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PRESSURE AND CAPACITY FORCES IN SLIDE JOURNAL PLANE BEARING BY LAMINAR UNSTEADY LUBRICATION

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Abstract

This paper shows results of numerical solutions an modified Reynolds equations for laminar unsteady oil flow in slide journal bearing with planar linear gap. This solution example apply to isothermal bearing model with infinity breadth. Lubricating oil used in this model has Newtonian properties and dynamic viscosity in dependence on pressure. It shows a preliminary analysis change of pressure and capacity forces in the bearing by laminar, unsteady lubrication caused by velocity perturbations of oil flow in the longitudinal direction of a bearing. Described effect can be used as an example of modeling the bearing friction node operations in reciprocating movement during exploitation of engines and machines. Plane crossbar journal bearing occur in ship combustion engine as a crosshead bearing. Results are presented in the dimensionless hydrodynamic pressure and capacity force diagrams.

Keywords: *journal plane bearing, lubrication, unsteady laminar oil flow, pressure distribution, capacity forces*

1. Introduction

Presented subject matter apply to unsteady laminar flows [1],[4],[5] where modified Reynolds number Re* is smaller or equal to 2. This flows are also determine by Taylor number Ty which is smaller or equal to 41,1. Laminar and unsteady flow of lubricant factor may occur during periodic or randomness non-periodic load perturbation. This kind of perturbation can occur during transient states of machines, but mostly during starts and stops. Presented work analyse change of oil lubricating flow perturbation in longitudinal direction on the slide plane and on the radial race of slide bearing. Plane bearing can be used as a work model of bearing friction node in kinematic pair in translational motion. As an example the crosshead bearing of slow-speed engine. Reynolds equation system for unsteady, laminar Newtonian oil flow in the cylinder radial bearing is presented in work [1] and in the plane slide bearing in work [3]. Stationary model of plane journal bearing lubrication is presented in work [2]. Velocity flow perturbations of lubricating oil on the slide can be caused by longitudinal vibrations during of reciprocating slide motion. Axial vibrations overlay on slide motion and this causes oil velocity perturbation on the slide bearing surface. Values of the perturbation are proportional to the longitudinal amplitude of perturbation and to forced frequency. Longitudinal vibration in the slide bearing elements can be caused by torsional vibration of the crankshaft. Oil flow velocity perturbations in the longitudinal direction on the bearing race can be caused by axial vibration of the race coming from vertical vibration of the engine. Isothermal bearing model can act as work model of bearing friction node by steady-state conditions of thermal load.

2. Modified Reynolds Equation

Lubricating gap is characterize by following geometric parameters: maximal gap height h_o, minimal gap height h_e , gap length L and gap width b (Fig.1). In presented model the following assumption were made: lubricating gap dimensions along it's width of mating surfaces remain identical. Lubricating gap height after gap length was described in cartesian co-ordinate system by the following dimensionless form:

$$h_1(x_1) = \varepsilon - (\varepsilon - 1)x_1 \qquad \text{for} \qquad 0 \le x_1 \le 1 \tag{1}$$

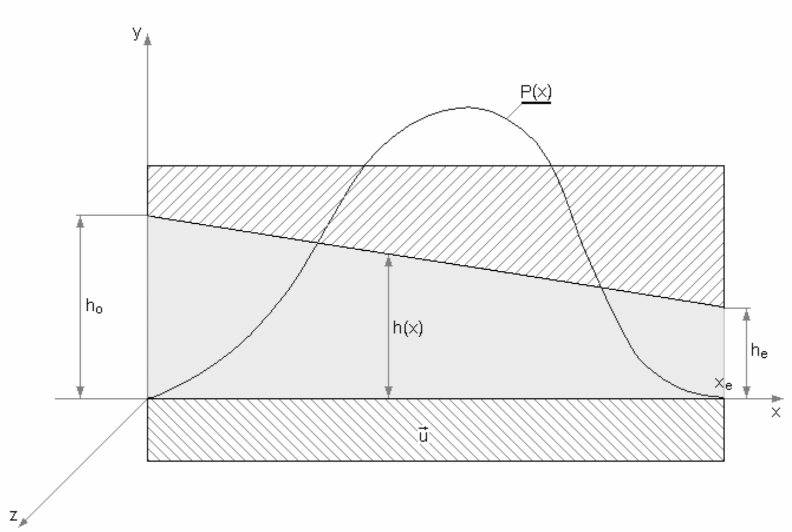

Fig.1. Geometry schema of the slide journal plate bearing gap

Dimensionless values [2],[3] that characterize lubricating gap are: length coordinate x_1 , gap height coordinate h_1 and gap convergence coefficient ε :

$$h_1 = \frac{h}{h_e}; \quad x_1 = \frac{x}{L}; \quad \varepsilon = \frac{h_0}{h_e} \tag{2}$$

In considered model we assume small unsteady disturbances and in order to maintain the laminar flow, oil velocity V_i^* and pressure p_1^* are total of dependent quantities $\tilde{V}_i$; $\tilde{p}_1$ and independent quantities $V_i; p_1$ from time [3],[5] according to equation (3).

$$\begin{aligned} V_i^* &= V_i + \tilde{V}_i \qquad i = 1,2,3 \\ p_1^* &= p_1 + \tilde{p}_1 \end{aligned} \tag{3}$$

Unsteady components of dimensionless oil velocity and pressure we [4] in following form of infinite series :

$$\tilde{V}_i(x_1; y_1; z_1; t_1) = \sum_{k=1}^{\infty} V_i^{(k)}(x_1; y_1; z_1) \exp(j k \omega_0 t_0 t_1) \qquad i=1,2,3$$

$$\tilde{p}_1(x_1; z_1; t_1) = \sum_{k=1}^{\infty} p_1^{(k)}(x_1; z_1) \exp(j k \omega_0 t_0 t_1) \tag{4}$$

where:
ω_0 – angular velocity perturbations in unsteady flow,
j - imaginary unit $j = \sqrt{-1}$.

Reynolds equation describing dimensionless total pressure p_1^* in the lubricating gap of a plane journal bearing [3] by unsteady, laminar, isothermal, Newtonian flow. Together with longitudinal velocity perturbations V_{10} on the race surface and V_{1h} on the slide. Velocity perturbation V_{30} along bearing width on the race and V_{3h} on the slide also occur in this model, as follows:

$$\frac{\partial}{\partial x_1}\left\{\frac{h_1^3}{\eta_{1B}e^{Kp_1}}\left[\frac{\partial p_1^*}{\partial x_1} - K\left(p_1^* - p_1\right)\frac{\partial p_1}{\partial x_1}\right]\right\} + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left\{\frac{h_1^3}{\eta_{1B}e^{Kp_1}}\left[\frac{\partial p_1^*}{\partial z_1} - K\left(p_1^* - p_1\right)\frac{\partial p_1}{\partial z_1}\right]\right\} =$$
$$= 6\frac{\partial h_1}{\partial x_1} + \frac{1}{2}\rho_1 \operatorname{Re}^* n\left\{\frac{\partial}{\partial x_1}\left[\frac{h_1^3}{\eta_{1B}e^{Kp_1}}\left(V_{10}+V_{1h}\right)\right] + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[\frac{h_1^3}{\eta_{1B}e^{Kp_1}}\left(V_{30}+V_{3h}\right)\right]\right\}\sum_{k=1}^{\infty} A_k + \tag{5}$$
$$-6\left\{\frac{\partial}{\partial x_1}\left[h_1\left(V_{10}+V_{1h}\right)\right] + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left[h_1\left(V_{30}+V_{3h}\right)\right] - 2\left(V_{1h}\frac{\partial h_1}{\partial x_1} + \frac{1}{L_1^2}V_{3h}\frac{\partial h_1}{\partial z_1}\right)\right\}\sum_{k=1}^{\infty} B_k$$

for $0 \le x_1 \le 1;\ 0 \le y_1 \le h_1\ ; -1 \le z_1 \le 1;\ 0 \le t_1 \le t_k\ ;\ p_1^* = p_1^*(x_1; z_1; t_1)$

Oil vector velocity components in dimension form V_x, V_y, V_z and in dimensionless form V_1, V_2, V_3 are described as follows:

$$V_x = UV_1 \qquad V_y = \psi U V_2 \qquad V_z = \frac{U}{L_1}V_3 \tag{6}$$

where:
U – linear velocity of slide bearing,
ψ – relative dimensionless clearance of bearing $(10^{-4} \le \psi \le 10^{-3})$,
b – bearing breadth,
L_1 – relative bearing breadth:

$$\psi = \frac{h_e}{L}\ ; \qquad L_1 = \frac{b}{L} \tag{7}$$

Oil dynamic viscosity η in dependence on pressure was taken according to the Barrus formula [5] and presented [1] in the dimension form η and in dimensionless form η_1:

$$\eta = \eta_0 e^{\alpha(p - p_a)} \approx \eta_0 e^{\alpha p}\ ; \qquad \eta_1 = \frac{\eta}{\eta_0} = \exp(\alpha p) \tag{8}$$

where:
η_o- oil dynamic viscosity by atmospheric pressure $p = p_a \approx 0$,
α – piezocoefficient taking into account viscosity changes in dependence on pressure.

Additional assumptions were made [2]: the dimensionless value for density ρ_1, pressure p_1, time t_1 and for remaining coordinates y_1 and z_1 according to the following designation:

$$\rho = \rho_0 \rho_1\ , \quad p = p_0 p_1\ , \quad t = t_0 t_1$$
$$z = b z_1\ , \quad y = h_e y_1\ , \quad K = \alpha p_0 \tag{9}$$

Density, pressure and time values with zero index are equivalent to basic sizes. Constant value K characterize dynamic viscosity in dependence on pressure. Pressure p_0 , Reynolds number Re, modified Reynolds number Re^* has the following form [2]:

$$p_0 = \frac{U\eta_0}{\psi^2 L} \quad ; \qquad Re = \frac{U\,\rho_0\,h_e}{\eta_0} \quad ; \qquad Re^* = \psi\,Re \tag{10}$$

Sums of a series $\sum_{k=1}^{\infty} A_k$ and $\sum_{k=1}^{\infty} B_k$ in Reynolds equation (5) were defined in works [1],[3], has form

$$\sum_{k=1}^{\infty} A_k = \sum_{k=1}^{\infty} \frac{\sin(k\omega_0 t_0 t_1)}{k} = \begin{cases} \dfrac{\pi - \omega_0 t_0 t_1}{2} & 0 < t_1 < 1 \\ 0 & t_1 = 0;\,1 \end{cases} \tag{11}$$

$$\sum_{k=1}^{\infty} B_k = \sum_{k=1}^{\infty} \frac{\cos(k\omega_0 t_0 t_1)}{k^2} = \frac{1}{4}\left[(\pi - \omega_0 t_0 t_1)^2 - \frac{\pi^2}{3}\right] \quad \text{dla} \quad 0 \le t_1 \le 1$$

In the further numerical analysis relation time was taken into account as a propagation period of axial velocity perturbation of lubricating oil. In case where oil velocity perturbations are caused by forced vibrations of engine then the number n in equation (5) define multiplication of perturbation frequency ω_0 to angular velocity of engine crankshaft ω.

3. Hydrodynamic pressure

Equation solution (3) for infinity breadth bearing with assumption that velocity perturbation does not depend on coordinate x_1 can be present [3] in total dimensionless hydrodynamic pressure p_1^* .

$$\begin{aligned} p_1^*(x_1) = {} & p_{1K} - \frac{p_{10}}{1-Kp_{10}}(V_{10} - V_{1h})\sum_{k=1}^{\infty} B_k + \frac{x_1}{4\varepsilon^2}\frac{\rho_1 Re^* n}{1-Kp_{10}}(V_{10}+V_{1h})\left(\varepsilon + 1 - \frac{\varepsilon + h_1}{h_1^2}\right)\sum_{k=1}^{\infty} A_k + \\ & + \frac{3K}{(\varepsilon-1)^2}\frac{\rho_1 Re^* n}{1-Kp_{10}}(V_{10}+V_{1h})\left\{(x_1 - 1)\ln\varepsilon - \ln h_1 + \frac{1}{\varepsilon+1}\left[(\varepsilon-1)(1-2x_1) + \frac{\varepsilon}{h_1} - h_1\right]\right\}\sum_{k=1}^{\infty} A_k \end{aligned} \tag{12}$$

The p_{10} value is a pressure value in the disccused lubricating gap by the steady flow with the constant lubricating oil dynamic viscosity. On the other hand p_{1K} value is a stationary pressure value for viscosity, in dependence on pressure and for discussed form of lubricating gap it was mentioned in work [2]:

$$p_{10} = \frac{6(\varepsilon-1)(1-x_1)x_1}{(\varepsilon+1)(\varepsilon - \varepsilon x_1 + x_1)^2} \quad ; \qquad p_{1K} = -\frac{1}{K}\ln\left|1 - Kp_{10}\right| \tag{13}$$

Perturbation pressure $\tilde{p}_1$ in the unsteady part of the flow can be presented as a difference of total pressure p_1^* and stationary pressure p_{1K}.

On the basis of presented dependences for isothermal bearing model with infinity width, the calculations of hydrodynamic pressure distribution in the lubricating gap were made. In the example calculations the following assumption were made: oil with constant density and value of the expression $n\rho_1 Re^*=12$, which approximately comply to longitudinal velocity perturbation function in the engine crosshead bearing after the first frequency force from two-stroke, six cylinder engine crankshaft torsional vibration. Hydrodynamic pressure distribution and other pressure parameters are in dependence on lubricating gap convergence coefficient ε [2]. Optimum gap convergence $\varepsilon_{opt} = 1+\sqrt{2}$ comply to maximal hydrodynamic pressure. Pressure in the optional point of lubricating gap changes due the perturbation time and its distribution along the gap length reach the maximal and minimum values. On the Fig.2 example of the hydrodynamic total pressure distribution along the gap length for bearing with the optimal convergence ε_{opt} and for the

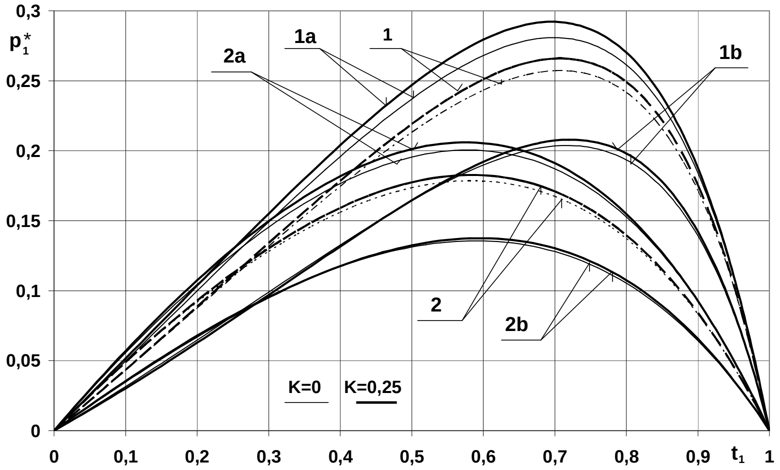

Fig.2 Total maximal (a) and minimal (b) pressure distributions p_1^* in direction x_1 for ε: 1) $\varepsilon = \varepsilon_{opt}$; 2) $\varepsilon=1{,}4$ by velocity perturbations: $V_{10}=0{,}05$

convergence $\varepsilon=1{,}4$ marked with numbers 1 and 2 by the constant viscosity (K=0) in dependence on pressure (K=0,25) marked with thin and thick lines. Maximal pressure distribution were marked with symbols a and the minimal pressure distribution were marked with b. Pressure quantities by stationary flow were marked with broken line. Unsteady flow on the Fig.2 is caused by longitudinal velocity perturbation only on the bearing race $V_{10}=0{,}05$. In the case where oil dynamic viscosity depends on pressure then pressure perturbations are higher than in the case where oil has constant viscosity. Pressure perturbation quantity depends on lubricating oil

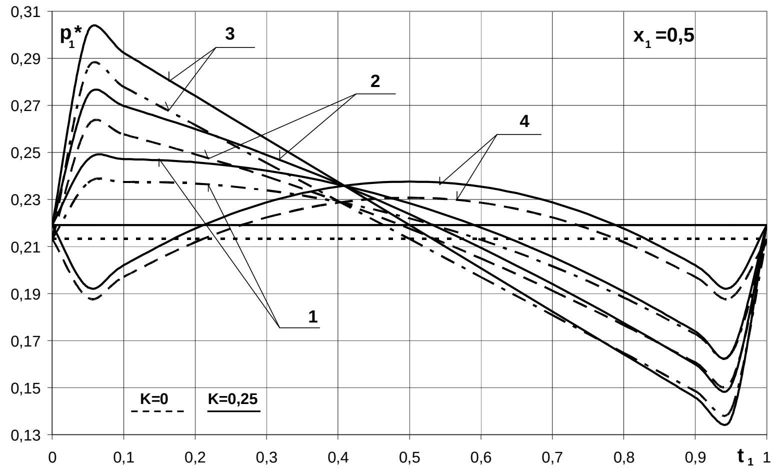

Fig.3 Pressure distributions p_1^* in place $x_1=0{,}5$ in the time t_1 by velocity perturbations: 1)$V_{10}=0{,}05$; $V_{1h}=0$; 2) $V_{10}=0{,}05$; $V_{1h}=0{,}025$;3) $V_{10}=0{,}05$; $V_{1h}=0{,}05$;4) $V_{10}=0{,}05$; $V_{1h}=-0{,}05$

convergence ε and is the maximum for optimal convergence. It apply also to stationary pressure increase in the lubricating gap. Further analysis of pressure distribution were made for gap with optima convergence ($\varepsilon=\varepsilon_{opt}$).

Total pressure distribution along the bearing gap and perturbation pressure distribution in the time function in the optional point on the race surface were analyzed. Numerical calculation results were presented by following longitudinal velocity perturbations: 1) $V_{10}=0{,}05$, $V_{1h}=0$; 2) $V_{10}=0{,}05$,

V_{1h}=0,025; 3) V_{10}=0,05, V_{1h}=0,05; 4) V_{10}=0,05, V_{1h}= -0,05. Unsteady pressure is changing at the time of velocity perturbation and its course is a function of time and its location along the bearing length. It is the temporary function of a period of velocity perturbation. Total pressure course p_1* in the point located in the half way of bearing length x_1=0,5 on the surface of the race in dimensionless time function is presented on the Fig. 3 for fourth different velocity perturbation. Steady pressure is marked with the misfiring line. When the velocity perturbation of oil on the race is harmonious with the slide velocity, the perturbation pressure increases. In the opposite situation it decreases and the drop is much higher than the rise. It lasts shorter than the half perturbation period. The opposite case is when velocity perturbation takes place on the slide. This case has not been presented on the figure. The periods of drop and rise of pressure are asymmetric in the case of different levels of velocity perturbation (Fig. 3). The level of velocity perturbation is higher for both options when the viscosity depends on the pressure.

4. Capacity forces

Hydrodynamic capacity force in the bearing comes from the hydrodynamic pressure integral on bearing surface slide. In dimensionless form:

$$W_1^* = \frac{W^*}{W_0} = \int_0^1 p_1^*(x_1)dx_1 \;; \qquad W_0 = bLp_0 \tag{14}$$

where:
W_0 – characteristic value of capacity force

Capacity load changes during the time of velocity perturbation. Capacity load change $\tilde{W}_{1K}$ is calculated as a difference between capacity in unsteady flow W_1^* and stationary flow W_{1K} :

$$\tilde{W}_{1K} = W_1^* - W_{1K} \;; \qquad W_{1K} = \int_0^1 p_{1K}(x_1)\,dx_1 \tag{15}$$

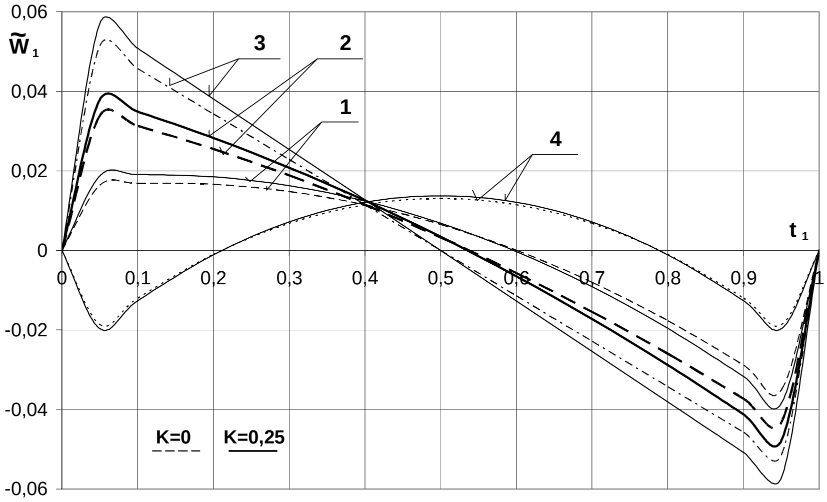

Fig. 4 The capacity forces $\tilde{W}_1$ of slide journal bearing in the time t_1 by velocity perturbations: 1) V_{10}=0,05; V_{1h}=0; 2) V_{10}=0,05; V_{1h}=0,025;3) V_{10}=0,05; V_{1h}=0,05;4) V_{10}=0,05; V_{1h}=-0,05

In case of lubricating oil flow with constant viscosity independent from pressure (K=0), the capacity load by stationary flow W_{10} is determined [2] by equation:

$$W_{10} = \frac{6}{(\varepsilon - 1)^2}\left(\ln\varepsilon - 2\frac{\varepsilon - 1}{\varepsilon + 1}\right) \tag{16}$$

Fig.4 presents hydrodynamic capacity change of plain bearing during the viscosity perturbation time for both considered perturbation alternatives. Capacity load change is similar to total pressure change resulting from Fig. 3. Capacity force decrease caused by velocity perturbations is greater than capacity force increase and it depends on perturbation variant.
Pressure perturbation and capacity load drop and is caused by appearance of counter flow velocity to the direction of stationary flow. In case of oil dynamic viscosity depends on pressure, capacity load by stationary condition is greater than by constant viscosity.

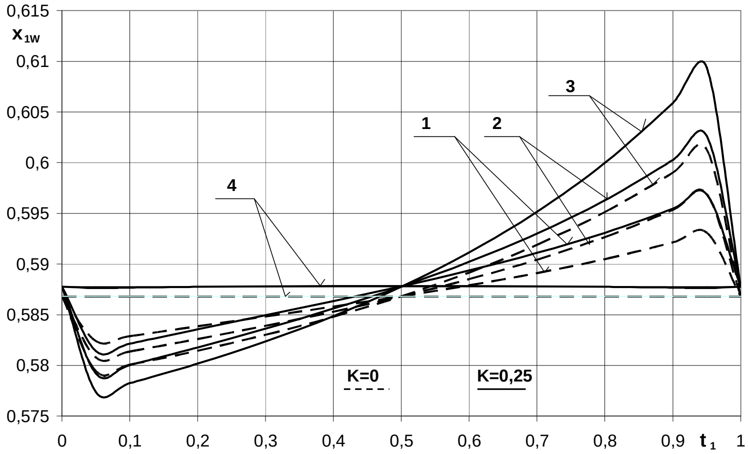

Fig. 5 Coordinate x_{1W} situated capacity force W_1^* in the time t_1 by perturbations:
1) V_{10}=0,05; V_{1h}=0; 2) V_{10}=0,05; V_{1h}=0,025;3) V_{10}=0,05; V_{1h}=0,05;4) V_{10}=0,05; V_{1h}=-0,05

In that case this lubricating oil quality causes that capacity load decrease as a result of velocity perturbation gives greater capacity load margin in both velocity perturbation cases. Capacity load position on the bearing lengthwise can be specified with coordinate – coordinate of centre elementary hydrodynamic pressure surface forces.

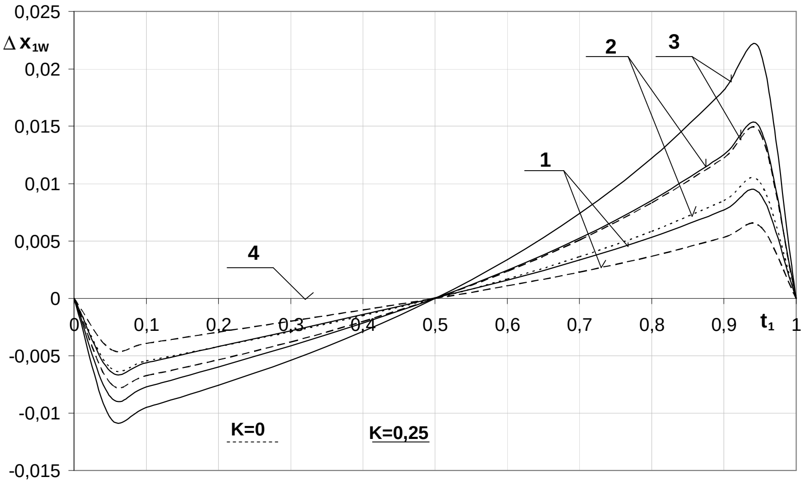

Fig. 6 Change of coordinate Δx_{1W} situated capacity force W_1^* in the time t_1 by perturbations:
1) V_{10}=0,05; V_{1h}=0; 2) V_{10}=0,05; V_{1h}=0,025;3) V_{10}=0,05; V_{1h}=0,05;4) V_{10}=0,05; V_{1h}=-0,05

This coordinate changes the position during velocity perturbation of oil flow. Capacity load diagrams for both considered velocity perturbation are presented on Fig. 5 and Fig. 6. Position change of capacity force is symmetrical in time, that is it crosses through stationary position by t_1=0,5 in spite of that capacity force is not achieving by stationary flow (Fig. 5). Capacity force shift is greater in the end of the bearing direction and occurs when the bearing load capacity decrease according to stationary flow. The capacity force coordinate by stationary flow, and all the coordinate changes in perturbation flow is greater in case when oil dynamic viscosity depends on pressure.

4. Conclusions

Discussed case of the solution to the Reynolds equation for the unsteady laminar Newtonian flow of lubricating factor allows initial estimation of hydrodynamic pressure distribution and its capacity as a basic operational parameter slide bearing. Unsteady axial velocity perturbation on the race surface and slide has influence on the hydrodynamic pressure distribution of the capacity of the lubricated gap. Pressure changes in the bearing are seasonal and equal to the lasting period of velocity perturbation. The level of changes and its nature depends on the kind of perturbation. The author bears in mind of the number of simplifying assumptions used in the presented model of bearing node and applying to the acceptance of Newtonian oil as well as examining isothermal model of bearing. The presented analytical example applies to the bearing of infinite length, however, the conclusions can be useful for the estimation of the pressure distribution and force with laminar, unsteady lubrication of slide nodes of the finished length. Presented results can be used as the comparative values in the case of numerical modeling of laminar, unsteady flows of liquids non-Newtonian in lubricating gaps slide journal bearings.

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