RELIABILITY OF THREE-STATE RESTORABLE SYSTEMS WITH REGARD TO THE UNRELIABILITY OF THE SAFETY IN THE RESTORABLE STATE

Jerzy Jaźwiński

Air Force Institute of Technology
ul. Księcia Bolesława 6, 01-494 Warszawa, PO BOX 96, Poland
tel.: +48 22 6852161, fax: +48 22 6852163
e-mail: jjur@wp.pl

Abstract

A model of a three-state system has been considered in the work. The system can pass from the state of full ability into the state of safety unreliability (irreversible state) or into the state of efficiency unreliability (reversible state). From this state the system can also pass to the state of safety unreliability. To the system's modeling Kołmogorov-Chapman differential equation has been used. The obtained results have been presented with an example. Analysis with the obtained equations has been done.

Keywords: *Systems reliability, restorable systems, restorable state, nonefficiency failure rate, danger failure rate, recurrence rate*

1. Introduction

The contemporary systems are characterized by a large surplus in the reliability sense. The failures of such systems cause not very important effects for the realization of a task, but the effects for safety are of course very important. Then the analysis of the system must take into account the safety factors. The author considered problem of safety in many works devoted to models of un restorable systems.

In this paper I would like to present a model of restorable system in safety aspects.

The system in which two kinds of failures can occur. The first kind of failures causes system efficiency unreliability and in this case the system is restorable. This restoration can be dangerous and the system can pass to safety unreliability. The second kind of failures causes system safety unreliability. In this case I assume that the system is not restorable.

To describe such models of the systems, the reliability and safety factors will be formulated.

2. Basic concepts and definitions

Taking into account that this field of knowledge is very young and the terminology is not fully unificated, that is up to now the object of discussion.

Due to limited length of this paper, only basic concepts used in this work, are presented.

$R(t)$ - Function of the system reliability - the probability of fulfilling requirements from the point of view of safety and reliability of the system;

$Q_B(t)$ - Function of the system safety unreliability - the probability of occurrence of event causing a dangerous situation.

$Q_S(t)$ - Function of the system efficiency unreliability - the probability of occurrence of event causing an unreliable inefficient work of the system.

Of course for all these quantities both the exploitation conditions (in a broad sense) and exploitation time interval must be strictly defined.
In practice we have most often dealt with systems which can be in the fitness-for-use state or in the efficiency unreliability state or in the safety unreliability state, In the case when the system passes to the efficiency unreliability state, it is restorable but can be dangerous.

Let us denote the additional reliability factors as follows:

λ_S - intensity of the passing from the ability state to the unreliable efficiency state;
λ_B - intensity of the passing from the ability state to the safety unreliability state;
μ_S - intensity of the passing from the state of efficiency unreliability to the ability state;
μ_B - intensity of the passing from the unreliability efficiency state to the unreliability of the safety state.

3. The system with restorable state of efficiency unreliability

The graph of the system is shown in fig. 1.

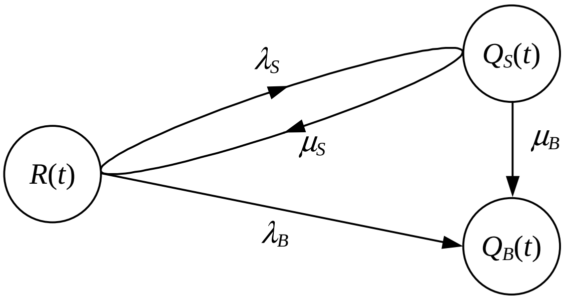

Fig. 1. The graph of the system with restorable efficiency unreliability state with regard to the safety unreliability with the restorable state

The system may be described by the Kołmogorov-Chapman differential equation set:

$$R'(t) = -(\lambda_S + \lambda_B) R(t) + \mu_S Q_S(t),$$

$$Q_S'(t) = \lambda_S R(t) - (\mu_S + \mu_B) Q_S(t),$$

$$Q_B'(t) = \lambda_B R(t) + \mu_B Q_S(t), \tag{1}$$

$$R(0) = 1; \; Q_S(0) = Q_B(0) = 0,$$

$$R(t) + Q_S(t) + Q_B(t) = 1.$$

The Laplace transforms of equation set (1) are:

$$\tilde{R}(p) = \frac{\mu_S + \mu_B + p}{p^2 + p(\lambda_S + \lambda_B + \mu_S + \mu_B) + \mu_S\lambda_B + \mu_S\mu_B + \lambda_B\mu_B} = \frac{\mu + p}{p^2 + pA + B}, \tag{2}$$

$$\tilde{Q}_S(p) = \frac{\lambda_S}{p^2 + pA + B}, \tag{3}$$

$$\tilde{Q}_B(p) = \frac{1}{p}\left[\lambda_B R(p) + \mu_B Q_S(p)\right], \tag{4}$$

where:

$\mu = \mu_S + \mu_B$,
$A = \lambda_S + \lambda_B + \mu_S + \mu_B$,
$B = \mu_S \mu_B + \mu_S \lambda_B + \lambda_B \mu_B$.

Solving the equation:

$$p^2 + pA + B = 0, \tag{5}$$

we obtain:

$$x_{1/2} = \frac{1}{2}\left(-A \pm C\right), \tag{6}$$

$$C = \sqrt{A^2 - 4B}. \tag{7}$$

Taking Laplace reciprocal transforms, after simple modifications we have got formulae as follows:

$$R(t) = \frac{1}{C} e^{x_1 t}\left[\left(x_1 + \mu_S\right) - \left(x_2 + \mu\right) e^{-Ct}\right], \tag{8}$$

$$Q_S(t) = \frac{\lambda_S}{C} e^{x_1 t}\left(1 - e^{-Ct}\right), \tag{9}$$

$$Q_B(t) = 1 - R(t) - Q_S(t). \tag{10}$$

The mean time of work of the system to the occurrence of its safety unreliability is given by the formula:

$$\overline{T}_B = -\frac{d\left[p\tilde{Q}_B(p)\right]}{dp}\bigg|_{p=0} = \frac{\mu + \lambda_S}{\mu_S\,\lambda_B + \mu_B\,\lambda_S + \mu_B\,\lambda_B}. \tag{11}$$

Example

In fig. 2 the relation of the mean time of work T_B has been shown to the occurrence of the safety unreliability state from the intensity μ_S at different values of μ_B=0, 0.1, 0.2, 0.5; λ_S=0.1; λ_B=0.01.

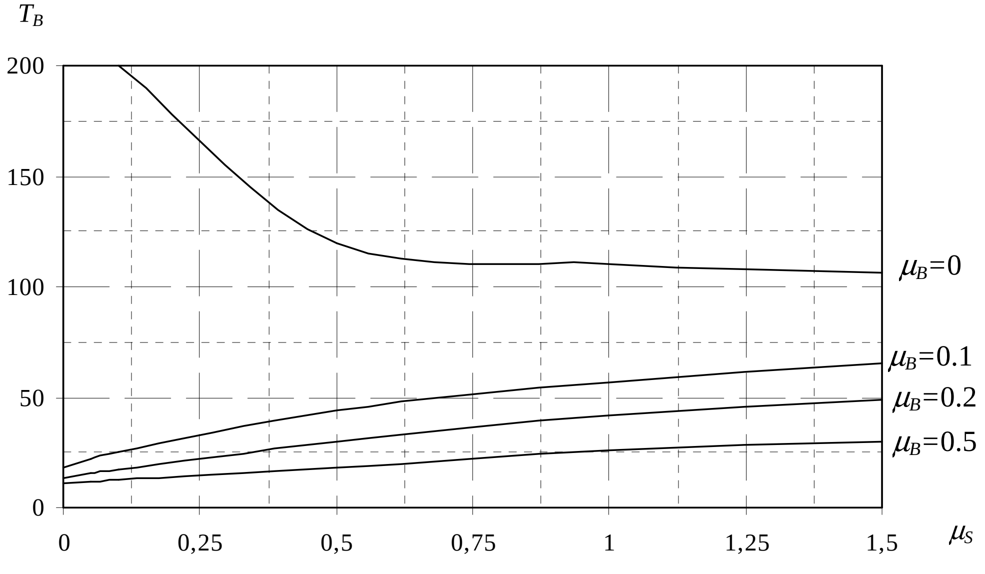

Fig. 2. Diagram of relation of $T_B = f(\mu_S)$ for the three state restorable system

The following conclusions result from the presented graph:

- when $\mu_B = 0$ - then with the increase of μ_S, T_B decreases;
- when $\mu_B \neq 0$ - then with the increase of μ_S, T_B increases and with the increase of μ_B - T_B decreases.

The equivalent danger failure rate of the whole system is given by:

$$\lambda_B(t) = \frac{\lambda_B R(t) + \mu_B\, Q_S(t)}{R(t) + Q_S(t)}. \tag{12}$$

Taking into account (8) and (9) we can determine $\lambda_B(t)$. Analysing this function we can show that:

$$\lambda_B(0) = \lambda_B$$

$$\lambda \lim_{t \to \infty} \lambda_B(t) = \frac{\lambda_B(x_1 + \mu) + \mu_B\, \lambda_S}{x_1 + \mu + \lambda_S}. \tag{13}$$

We can see that in consequence, $\lambda_B(t)$ is the decreasing function and the time $\overline{T}_B$ depends on the ratio λ_S / μ_S.

4. Conclusion

A dangerous restoration can essentially reduce the system's safety.

To apply in practice safety and reliability factors proposed in this paper, the investigations to obtain credible data for their calculation are necessary.

References

[1] Jaźwiński, J., *Reliability and safety of restorable systems*, Journal of Explo-Diesel & Gas Turbine'05, IV International Scientifically-Technical Conference, Gdańsk – Międzyzdroje – Kopenhaga 2005.

[2] Jaźwiński, J., Grabski, F., *Niektóre problemy modelowania systemów transportowych*, Biblioteka Problemów Eksploatacji, Warszawa-Radom 2003.

[3] Jaźwiński, J., Ważyńska-Fiok, K., *Bezpieczeństwo systemów czterostanowych z odnawialnym stanem zawodności sprawności*, Zagadnienia Eksploatacji Maszyn, z. 2.1, 1971.

[4] Jaźwiński, J., Ważyńska-Fiok, K., *Bezpieczeństwo systemów*, WNT, Warszawa 1993.

[5] Jaźwiński, J., Ważyńska-Fiok, K., *Niezawodność systemów transportowych*, PWN, Warszawa 1990.

[6] Jaźwiński, J., Ważyńska-Fiok, K., *Problems of System Safety with Functional and Time Surplus*, Second European Simposion on System Safety, La Baule, France 1982.

[7] Jaźwiński, J., Ważyńska-Fiok, K., *Reliability and Safety of Transport Systems*, 8 Zuverlassig-keitstagung, Leipzing – DDR 1987.