STOCHASTIC CONTRIBUTIONS ON THE PRESSURE IN SLIDE BEARING GAPS AFTER IMPULSE

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Abstract

This paper concerns the derivation of the stochastic modified Reynolds equations describing the hydrodynamic pressure for non-Newtonian, viscoelastic, lubricant inside the curvilinear orthogonal (spherical, conical, cylindrical, parabolic) slide bearing. Non- isothermal, unsteady and random flow conditions and thermal deformations of the bearing and its bearing sleeve are taken into account. This problem finds application in ship power plants, electric locomotive designing, and precision engineering. In particular case are determined pressure and capacity distributions in spherical bearing.

Keywords: *time depended stochastic hydrodynamic theory, bearing surfaces with curvilinear sections*

1. Introduction and basic equations

This paper presents the general analysis of the influence of unsteady non isothermal flow of, visco-elastic oil in magnetic field on the pressure, within deformed curvilinear bearing gaps between two rotational surfaces in random conditions (see Fig.1).
The flow analysis of the viscoelastic lubricant, will be performed by means of the Helmholtz Equation and the equations of continuity, motion and energy Ref.[1],[2],[3]:

$$\nabla^2 \mathbf{H} = \mu\mu_e\, \partial^2\mathbf{H}/\partial t^2, \tag{1}$$

$$div(\rho\mathbf{v})=0, \tag{2}$$

$$Div\mathbf{S}+\mu(\mathbf{N}\,\nabla)\mathbf{H}=\rho d\mathbf{v}/dt, \tag{3}$$

$$div(\kappa gradT)+div(\mathbf{vS})-\mathbf{v}Div\mathbf{S}-\mu T\Xi d\mathbf{H}/dt=\rho dc_v T/dt, \tag{4}$$

where:

v - lubricant velocity vector,
T - temperature of lubricant,
N - magnetization vector of lubricant,
H - magnetic intensity vector,
Ξ - first derivative of the magnetization vector with respect to the temperature,
c_v - specific heat,
μ,μ_e - magnetic and electric permeability coefficient of lubricant,
S - stress tensor in the lubricant,

∇ - Del vector,
κ - thermal conductivity coefficient of the lubricant.

The magnetic susceptibility coefficient of the lubricant has constant scalar value. The relationship between the stress tensor $\mathbf{S} \equiv ||\tau_{ij}||$ and the strain Rivlin-Ericksen tensor $\mathbf{A_1}$, $\mathbf{A_2}$ in the lubricant is as follows [3]:

$$\boldsymbol{S} = -p\boldsymbol{I} + \eta\, \boldsymbol{A_1} + \alpha \boldsymbol{A_1}\boldsymbol{A_1} + \beta \boldsymbol{A_2}, \tag{5}$$

where:

$\mathbf{I} = ||\delta_{ij}||$ - unit tensor,
δ_{ij} - Kronecker delta.

Symbols α and β denote empirical coefficients of lubricant, which describes viscoelastic properties of the fluid in Pas^2 units. Lubricant dynamic viscosity depends on magnetic induction and temperature, i.e. $\eta = \eta(H,T)$.

The stationary equation of motion and heat equation for the elastic bearing sleeve have the following form:

$$Div\boldsymbol{S}^* + \mu^*(\boldsymbol{N}^* \boldsymbol{\nabla})\boldsymbol{H}^* = \rho^* \partial^2 u/\partial t^2 + (\xi/T_o) gradT^*, \tag{6}$$

$$\mathrm{div}(\kappa^* \mathrm{grad}T^*) = (c_v\, \rho)^* \partial T^*/\partial\, t + \xi\, \partial(\mathrm{div}u)\partial\, t, \tag{7}$$

where:

$$\xi \equiv 3K \times (\alpha_T)^* T_o,$$

K - bulk modulus,
T_o - value of characteristic temperature,
$\boldsymbol{N}^*$ - magnetization intensity,
$\boldsymbol{H}^*$ - vector of magnetic intensity vector in elastic body,
μ^* - magnetic permeability coefficient of hyper-elastic body.

The following symbols: ρ^*, $(c_v)^*$, κ^*, $(\alpha_T)^*$ denote: density, specific heat, thermal conductivity, linear expansion coefficient for elastic body material respectively. We take into account the Duhamel-Neumann relations between the components τ^*_{ij} of the stress tensor S^* in the elastic body and the components ε_{ij} of the strain tensor [1]. Moreover we consider strain-displacement dependencies [2] for $u_{\alpha 1}$, $u_{\alpha 2}$, $u_{\alpha 3}$ - components of displacement vector u of elastic body. The characteristic dimensional height of elastic layer ε_s is about thousand times smaller than radius of body curvature and other quantities occurring in the friction region. The total dimensional gap height has the following form:

$$\varepsilon_T = \varepsilon_p + u_{\alpha 2} + u_{\alpha 2B}, \tag{8}$$

where:

ε_p - primary height of the gap,
$u_{\alpha 2}$ - dimensional temperature and pressure displacement in gap height direction of the external elastic layer surface being in contact with the lubricant,
$u_{\alpha 2B}$ - dimensional magnetic displacement in gap height direction of the external elastic layer surface being in contact with the lubricant.

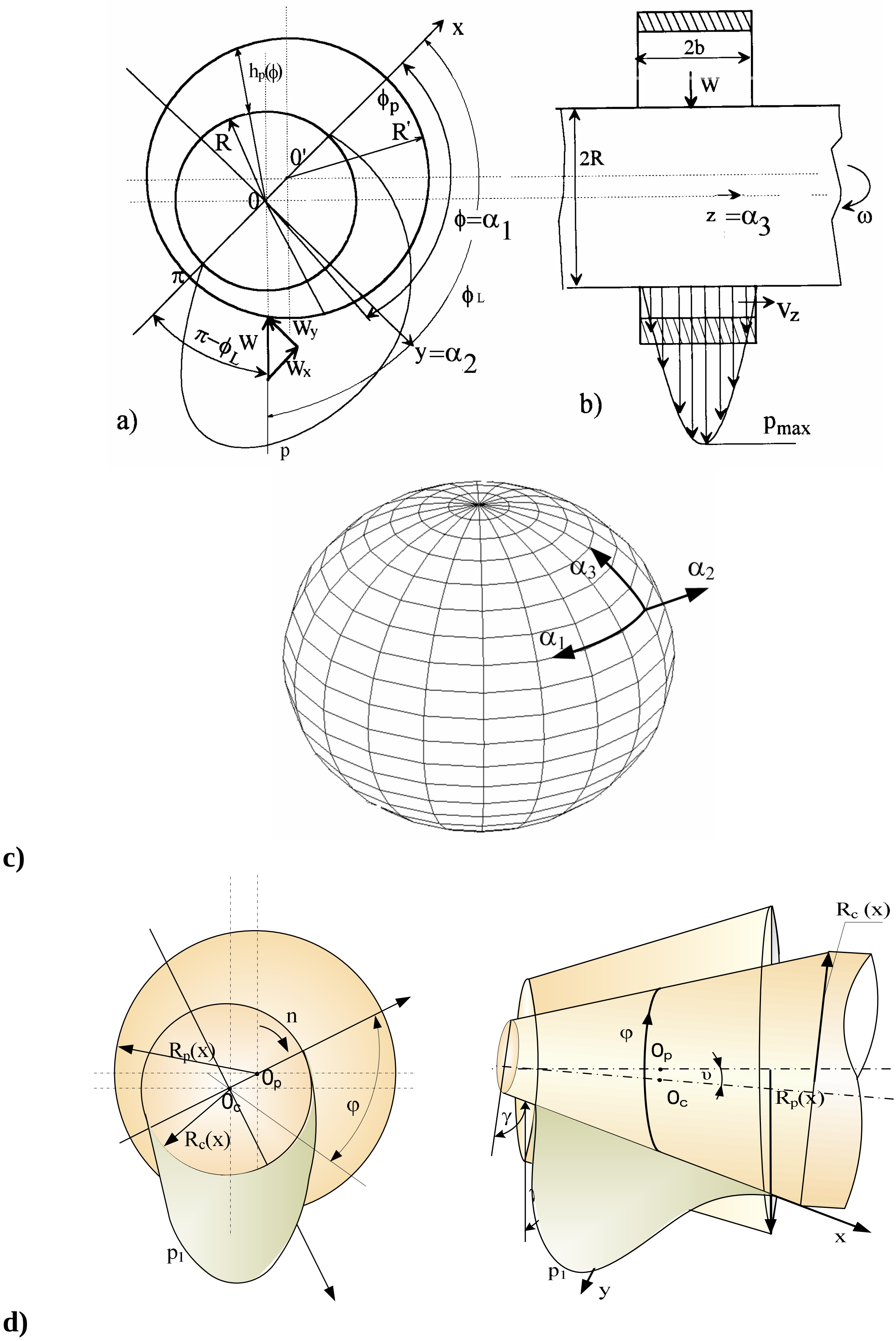

Fig. 1. Slide bearing geometries: a) cylindrical surface, b) cylindrical bearing gap, c) spherical surface, d) conical surface, where $x=\alpha_3$, $y=\alpha_2$, $\phi=\alpha_1$

The system of Eqs.(1), (2), (3), (4), (6), (7) has the following unknowns: $v_{\alpha 1}$, $v_{\alpha 2}$, $v_{\alpha 3}$ – three dimensional components of lubricant velocity vector in three curvilinear, orthogonal dimensional α_{1d}, α_{2d}, α_{3d} directions, p - hydrodynamic dimensional pressure, T - dimensional temperature in lubricant, T^* - dimensional temperature in the superficial layer of the sleeve body and $u_{\alpha 1}$, $u_{\alpha 2}$, $u_{\alpha 3}$ three components displacements of displacement vector $\boldsymbol{u}$ in elastic body. Symbol α_{1d} indicates circumferential direction in each coordinate system for example φ in cylindrical or spherical bearings. Symbol α_{2d} indicates gap height direction for example r in cylindrical or spherical bearings. Symbol α_{3d} indicates longitudinal direction in each coordinate system for example z in cylindrical and ϑ in spherical ones. Hence in cylindrical and spherical coordinates we have: $(v_{\alpha 1}, v_{\alpha 2}, v_{\alpha 3})=(v_{\phi}, v_r, v_z)$, $(v_{\alpha}, v_{\alpha 2}, v_{\alpha 3})=(v_{\phi}, v_r, v_{\vartheta})$.

2. Estimation of hydrodynamic equations

In order to simplify the boundary conditions for lubricant flow equations, we introduce following dimensionless components of velocity vector in the circumferential, radial and longitudinal directions v_1, v_2, v_3 respectively. And, the dimensionless components of magnetic induction vector, magnetization vector, magnetic intensity vector, have here the following form: B_1,B_2,B_3; N_1,N_2,N_3; H_1,H_2,H_3, respectively and they depend on the dimensionless variables α_1 and α_3. Dimensionless components of displacement vector in the dimensionless variables $(\alpha_1, \alpha_2, \alpha_3)$ have the form u_1, u_2, u_3. Symbols T_1, p_1, t_1, η_1, κ_1, h_{p1} denote dimensionless temperature, dimensionless pressure, time, dimensionless viscosity, dimensionless thermal conductivity coefficient and dimensionless gap height, respectively. Between dimensional and dimensionless quantities we have following relationship [1], [3]:

$$p=p_o p_1,\ v_{\alpha 1}=Uv_1,\ v_{\alpha 2}=\psi U v_2,\ v_{\alpha 3}=Wv_3,\ T=T_o+BrT_oT_1,\ \eta=\eta_o\eta_1,\ \kappa=\kappa_o\kappa_1,\ \varepsilon_p=\varepsilon\varepsilon_{p1},\ u_{\alpha 2}=\varepsilon u_2, \quad (9)$$

$$\alpha_{2d}=R(1+\psi\alpha_2),\ \ \alpha_{3d}=R^*\alpha_3,\ \alpha_{1d}=R\alpha_1. \quad (10)$$

We denote: R - radius of the curvature in α_1 direction, R^* - radius of the curvature in α_3 direction or length of the bearing of cylindrical bearing, $U=\omega R$ surface linear dimension velocity in α_1 direction, $W=U/L_1$ - surface linear dimension velocity in α_3 direction, $L_1=R^*/R$, ε - average dimensional gap height, ω - angular journal velocity, $B_o=\mu H_o$, $p_o=\omega\eta_o/\psi^2$, $\psi\equiv\varepsilon/R$. Other dimensional characteristic values are distinguished by symbol (o) in lower index namely: η_o, κ_o, ρ_o, H_o, T_o, N_o, t_o. We determine the quantities: dimensionless radial clearance, dimensionless length, Magnetic, Brinkman, Reynolds, Strouhal, and two Deborah Numbers, in the following form, respectively:

$$\psi\equiv\frac{\varepsilon}{R}\cong 10^{-3}, L_1\equiv\frac{R^*}{R}, R_f\equiv\frac{N_oB_o}{p_o}, Br\equiv\frac{U^2\eta_o}{\kappa_oT_o},$$
$$Re\equiv\frac{U\varepsilon\rho_o}{\eta_o}, Str\equiv\frac{R}{Ut_o}, D_\alpha\equiv\frac{\alpha U}{\eta_oR}, D_\beta\equiv\frac{\beta U}{\eta_oR}, \quad (11)$$

where:

$0\leq D_\alpha\leq 1$, $0\leq D_\beta\leq 1$, $0\leq Q_{Br}\equiv BrT_o\delta_T\leq 1$,
δ_T - dimensional coefficient which describes the influence of temperature on oil viscosity.

3. Boundary conditions with random effects

The lubricant flow in bearing gap is generated by the rotation of a cylindrical spherical, conical, or parabolic journal. Bearing sleeve is motionless. Hence the boundary conditions for the lubricant velocity components have the form:

$$v_1=h_1 \quad for \quad \alpha_2=0, \quad v_1=0 \quad for \quad \alpha_2=\varepsilon_{T1},$$
$$v_2=0 \quad for \quad \alpha_2=0, \quad v_2=0 \quad for \quad \alpha_2=\varepsilon_{T1}, \tag{12}$$
$$\mathrm{v}_3=0 \quad \text{for} \quad \alpha_2=0, \quad \mathrm{v}_3=0 \quad \text{for} \quad \alpha_2=\varepsilon_{T1},$$

where

$\varepsilon_{T1} \equiv \varepsilon_{p1} + u_{or1} + u_{o2B}$,
h_1 - Lame coefficient.

In cylindrical and spherical coordinates the dimensionless oil velocity components (v_1,v_2,v_3) have the dimensional form: $(v_{\alpha1},v_{\alpha2},v_{\alpha3})=(v_\phi,v_r,v_z)$, $(v_{\alpha1},v_{\alpha2},v_{\alpha3})=(v_\phi,v_r,v_\vartheta)$. Hence the first dimensionless condition (12.1) in cylindrical $(h_1=1)$ and spherical $(h_1=\sin\vartheta)$ coordinates has the following dimensional forms: $v_\phi=\omega R$ *for* $\alpha_{2d}=0$, $v_\phi=0$ for $\alpha_{2d}=\varepsilon_T$ and $v_\phi=\omega R\sin\vartheta$ for $\alpha_{2d}=0$, $v_\phi=0$ for $\alpha_{2d}=\varepsilon_T$. Symbol R denotes radius of cylindrical of spherical journal and symbol ω denotes angular velocity of cylindrical journal and spherical journal in circumferential direction.
Dimensionless temperature distribution along the bearing gap has the constant dimensionless value $f_{c1}=1$ on the journal surface and on the sleeve surface it changes into the form $f_{p1}(\alpha_1,\alpha_3)$. Hence:

$$T_1(\alpha_1,\alpha_2,\alpha_3)=1 \quad \text{for} \quad \alpha_2=0,$$
$$T_1(\alpha_1,\alpha_2,\alpha_3)= f_{p1}(\alpha_1,\alpha_3) \quad \text{for} \quad \alpha_2=\varepsilon_{T1}. \tag{13}$$

Heat flux is transferred from the journal (rotational bearing surface) into the lubricant, hence we obtain boundary condition in the following form:

$$\frac{\partial T_1}{\partial \alpha_2} = -q_{c1} \quad for \quad \alpha_2 = 0. \tag{14}$$

For free heat exchange between the journal and lubricant and by virtue of Newton-Fourier law, then the dimensionless heat flux obtains the following form:

$$q_{c1} \equiv \frac{\zeta\varepsilon}{\kappa_o}\Delta f_1, \tag{15}$$

where:

ζ - heat transfer coefficient,
Δf_1 - dimensionless difference of temperature between journal temperature and ambient oil temperature.

By using the optimal function f of probability density distribution for the stochastic gap changes caused by the roughness, then the mean value of total film thickness $E(\varepsilon_{T1})$ and mean the value of pressure function $E(p_{10})$ are presented by virtue of the expectancy operator in the following form:

$$E(*)=\int_{-\infty}^{+\infty}(*)\times f(\delta_1)d\delta_1,\quad \sigma_1=\frac{c_1}{\sqrt{13}}=0{,}375,$$

$$f(\delta_1)\equiv\begin{cases}\left(1-\dfrac{\delta_1^2}{c_1^2}\right)^5 & for\ -c_1\le\delta_1\le+c_1,\\ 0 & for\quad |\delta_1|>c_1,\end{cases}\tag{16}$$

where symbol $c_1=1{,}353515$ denotes the half total range of random variable of the thin layer thickness for normal hip joint. Symbol $\sigma_1=0{,}37539$ denotes the dimensionless standard deviation.

4. Particular solution of non-isothermal problem

Now we consider a particular case of the system of equations (1)-(4) in curvilinear coordinates for steady flow, various viscosity $\eta_1(\alpha_1,\alpha_2,\alpha_3)$, constant thermal conductivity coefficient $\kappa_1=1$, and Newtonian flow without magnetic effects, where centrifugal forces and convection forces are neglected and $(0<Re\psi Str<<1,\ 0<D_\beta Str<<1,\ 0<Re\psi<<1,\ 0<Gz<<1)$.

The particular dimensionless solution of the energy equation (4) under boundary conditions (12), (13), (14) has the following form:

$$T_1(\alpha_1,\alpha_2,\alpha_3)=1-q_{c1}\alpha_2-\int_0^{\alpha_2}\int_0^{\alpha_2}\eta_1\left[\left(\frac{\partial v_1}{\partial\alpha_2}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial v_3}{\partial\alpha_2}\right)^2\right]d\alpha_2 d\alpha_2,\tag{17}$$

$0\le\alpha_1\le2\pi\theta_1$, $0\le\theta_1\le1$, $b_{m1}\le\alpha_3\le b_{s1}$, $0\le\alpha_2\le\varepsilon_{T1}\equiv\varepsilon_{p1}+u_{o2}+u_{o2B}$, $s\equiv\alpha_2/\varepsilon_{T1}$, $\varepsilon_{T1}=\varepsilon_{T1}(\alpha_1,\alpha_3)$, η_1 - const.

Dimensionless temperature on the internal sleeve surface has the following form:

$$f_{p1}(\alpha_1,\alpha_3)=T_1(\alpha_1,\alpha_2=\varepsilon_{T1},\alpha_3)=1-q_{c1}\varepsilon_{T1}-\int_0^{\varepsilon_{T1}}\int_0^{\alpha_2}\eta_1\left[\left(\frac{\partial v_1}{\partial\alpha_2}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial v_3}{\partial\alpha_2}\right)^2\right]d\alpha_2 d\alpha_2\tag{18}$$

where:

$0\le\alpha_1\le2\pi\theta_1$, $0\le\theta_1\le1$, $b_{m1}\le\alpha_3\le b_{s1}$.

For the rotational journal we have $h_1=h_1(\alpha_3)$, $h_3=h_3(\alpha_3)$. Solutions of the partial differential equations (3) under the boundary conditions (12) have the following form [4], [5]:

$$v_1(\alpha_1,\alpha_2,\alpha_3)=\frac{1}{h_1}\frac{\partial p_1}{\partial\alpha_1}A_\eta+(1-A_s)h_1,\tag{19}$$

$$v_3(\alpha_1,\alpha_2,\alpha_3)=\frac{1}{h_3}\frac{\partial p_1}{\partial\alpha_3}A_\eta,\tag{20}$$

$$A_s \equiv \frac{\int_0^{\alpha_2} \frac{1}{\eta_1} d\alpha_2}{\int_0^{\varepsilon_{T1}} \frac{1}{\eta_1} d\alpha_2}, \quad A_\eta \equiv \int_0^{\alpha_2} \frac{\alpha_2}{\eta_1} d\alpha_2 - A_s \int_0^{\varepsilon_{T1}} \frac{\alpha_2}{\eta_1} d\alpha_2, \tag{21}$$

where:

$0 \le \alpha_1 \le 2\pi\theta_1$, $0 \le \theta_1 \le 1$, $b_{m1} \le \alpha_3 \le b_{s1}$, $0 \le \alpha_2 \le \varepsilon_{T1} \equiv \varepsilon_{p1} + u_{or1} + u_{o2B}$, $\varepsilon_{T1} = \varepsilon_{T1}(\alpha_1, \alpha_3)$, $\eta_1(\alpha_1, \alpha_2, \alpha_3)$.

Solutions of the continuity equations (15) under the boundary conditions (12.2) $v_2=0$ for $\alpha_2=0$ has the following form:

$$v_2(\alpha_1,\alpha_2,\alpha_3) = -\int_0^{\alpha_2} \frac{1}{h_1} \frac{\partial v_1}{\partial \alpha_1} d\alpha_2 - \frac{1}{L_1^2} \int_0^{\alpha_2} \frac{1}{h_1 h_3} \frac{\partial (h_1 v_3)}{\partial \alpha_3} d\alpha_2. \tag{22}$$

We put the solutions (19), (20) in solution (22) and we take expected value of the both sides of the equation using the expectancy operator E. If we impose second boundary condition (12.2) for radial component of lubricant velocity i.e. $v_2=0$ for $\alpha_2=\varepsilon_{T1}$ then it is easy to see that the pressure function p_1 in the curvilinear coordinates (α_1, α_2, α_3) satisfies the following modified stochastic Reynolds equation:

$$\frac{1}{h_1}\frac{\partial}{\partial \alpha_1}\left[\frac{\partial p_1}{\partial \alpha_1} E\left(\int_0^{\varepsilon_{T1}} A_\eta\right) d\alpha_2\right] + \frac{1}{L_1^2}\frac{1}{h_3}\frac{\partial}{\partial \alpha_3}\left[\frac{h_1}{h_3}\frac{\partial p_1}{\partial \alpha_3} E\left(\int_0^{\varepsilon_{T1}} A_\eta\right) d\alpha_2\right] = h_1 \frac{\partial}{\partial \alpha_1}\left[E\left(\int_0^{\varepsilon_{T1}} A_s\right) d\alpha_2 - E(\varepsilon_{T1})\right]. \tag{23}$$

If oil dynamic viscosity is constant in gap height direction i.e. $\eta_1(\alpha_1,\alpha_3)$, then:

$$A_s = \frac{\alpha_2}{\varepsilon_{T1}} \equiv s, \quad \int_0^{\varepsilon_{T1}} A_\eta d\alpha_2 = -\frac{\varepsilon_{T1}^3}{12\eta_1}, \quad \int_0^{\varepsilon_{T1}} A_s d\alpha_2 - \varepsilon_{T1} = -\frac{1}{2}\varepsilon_{T1}. \tag{24}$$

Hence the stochastic Reynolds equation (23) tends to the following form:

$$\frac{1}{h_1}\frac{\partial}{\partial \alpha_1}\left[\frac{E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial p_1}{\partial \alpha_1}\right] + \frac{1}{L_1^2}\frac{1}{h_3}\frac{\partial}{\partial \alpha_3}\left[\frac{h_1}{h_3}\frac{E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial p_1}{\partial \alpha_3}\right] = 6h_1 \frac{\partial E(\varepsilon_{T1})}{\partial \alpha_1}. \tag{25}$$

For cylindrical bearing we have $\alpha_1=\phi$, $\alpha_2=r_1$, $\alpha_3=z_1$ and dimensionless Lame coefficients are as follows $h_1=h_3=1$, where $L_1=b/R$, b - bearing length, R - radius of the journal. In this case the stochastic Reynolds equation (23) has the form:

$$\frac{\partial}{\partial \phi}\left[\frac{E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial E(p_1)}{\partial \phi}\right] + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\frac{E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial E(p_1)}{\partial z_1}\right] = 6\frac{\partial E(\varepsilon_{T1})}{\partial \phi}, \tag{26}$$

where:

$p_1(\varphi, z_1)$, $0 \le \varphi \le 2\pi$, $-1 \le z_1 \le 1$, $\varepsilon_{T1} = \varepsilon_{T1}(\varphi_1, z_1)$.

For the spherical bearing we have $\alpha_1=\phi$, $\alpha_2=r_1$, $\alpha_3=\vartheta_1$ and dimensionless Lame coefficients are as follows: $h_1=\sin\vartheta_1$, $h_3=1$, $L_1=1$. In this case the dimensionless pressure function p_1 in the spherical coordinates (φ,ϑ_1) satisfies the modified Reynolds equation (25) in the form:

$$\frac{1}{\sin\vartheta_1}\frac{\partial}{\partial\phi}\left[\frac{E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial E(p_1)}{\partial\phi}\right]+\frac{\partial}{\partial\vartheta_1}\left[\frac{E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial E(p_1)}{\partial\vartheta_1}\sin\vartheta_1\right]=6\frac{\partial E(\varepsilon_{T1})}{\partial\phi}\sin\vartheta_1, \tag{27}$$

where:

$$0\le\phi<2\pi\theta_1,\ \ 0\le\theta_1<1,\ \ \pi/8\le\vartheta_1\le\pi/2.$$

For the conical bearing we have $\alpha_1=\phi$, $\alpha_2=y_1$, $\alpha_3=x_1$ and dimensionless Lame coefficients are as follows: $h_1=x_1\cos\alpha$, $h_3=1$, $L_1=b_c/R$, where α - angle between conical surface and the plane of cross section of the journal, b_c - the length of the cone generating line, R - radius of the journal. In this case the dimensionless pressure function p_1 in the conical coordinates $(\varphi,\ y_1,\ x_1)$ satisfies the modified Reynolds equation (25) in the form:

$$\frac{1}{x_1}\frac{\partial}{\partial\phi}\left[\frac{E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial E(p_1)}{\partial\phi}\right]+\frac{\cos^2\alpha}{L_1^2}\frac{\partial}{\partial x_1}\left[\frac{x_1E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial E(p_1)}{\partial x_1}\right]=6\frac{\partial E(\varepsilon_{T1})}{\partial\phi}x_1\cos^2\alpha, \tag{28}$$

where:

$$0\le\phi<2\pi\theta_1,\ \ 0\le\theta_1<1,\ \ 0\le x_1\le 1,\ \ x_1=x/b_c.$$

In parabolic bearing we have the non monotone generating line of the journal in length direction. For the conical bearing we have: $\alpha_1=\phi$, $\alpha_2=y_1$, $\alpha_3=\chi_1$ and dimensionless Lame coefficients are as follows:

$$h_1=\cos^2(\Lambda_1\chi_1),\quad h_3=\sqrt{1+4\Lambda_1^2\sin^2(\Lambda_1\chi_1)}\,\cos(\Lambda_1\chi_1),\quad \Lambda_1\equiv\frac{1}{L_1}\sqrt{\frac{R-a}{R}},\ L_1\equiv\frac{b_p}{R}, \tag{29}$$

where:

R - the largest radius of the parabolic journal,
a - the smallest radius of the parabolic journal,
$2b_p$ - the bearing length.

In this case the dimensionless pressure function p_1 in the conical coordinates $(\varphi,\ y_1,\ \chi_1)$ satisfies the modified Reynolds equation (25) in the following form:

$$\frac{\partial}{\partial\phi}\left[\frac{E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial E(p_1)}{\partial\phi}\right]+\frac{\cos(\Lambda_1\chi_1)}{L_1^2\sqrt{1+4\Lambda_1^2\sin^2(\Lambda_1\chi_1)}}\frac{\partial}{\partial x_1}\left[\frac{\cos(\Lambda_1\chi_1)}{\sqrt{1+4\Lambda_1^2\sin^2(\Lambda_1\chi_1)}}\frac{E(\varepsilon_{T1}^3)}{\eta_1}\frac{\partial E(p_1)}{\partial\chi_1}\right]=$$

$$=6\frac{\partial E(\varepsilon_{T1})}{\partial\phi}\cos^4(\Lambda_1\chi_1),\quad |\chi_1|\le\frac{1}{2\Lambda_1}\arccos\sqrt{\frac{a}{R}}, \tag{30}$$

where:

$$0\le\phi<2\pi\theta_1,\ \ 0\le\theta_1<1,\ \chi_1=\alpha_3/R.$$

The dimensionless gap height ε_{T1} depends on the variable α_1 and α_3 and consists of two parts:

$$\varepsilon_{T1}=\varepsilon_{T1s}(\alpha_1\alpha_3)+\delta_1(\alpha_1\alpha_3,\xi), \tag{31}$$

where ε_{T1s} denotes the total dimensionless nominal smooth part of the geometry of thin fluid layer. This part of gap height contains dimensionless corrections of gap height caused by the hyper elastic deformations. Symbol δ_1 denotes the dimensionless random part of changes of gap height resulting from the vibrations, unsteady loading and surface roughness and asperities measured from the nominal mean level. Symbol ξ describes the random variable, which characterizes the roughness arrangement.

Using the probability function (31), then after calculations we obtain:

$$\begin{gathered} E(\varepsilon_{T1})=\int_{-c_1}^{+c_1}(\varepsilon_{T1s}+\delta_1)f(\delta_1)d\delta_1=\varepsilon_{T1s}, \\ E(\varepsilon_{T1}^3)=\int_{-c_1}^{+c_1}(\varepsilon_{T1s}+\delta_1)^3 f(\delta_1)d\delta_1=\varepsilon_{T1s}^3+3\sigma_1^2\varepsilon_{T1s}\,. \end{gathered} \tag{32}$$

5. Conclusions

The main achievements of this paper are the hydrodynamic pressure derivations for slide bearing journals in arbitrary curvilinear orthogonal coordinates and in particular case for cylindrical, spherical, conical, parabolic bearings with random conditions during the lubrication.

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References

[1] Wierzcholski, K., *Analytical simulations of deformations for journal bearing gap*, Proc. of Third Inter. Congress on Thermal Stresses, Vol. 1, pp. 391-394, Cracow 1999.
[2] Wierzcholski, K., *Non Isothermal Lubrication for Viscoelastic Ferromagnetic Oils*, Proc. of Thermal Stresses, Fourth Inter. Congress on Thermal Stresses, pp. 613-616, Osaka 2001.
[3] Wierzcholski, K., *Ferromagnetic Turbulent Lubrication for Thermoelasticity Deformations of Bearing Gap*, Proc. of Thermal Stresses, The Fifth Inter. Congress on Thermal Stresses, WA4-3, pp. WA4-3-1– WA4-3-4, Blacksburg, Virginia USA 2003.
[4] Wierzcholski, K., The *Method of Solving of the System of non-Linear Differential Equations for non-Isothermic Laminar non-Newtonian Flow in the Thin Layer Between Two Certain Surfaces*, (in Polish), Scientific Papers of the Institute of Machine Design and Operation Studies and Research - *The boundary integral equation method in fluid mechanics*, Technical University of Wrocław, Nr 59, Ser. Nr 26, pp. 134 -144, Wrocław 1993.
[5] Wierzcholski, K., *Contribution to the analytical solutions of non linear basic equations for ferrolubricant flow in a film between two non rotational surfaces*, 7-th International Symposium on System Modelling Control, Polish Cybernetical Society, Vol. 2, pp. 286-291, Zakopane 1993.