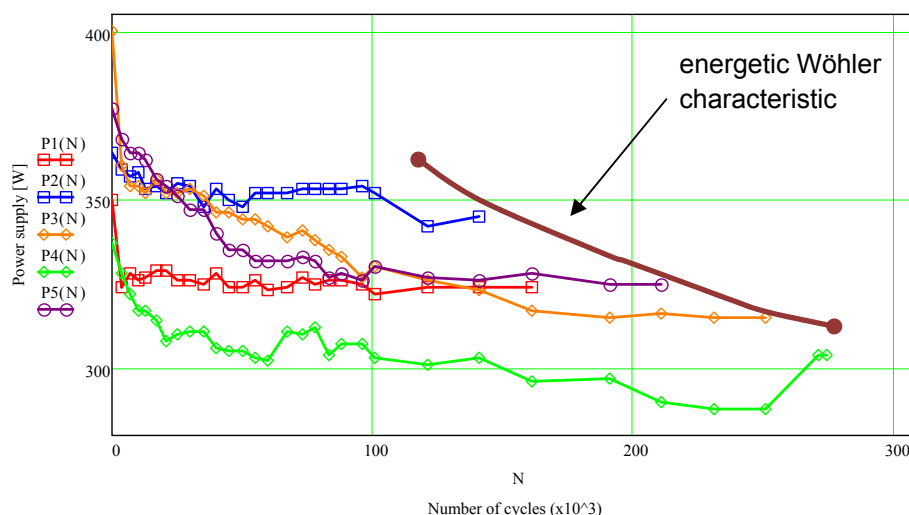


Fig. 7. Wöhler characteristic as function of power supply (delivered energy)

This test allows to predict a behaviour of diagnostic parameters in the next part of experiment. Thanks to this test is making now a method of research of the future experiment and a statistical analysis.

The next tests will be doing for higher and smaller loading that allow to observe the parameters of drive system (especially a rotary speed) depending on delivered power and a influence on fatigue life of material. This all results will be necessary to elaborate a statistical analysis for a whole experiment. This allows elaborate characteristic of behaviour of drive system and make mathematical model of accumulation and dissipation of energy in this system.

## References

- [1] Kaczmarek A.: *Application of laboratory stand for multi-symptoms tests for high cyclic fatigue of constructional material*, *Journal of POLISH CIMAC*, Vol. 7, No. 3, Gdańsk 2012.
- [2] Kaczmarek A.: *Stanowisko laboratoryjne do badania wytrzymałości zmęczeniowej materiałów konstrukcyjnych na podstawie energetycznych procesów reszkowych. Współczesne technologie i konwersja energii*, Wydawnictwo Politechniki Gdańskiej, Gdańsk 2012.
- [3] Korczewski Z.: *The conception of energetic investigations of the multisymptom fatigue of the simple mechanical systems' constructional materials*. *Journal of POLISH CIMAC*, Vol. 7, No. 1, Gdańsk 2012.
- [4] Kocańda S.: *Badanie własności mechanicznych metali*, WNT, Warszawa 1954.
- [5] Plamitzer A.: *Maszyny elektryczne*, WNT, Warszawa 1982.
- [6] Szala J., Boroński D.: *Ocena stanu zmęczenia materiału w diagnostyce maszyn i urządzeń*, Wydawnictwo Naukowe Instytutu Technologii Eksploatacji – PIB, Radom 2008.
- [7] Kaleta J.: *Metody doświadczalne w zmęczeniu materiałów i konstrukcji. Badania podstawowe. Zbiór monografii pod redakcją Józefa Szali*, WU ATR w Bydgoszczy, Bydgoszcz 2000.