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MULTIEQUATION MODELS AS DIAGNOSTIC TOOLS FOR MEASURING THE OPERATIONAL FLUIDITY OF THE CONTAINER HANDLING TERMINAL

Ryszard ZADRĄG[1]
Bohdan PAC[2]

[1]*Gdank University of Technology,*
[2]*WSB University in Gdansk*
e-mail: ryszadra@pg.gda.pl, b.pac@wsb.gda.pl

Abstract

The article presents the continuation of the authors' research on the problem of the operational fluidity of cargo handling sea terminals, based on the example of the DCT. The article presents a solution based on the multi-equation models and applied to analyze the operational fluidity of terminals in the transshipment and feeder relation. To provide a solution to the problem of the functioning of sea-land supply chains at the terminal, the GRETL software has been applied.

Keywords*: multi-equation models, simulation, container terminal*

1. Introduction

The question of providing the operational fluidity to sea-land supply chains in operations of container terminals is connected with the coordination of the processes which take place there, the capacity of cargo handling equipment and the capacity of the storage yards. The article comes as the continuation of the authors' research on modelling sea-land supply chains with the use of simulation methods. Maintaining the operational fluidity depends on the storage yards and stockpiling areas for the containers. They play the role of the stability absorbers in a supply chain because the character of the turnover at the terminal negatively affects the regularity of the process (considering the deadweight of large vessels in particular), and it generates its periodical fluctuations. Therefore, the monitoring and assessment of this process is necessary, and it is realized in many ways. Most frequently, it means defining the occupancy level of a storage yard. The occupancy indicator, which describes this particular feature, is of typical static character, and it does not consider the dynamics of the changes which are connected with loading and unloading of vessels characterized with significant deadweight (such as Maersk vessels). The changes of this indicator present some noticeable deviations, however they do not come as the measure of the process.

As such, applying the static models, where the handling operation of the terminal is recorded on a daily basis, resulted in a significant irregularity of processes related to large vessel handling. It negatively affected the correlation of indicators concerning the occupancy of the quay and storage yard. Considering the abovementioned facts, the real hourly-based form of recording handling operations was assumed for further research. This form guarantees not only the operational fluidity of the terminal, but it also eliminates significant deviations concerning the occupancy of its storage yards.

The article aims at the construction of a model concerning the functioning of a supply chain in a sea container terminal, in the following relations: export-import, import-export. The assumed level of transshipment transportation must be also taken into consideration with the abovementioned hourly-based form of container handling records. The model could also pertain to a situation in which the dynamics of material fluidity must be considered when the delays of the means of road transport supporting the container supply chain appear, and there might be some cargo pile-up. In such cases, it is necessary to provide tools which will allow us to monitor the processes and to manage them in a better way by making decisions which involve less risk. The authors decided to solve the following research problem: how to formulate a task which refers to process modelling in such a way that could satisfactorily adjust the model to the real processes. Considering its simplicity, the multi-equation model method has been applied. The working hypothesis to be proved was stated as follows: the application of the multi-equation model method to the analysis of the operational fluidity of sea container terminals allows us to provide satisfying forecasts of the occupancy levels of storage yards, depending on the occupancy levels of the quays where vessel handling operations take place. To solve this research problem and to prove the working hypothesis, the following research tasks were carried out:

- The multi-equation model of dynamic processes was identified;
- The model of storage yard occupancy of a sea container terminal was constructed;
- The operational fluidity of a terminal in the sea-land supply chain was analyzed with the use of the multi-equation methods.

2. The identification of the multi-equation model of dynamic processes

A sea-land supply chain at a container terminal – as a whole and as its particular partial processes, that is namely: vessel loading-unloading, cargo feeder transport realized by the road and railway means of transport from the terminal – is realized in time, and it has a dynamic character. Applying the multi-equation model allows us to describe it with a system of linear difference equations [1]. Since the measurement of the supply chain and its processes is of discrete character (daily or hourly flows are measured), a discrete signal in time (a time series) is a function, the domain of which is a set of integers. So, a discrete signal in time is a sequence of numbers. The sequences of this type will be presented further in the function notation $x[k]$.

The $x[k]$ signal which is discrete in time is often determined by sampling of the $x(t)$ signal which is continuous in time. If sampling is of the even periodic character, then $x[k] = x(kT)$. The T constant is referred to as the sampling interval. The course of a dynamic process in time depends not only on the value of perturbations at the present moment but also on the value of such perturbations in the past. Therefore, a dynamic process/system has memory in which the results of past impacts are collected.

The relations between the input signals $x_1[k], x_2[k], \cdots, x_n[k]$ and the output signals $y_1[k], y_2[k], \cdots, y_m[k]$, $k = 0,1,2,\ldots$ can be described with the use of a system of the linear difference equations.

$$\begin{cases} y_1[k+1] = a_{11}y_1[k] + a_{12}y_2[k] + \cdots + a_{1m}\, y_m[k] \ + b_{11}x_1[k] + b_{12}x_2[k] + \cdots + b_{1n}x_n[k] + \xi_1 \\ y_2[k+1] = a_{21}y_1[k] + a_{22}y_2[k] + \cdots + a_{2m}\, y_m[k] \ + b_{21}x_1[k] + b_{22}x_2[k] + \cdots + b_{2n}x_n[k] + \xi_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ y_m[k+1] = a_{m1}y_1[k] + a_{m2}y_2[k] + \cdots + a_{mm}y_m[k] + b_{m1}x_1[k] + b_{m2}x_2[k] + \cdots + b_{mn}x_n[k] + \xi_m \end{cases} \qquad (1)$$

where:

$y_i[k], i = 1,2,\ldots,m$ - the values of the output signals at the k moment,

$x_j[k]$, $j = 1,2,\ldots,n$ - the values of the input signals at the k moment,

a_{ij} – is the coefficient which appears in the ith equation at the jth output signal, $i, j = 1,2,\cdots,m$

b_{ij} - is the coefficient which appears in the ith equation at the jth input signal, $i = 1,2,\cdots,m, \quad j = 0,1,\cdots,n$

ξ_i - is the non-observable random factor in the ith equation;

The system of equations (1) can be formulated as a matrix.

$$\mathbf{y}[k+1] = \mathbf{A}\mathbf{y}[\mathbf{k}] + \mathbf{B}\,\mathbf{x}[\mathbf{k}] + \boldsymbol{\xi} \qquad (2)$$

where:

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ b_{m1} & b_{m2} & \cdots & b_{mm} \end{bmatrix}, \qquad \mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m1} & \cdots & a_{mn} \end{bmatrix},$$

$$\mathbf{y}[k] = \begin{bmatrix} y_1[k] \\ y_2[k] \\ \cdots \\ y_m[k] \end{bmatrix}, \quad \mathbf{y}[k+1] = \begin{bmatrix} y_1[k+1] \\ y_2[k+1] \\ \cdots \\ y_m[k+1] \end{bmatrix}, \quad \mathbf{x}[\mathbf{k}] = \begin{bmatrix} x_1[k] \\ x_2[k] \\ \cdots \\ x_n[k] \end{bmatrix}, \quad \boldsymbol{\xi} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \cdots \\ \xi_m \end{bmatrix} \quad .$$

with further denotation:

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1m} & c_{1m+1} & c_{1m+2} & \cdots & c_{1m+n} \\ c_{21} & c_{22} & \cdots & c_{2m} & c_{2m+1} & c_{2m+2} & \cdots & c_{2m+n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m1} & \cdots & c_{mn} & c_{mm} & c_{mm+1} & \cdots & \pi_{mm+n} \end{bmatrix}, \quad \boldsymbol{\eta} = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_m \end{bmatrix},$$

$$c_{i\nu}[k] = b_{i\nu}[k], \text{ dla } \nu = 1,2,\ldots,m, \qquad c_{i\nu}[k] = a_{i\nu}[k], \quad \text{dla } \nu = m+1, m+2,\ldots,m+n, \qquad (3)$$

$$\mathbf{z}[k]=\begin{bmatrix} z_1[k] \\ \vdots \\ z_m[k] \\ z_{m+1}[k] \\ \vdots \\ z_{m+n}[k] \end{bmatrix} \quad ,$$

$$z_\nu[k]=y_\nu[k],\ \text{dla}\ \nu=1,2,\ldots,m, \qquad z_\nu[k]=x_{\nu-m}[k], \quad \text{dla}\ \nu=m+1,m+2,\ldots,m+n\,,$$

The equation system (1) and the functionals (3) can be presented in their reduced form as

$$\mathbf{y}[k+1]=\mathbf{C}\,\mathbf{z}[k]+\boldsymbol{\eta} \quad . \tag{4}$$

The identification of the equation system (1) will be understood as a question concerning the selection of coefficients, with the consideration of the values determined by the measurements performed on a real object. These are the values of:

$\tilde{x}_1[k],\tilde{x}_2[k],\cdots,\tilde{x}_n[k], \quad k=0,1,2,\cdots,N$ input signals $x_1,x_2,\ldots,x_n$

and the values of

$\tilde{y}_1[k],\tilde{y}_2[k],\cdots,\tilde{y}_m[k], \quad k=0,1,2,\cdots,N+1$ output signals $y_1,y_2,\cdots,y_m$,

at the moments $t_k=kT$.

The measurement values can be presented as the following matrix (5)

$$\tilde{\mathbf{X}}=[e_1\,|\,e_2\,|\ldots|\,e_m\,|\,e_{m+1}\,|\,e_{m+2}\,|\ldots|\,e_{m+n}]=\begin{bmatrix} \tilde{y}_1[0] & \tilde{y}_2[0] & \cdots & \tilde{y}_m[0] & & \tilde{x}_1[0] & \cdots & \tilde{x}_n[0] \\ \tilde{y}_1[1] & \tilde{y}_2[1] & \cdots & \tilde{y}_m[1] & & \tilde{x}_1[1] & \cdots & \tilde{x}_n[1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{y}_1[N] & \tilde{y}_2[N] & \cdots & \tilde{y}_m[N] & & \tilde{x}_1[N] & \cdots & \tilde{x}_n[N] \end{bmatrix}$$

The coefficients $b_{i1},\ldots,b_{i\,m},a_{im+1}\ldots,a_{im+n},\ i=1,2,\ldots,m$ of the equation system presented above will be selected so that the functionals (6)

$$J_i(b_{i1},\ldots b_{im,}a_{im+1},\ldots,a_{i(m+n)})=\sqrt{\sum_{k=1}^{N}(a_{i1}\tilde{y}_1[k]+\ldots+a_{im}\tilde{y}_m[k]+b_{im+1}\tilde{x}_1[k]\ldots+b_{i(m+n)}\tilde{x}_n[k]-\tilde{y}_i[k+1])^2}\,,$$

can reach the minimum for $i=1,2,\ldots m$

Considering the denotations (2) assumed above, the functionals (5) can be stated in the following form (6):

$$J_i(c_{i1},c_{i2},\ldots,c_{i(m+n)})=\sqrt{\sum_{k=0}^{N}(c_{i1}\tilde{z}_1[k]+\ldots+c_{im}y\tilde{z}_m[k]+c_{im+1}\tilde{z}_{m+1}[k]\ldots+c_{i(m+n)}\tilde{z}_{m+n}[k]-\tilde{y}_i[k+1])^2}\,,$$

$i=1,2,\ldots m$,

$\tilde{z}_\nu[k]=\tilde{y}_\nu[k]$, for $\nu=1,2,\ldots,m,$ $\qquad \tilde{z}_\nu[k]=\tilde{x}_{\nu-m}[k],$ for $\nu=m+1,m+2,\ldots,m+n$.

Actually, the matrix (4) comes as a system of vectors which are linearly independent in the Hilbert space

$$\mathbf{e}_1 = \begin{bmatrix} \tilde{y}_1[0] \\ \tilde{y}_1[1] \\ \vdots \\ \tilde{y}_1[N] \end{bmatrix}, \mathbf{e}_2 = \begin{bmatrix} \tilde{y}_2[0] \\ \tilde{y}_2[1] \\ \vdots \\ \tilde{y}_2[N] \end{bmatrix}, \cdots, \mathbf{e}_m = \begin{bmatrix} \tilde{y}_m[0] \\ \tilde{y}_m[1] \\ \vdots \\ \tilde{y}_m[N] \end{bmatrix}, \mathbf{e}_{m+1} = \begin{bmatrix} \tilde{x}_1[0] \\ \tilde{x}_1[1]_1 \\ \vdots \\ \tilde{x}_1[N] \end{bmatrix}, \cdots, \mathbf{e}_{m+n} = \begin{bmatrix} \tilde{x}_n[0] \\ \tilde{x}_n[1] \\ \vdots \\ \tilde{x}_n[N] \end{bmatrix}. \tag{7}$$

The question of selecting the best model from the group of equations (7), in terms of the minimization of the quality identifying indicators, was solved with the use of the orthogonal projection theorem [6]. Considering the extensive character of the question, the transformations leading to the equation in its matrix form were omitted.

$$\mathbf{G}(\mathbf{C}_i^0)^T = \mathbf{W}_i\,, \tag{8}$$

where:

$$G = \begin{bmatrix} \sum_{k=0}^{N} \tilde{y}_1[k]\tilde{y}_1[k] & \cdots & \sum_{k=0}^{N} \tilde{y}_m[k]\tilde{y}_1[k] & \sum_{k=0}^{N} \tilde{x}_1[k]\tilde{y}_1[k] & \cdots & \sum_{k=0}^{N} \tilde{x}_n[k]\tilde{y}_1[k] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sum_{k=0}^{N} \tilde{y}_1[k]\tilde{y}_m[k] & \cdots & \sum_{k=0}^{N} \tilde{y}_m[k]\tilde{y}_m[k] & \sum_{k=0}^{N} \tilde{x}_1[k]\tilde{y}_m[k] & \cdots & \sum_{k=0}^{N} \tilde{x}_n[k]\tilde{y}_m[k] \\ \sum_{k=0}^{N} \tilde{y}_1[k]\tilde{x}_1[k] & \cdots & \sum_{k=0}^{N} \tilde{y}_m[k]\tilde{x}_1[k] & \sum_{k=0}^{N} \tilde{x}_1[k]\tilde{x}_1[k] & \cdots & \sum_{k=0}^{N} \tilde{x}_1[k]\tilde{x}_1[k] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sum_{k=0}^{N} \tilde{y}_1[k]\tilde{x}_n[k] & \cdots & \sum_{k=0}^{N} \tilde{y}_m[k]\tilde{x}_n[k] & \sum_{k=0}^{N} \tilde{x}_1[k]\tilde{x}_n[k] & \cdots & \sum_{k=0}^{N} \tilde{x}_1[k]\tilde{x}_n[k] \end{bmatrix} = \tilde{\mathbf{X}}^T\tilde{\mathbf{X}}$$

$$(\mathbf{C}_i^0)^T = \begin{bmatrix} c_{i1}^0 \\ \vdots \\ c_{im}^0 \\ c_{im+1}^0 \\ \vdots \\ c_{im+n}^0 \end{bmatrix}, \qquad \mathbf{W}_i = \begin{bmatrix} \sum_{k=0}^{N} \tilde{y}_i[k+1]\tilde{y}_1[k] \\ \vdots \\ \sum_{k=0}^{N} \tilde{y}_i[k+1]\tilde{y}_n[k] \\ \sum_{k=0}^{N} \tilde{y}_i[k+1]\tilde{x}_1[k] \\ \vdots \\ \sum_{k=0}^{N} \tilde{y}_i[k+1]\tilde{x}_n[k] \end{bmatrix} = \tilde{\mathbf{X}}^T\tilde{\mathbf{y}}_i[k+1]$$

So, the matrix equation (8) can be presented in the following form:

$$(\tilde{\mathbf{X}}^T\tilde{\mathbf{X}})(\mathbf{C}_i^o)^T = \tilde{\mathbf{X}}^T\tilde{\mathbf{y}}_i[k+1] \tag{9}$$

Hence:

$$(\mathbf{C}_i^o)^T = (\tilde{\mathbf{X}}^T\tilde{\mathbf{X}})^{-1}\tilde{\mathbf{X}}^T\tilde{\mathbf{y}}_i[k+1], \quad i = 1,2,\cdots,m \quad . \tag{10}$$

Therefore, the optimal coefficients

$$c_{ij}^0, \quad i = 1,2,\ldots,m, \quad j = 1,\ldots,m,m+1,\ldots,m+n\,.$$

of the reduced form of the model (6) can be determined with the use of the equation:

$$(\mathbf{C}^0)^T = (\tilde{\mathbf{X}}^T\tilde{\mathbf{X}})^{-1}\tilde{\mathbf{X}}^T\tilde{\mathbf{Y}}[k+1] \tag{11}$$

$\tilde{\mathbf{X}}_{(N+1)\times(m+n)}$ - the matrix of the measurement values of the signals $y_1, y_2, \ldots, y_n, x_1, x_2, \ldots, x_m$,

$\tilde{\mathbf{X}}^T_{(m+n)\times(N+1)}$ - the matrix transposed to the matrix of the measurement values of the input signals,

$(\tilde{\mathbf{X}}^T\tilde{\mathbf{X}})^{-1}_{(N+1)\times(N+1)}$ - the matrix invertible to the Gram's $\mathbf{G}_{(N+1)\times(N+1)}$,matrix,

$\tilde{\mathbf{Y}}_{(N+1)\times m}[k+1]$-the matrix of the measurement values of the output signals $y_1, y_2, \ldots, y_m$,

$\tilde{\mathbf{Y}}^T_{m\times(N+1)}$ - the matrix transposed to the matrix of the measurement values of the output signals,

N – the number of measurements, n – the number of the input signals, m – the number of the output signals.

3. The model of the storage yard occupancy at a sea container terminal

The research was carried out at the DCT Gdańsk during the subsequent 16 days. The experimental material was collected in the result of direct observations, and a model of the cargo turnover at the terminal was constructed. Then the model was applied to the simulation of the vessel traffic with the use of the Monte Carlo method [4]. The assumed methodology allowed the authors to obtain significant convergence of the simulation results and the empirical material. The length of vessels was assumed as an independent variable of the model. It was also an argument which decided about the number of the incoming vessels and the time in which they were handled: loaded and unloaded. The reason for that was the fact that the analyzed cargo handling capacity of the container terminal mainly depends on the exploitation intensity of the cargo handling infrastructure. The capacity of this infrastructure results from the length of the quays and the number of cargo handling berths. Accepting the length of a vessel as an independent variable leads to analyzing the occupancy level of the storage yards of the terminal, depending on the size of daily container cargo turnovers, in terms of the import-export relation, during the forecast time. They determine the demand for the road means of transport [5].

Despite this, seeing the daily records of the container turnover, the sudden increase in their number will still take place; only the intensity of this operation will decrease. In fact, this increase in the volume of containers doesn't take place, as the loading and unloading is carried out fluidly. As such, it seems that the best solution in terms of modelling the dynamics of changes in storage yard occupancy will be an hourly-based operation record. In this way the use of the term "a stream of containers" will be well-grounded. The container streams will determine the intensity of the container turnover at the storage yard. It is therefore necessary to consider the fact that there are three streams of containers in the daily turnover, namely:

- the stream of containers which participate only in sea cargo handling (they do not leave the storage yard) – that is: they participate in transshipment – it was assumed that they come as 60% of the total number of containers;
- The stream of containers which are handled by the means of road transport – they come as 65% of the turnover of containers which are transported to and from the terminal by road transport;
- The stream of containers which are handled by the means of railway transport – they come as 35% of the turnover of containers which are transported to and from the terminal by railway transport.

Additionally, it is possible to consider some delays in the land turnover. They can result in some pile-up in the flow of the container streams at the storage yard. It is particularly significant when a big percentage of the storage yard occupancy or optimization of storage areas is concerned. For the modelling the maximal values of the pile-up coefficients were assumed: for the road transport the pile-up coefficient was – 1.5 and for the railway transport it was – 1.3. In the case of the stream of the containers x2 outgoing the storage yard, a minus sign (-) was applied to provide clear understanding.

Having all the above factors specified, a new model was presented (Fig. 1).

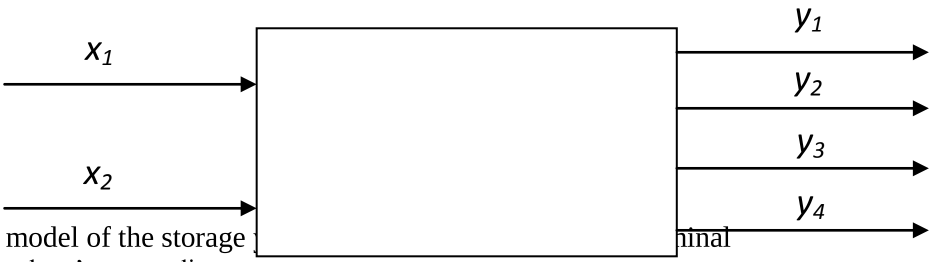

Fig. 1. The model of the storage [illegible] ninal
Source: the authors' own studies

In the above model, the following streams of containers have been identified:

- x_1- the number of containers incoming the storage yard [pcs];
- x_2- the number of containers outgoing the storage yard [pcs];
- y_1 - the occupancy of the storage yard at the initial occupancy at the level of 50% [pcs].
- y_2 - the container turnover in the maritime transit [pcs];
- y_3 - the container turnover in the road transport [pcs];
- y_4 - the container turnover in the railway transport [pcs].

4. The analysis of the operational fluidity of a cargo handling terminal.

The model presented in the previous section of the article was described by the system of equations which is analogical to (1):

$$
\begin{aligned}
y_1[k+1] &= b_0 + b_{11}x_1[k] + b_{12}x_2[k] + a_{13}y_3[k] + a_{14}y_4[k] + \xi \\
y_2[k+1] &= b_0 + b_{21}x_1[k] + b_{22}x_2[k] + a_{23}y_3[k] + a_{24}y_4[k] + \xi \\
y_3[k+1] &= b_0 + b_{31}x_1[k] + b_{32}x_2[k] + a_{31}y_1[k] + a_{34}y_4[k] + \xi \\
y_4[k+1] &= b_0 + b_{41}x_1[k] + b_{42}x_2[k] + a_{41}y_1[k] + a_{43}y_3[k] + \xi
\end{aligned} \tag{14}
$$

Also, as in the previously presented model, the estimation of the equation coefficients of the particular output variables was provided with the use of the least squares method. The parameters of the estimation are presented in Table 1, 2, 3, and 4.

Tab. 1. The estimation of the dependent y_1 variable with the use of the least squares method

	Coefficient		*Standard error*	*t-Student*	*p significance*
const	b_0	9654,38	4,25005	2271,5900	<0,00001
x1_1	b_{11}	0,570112	0,0210971	27,0232	<0,00001
x2_1	b_{12}	0,329675	0,0235174	14,0183	<0,00001
y3_1	a_{13}	0,879112	0,0767075	11,4606	<0,00001
y4_1	a_{14}	0,971462	0,155255	6,2572	<0,00001

Tab. 2. The estimation of the dependent y_2 variable with the use of the least squares method

		Coefficient	Standard error	t-Student	p significance
const	b_0	9653,64	3,94462	2447,2936	<0,00001
x1_1	b_{21}	0,555468	0,0195809	28,3678	<0,00001
x2_1	b_{22}	0,351168	0,0218273	16,0884	<0,00001
y3_1	a_{23}	0,0867545	0,0711947	1,2186	0,22376
y4_1	a_{24}	-0,264657	0,144097	-1,8367	0,06703

Tab. 3. The estimation of the dependent y_3 variable with the use of the least squares method

		Coefficient	Standard error	t-Student	p significance
const	b_0	-7811,23	211,746	-36,8896	<0,00001
x1_1	b_{31}	-0,492549	0,0152679	-32,2605	<0,00001
x2_1	b_{32}	-0,326617	0,00973906	-33,5368	<0,00001
y1_1	a_{31}	0,809479	0,0219116	36,9429	<0,00001
y4_1	a_{34}	-0,447171	0,0589745	-7,5824	<0,00001

Tab. 4. The estimation of the dependent y_4 variable with the use of the least squares method

		Coefficient	Standard error	t-Student	p significance
const	b_0	-8431,78	240,015	-35,1303	<0,00001
x1_1	b_{41}	-0,502781	0,0165117	-30,4499	<0,00001
x2_1	b_{42}	-0,368192	0,00917027	-40,1506	<0,00001
y1_1	a_{41}	0,873811	0,0248615	35,1472	<0,00001
y3_1	a_{43}	-0,890932	0,0330604	-26,9486	<0,00001

The final system of equation (14) is the following:

$$\mathrm{y}_1[k+1] = 9654{,}38 + 0{,}570112x_1 + 0{,}329675x_2 + 0{,}879112y_3 + 0{,}971462y_4$$

$$\mathrm{y}_2[k+1] = 9653{,}64 + 0{,}555468x_1 + 0{,}351168x_2 + 0{,}0867545y_3 - 0{,}264657y_4$$

$$\mathrm{y}_3[k+1] = -7811{,}23 - 0{,}492549x_1 - 0{,}326612x_2 + 0{,}809479y_1 - 0{,}447171y_4$$

$$\mathrm{y}_4[k+1] = -8431{,}78 - 0{,}502781x_1 - 0{,}368192x_2 + 0{,}873811y_1 - 0{,}890932y_3$$

The convergence of the model and the empirical results is proved by the measure of fitting, which was assumed in the previous analysis based on the value of the coefficient of the relative variance (13). Its values, especially for the y_1 i y_2 variables are respectively 0.4 and 0.3% which clearly indicates the accurate fitting. The graphic form of the fitting for the y_1 variable, that is the occupancy level of the storage yard at the initial 50% of the occupancy level of the yard, and the y_2, that is the turnover of containers in transshipment, is presented in Fig. 2 and Fig. 3.

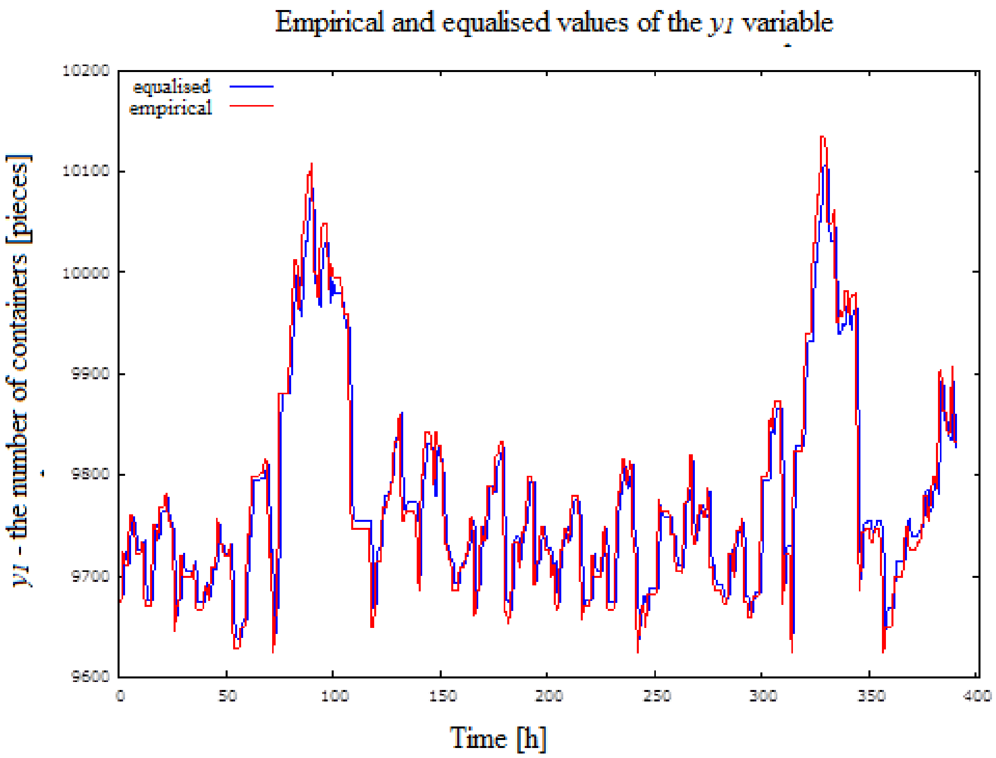

Fig. 2. Graphical presentation of the fitting of the y_1 variable showing the storage yard occupancy at the initial 50% of the storage yard occupancy

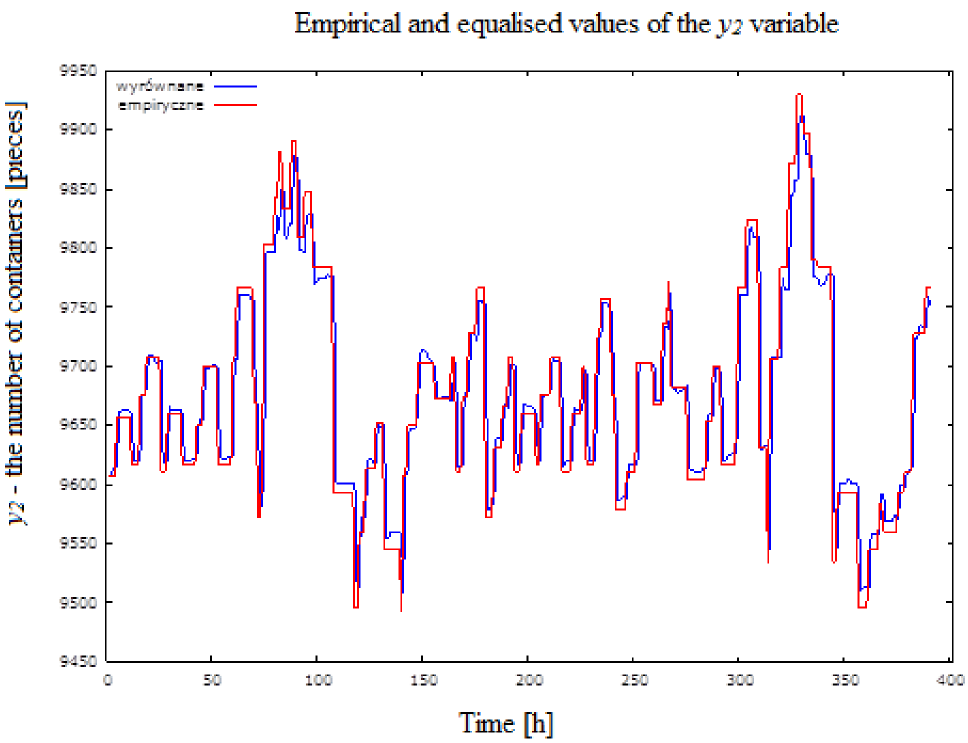

Fig. 3. Graphical presentation of the fitting of the y_2 variable showing the turnover of containers in transshipment

In the case of the y_3 and y_4 variables however, the values of the relative variance coefficient are much higher, at the level of 18%, but the values of the average standard deviation as well as the value of the standard error of the residuals are low. The average standard deviation for the y_3 and y_4 variables is respectively 40.88 and 22.04; the standard error of the residuals presents the values of 11,71 and 6,57. The coefficient of the relative variance presents high values because of the smaller streams (smaller turnover) of containers in the road and railway transport in comparison to the transshipment. The maximum hourly-based turnover of containers coming from the road and railway transport is respectively 152 and 82 containers. Therefore, considering the y_3 and y_4 variables and the characteristics of the populations mentioned above, the best measure of the model fitting may be the charts of regression residuals presented in Fig. 4 and 5.

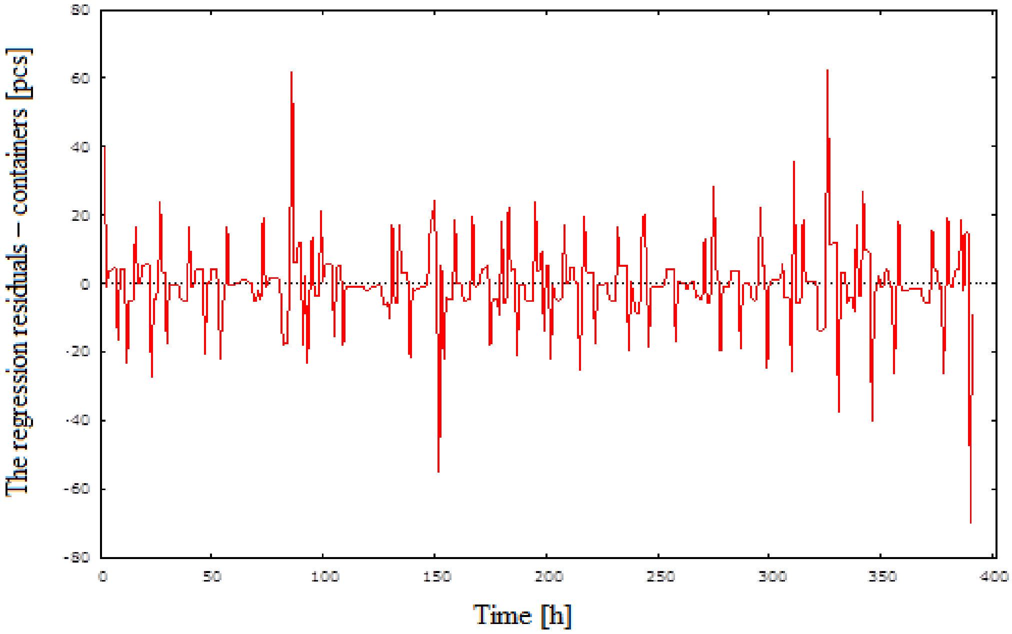

Fig. 4. The distribution of the regression residuals in the analyzed time period for the y_3 variable – the container turnover in the road transportation

The analysis of Figures 4 and 5 indicates that the highest regression residuals appear in the time periods when the largest vessel are loaded and unloaded. Despite this fact, the maximal forecast error in the case of the container turnover in the road transport is 63 containers (Fig. 4), whereas in the case of the container turnover in the railway transport, this number is 38 containers (Fig. 5). The values of the maximal forecast error come as a significant measure of the model fitting also when cargo handling capacity of the terminal is analyzed. In the analyzed case, the capacity of the particular loading and unloading facilities was never exceeded, with the consideration of the overestimation caused by the maximal forecast error. In this way, the suggested models become useful in any operational conditions of the container terminal.

As it has been already mentioned, the multi-equation models have another significant advantage. With the assumed error, they allow us to forecast changes in the value of the particular variables beyond the observation time period. The ability to predict is particularly useful in optimization of the operational capacity of the container terminal.

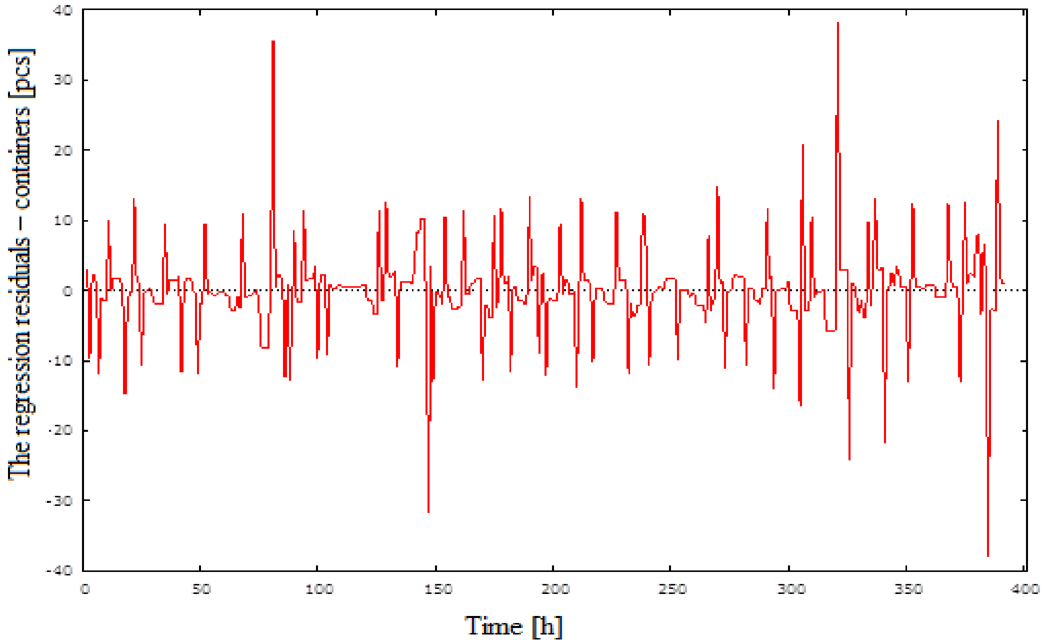

Fig. 5. The distribution of the regression residuals in the analyzed time period for the y_4 variable – the container turnover in the railway transportation

In the analyzed case, the k-period moving forecast, where k=4 (hours) was applied. The forecast period was assumed with the consideration of a possibility that there might appear decision-making situations (Fig. 6 and 7).

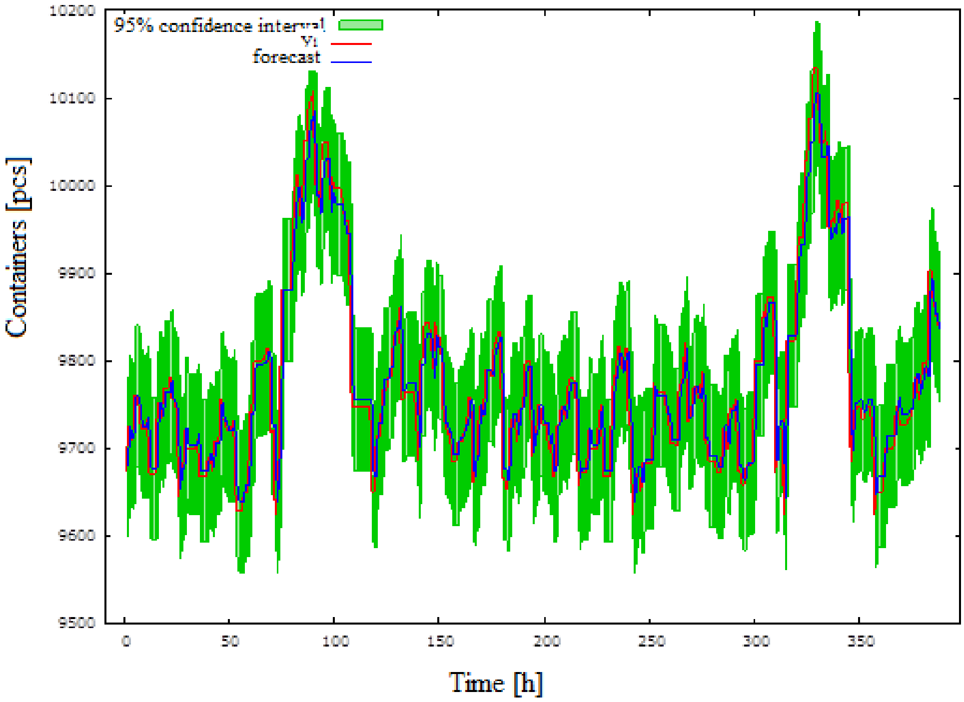

Fig. 6. The forecast of the variance pertaining to the value of the y_1 variable – the storage yard occupancy level at the initial 50% storage yard occupancy

For the y_1 variable, that is namely: the storage yard occupancy at the initial 50% of the storage yard occupancy, and the y_2 variable, that is: the container turnover in the maritime transit, the average forecast error was respectively: -1.62% and -1.48%. The average absolute error indicated similar low values for the y_1 variable: 26.277 and for the y_2 variable: 22,084. The similar values were also obtained for the y_3 i y_4 variables. Here the average forecast error was respectively - 0.40% and 0.27%, whereas the average absolute error indicated the following values: 7.505 and 3.920.

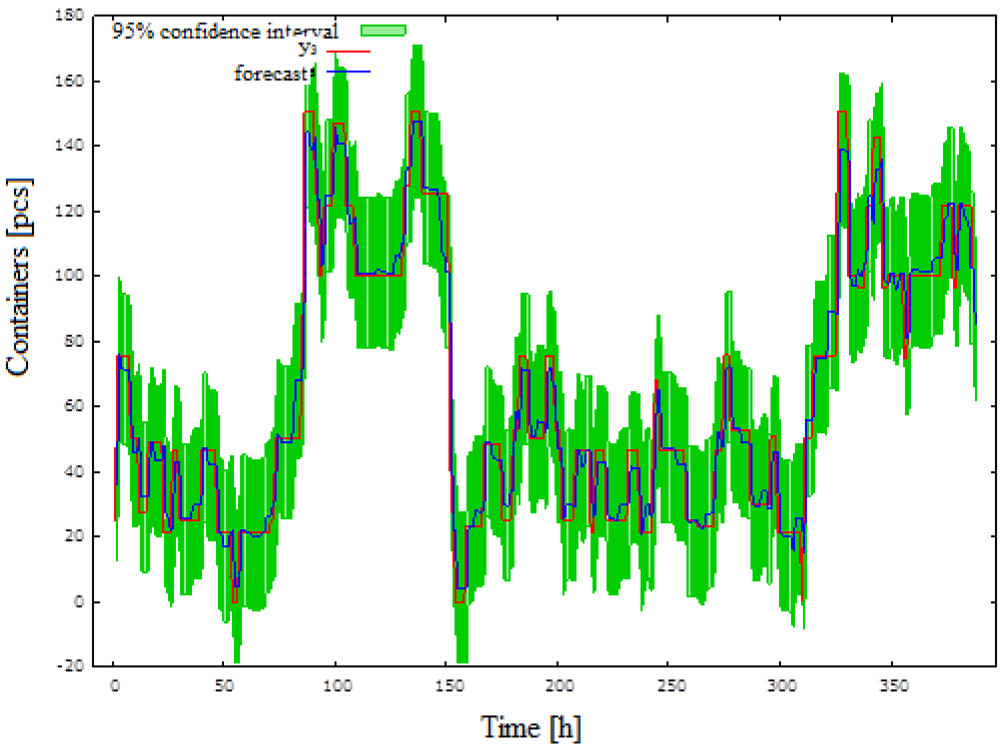

Fig. 7. The forecast of the variance pertaining to the value of the y_3 variable – the container turnover in the road transport

An example of the forecast provided in accordance with the assumed models is presented in Fig. 6 and 7 where, apart from the forecast values, the 95% confidence interval is marked in green.

5. Conclusions

The research indicates that applying the multi-equation models for the analysis of the operational fluidity of a container terminal is well justified because the model fitting to the empirical observations presents a satisfying level. It is caused by the results of the previous research in which the following relations were analyzed: the relation between the length of a vessel and the unloading time of a vessel; the length of a vessel and the number of the carried containers; the hourly-based operation record of the terminal in terms of loading and unloading containers. The hourly-based operation record makes the occupancy level of the storage yards real. This level indicates the values which are considerably below the maximal capacity of the storage yards, with the assumed initial occupancy level of 50%.

The hourly-based record of the terminal operation provides a safe reserve of unoccupied storage space in case of any delays in land transport and any subsequent pile-ups in cargo handling operations, with a real assumption that there will not be any congestion in maritime transport.

The satisfying results of the model fitting to the empirical observations come as a premise for applying them in planning storage surface for handled cargo streams. This is particularly important in the urban areas, considering the financial burden of the business entities which operate in commercial sea ports.

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