



## PRESSURE IN SLIDE JOURNAL BEARING LUBRICATED OIL WITH MICROPOLAR STRUCTURE

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### **Abstract**

*Present paper shows the results of numerical solution Reynolds equation for laminar, steady oil flow in slide bearing gap. Lubrication oil is fluid with micropolar structure. Properties of oil lubrication as of liquid with micropolar structure in comparison with Newtonian liquid, characterized are in respect of dynamic viscosity additionally dynamic couple viscosity and three dynamic rotation viscosity. Under regard of build structural element of liquid characterized is additionally microinertia coefficient. In modeling properties and structures of micropolar liquid one introduced dimensionless parameter with in terminal chance conversion micropolar liquid to Newtonian liquid. The results shown on diagrams of hydrodynamic pressure in dimensionless form in dependence on coupling number  $N^2$  and characteristic dimensionless length of micropolar fluid  $A_1$ . Presented calculations are limited to isothermal models of bearing with infinite length.*

**Keywords:** micropolar lubrication, journal bearing, hydrodynamic pressure

### **1. Introduction**

Presented article take into consideration the laminar, steady flow in the crosswise cylindrical slide bearing gap. Non-Newtonian fluid with the micropolar structure is a lubricating factor. Materials engineering and tribology development helps to introduce oils with the compound structure (together with micropolar structure) as a lubricating factors. Exploitation requirements incline designers to use special oil refining additives, to change viscosity properties. As a experimental studies shows, most of the refining lubricating fluids, can be included as fluids of non-Newtonian properties with microstructure [3],[4],[6]. They belong to a class of fluids with symmetric stress tensor that we shall call polar fluids, and include, as a special case, the well known Navier-Stokes model. Physically, the micropolar fluids may represent fluids consisting rigid randomly orientated spherical particles suspended in a viscous medium, where the deformation of fluid particles is neglected [3]. Presented work dynamic viscosity of isotropic micropolar fluid is characterized by five viscosities: shearing viscosity  $\eta$  (known at the Newtonian fluids), micropolar coupling viscosity  $\kappa$  and by three rotational viscosities  $\alpha$ ,  $\beta$ ,  $\gamma$  bounded with rotation around the coordinate axes. This kind of micropolar fluid viscosity characteristic is a result of essential compounds discussed in works [3] and [4]. Regarding of limited article capacity please read above works. In difference to classical oil with Newtonian properties, micropolar fluid is characterized by microinertia of the fluid part and by microrotation velocity field  $\vec{\Omega}$ . This fact determine the additional system of equation development describing micropolar fluid flow which is described by moment of momentum equation. In result of the above, the conjugation between fluid flow field and the microrotation velocity field. In presented flow, the

influence of lubricating fluid inertia force and the external elementary body force field were omit [3],[4].

## 2. Basic equations

Basic equation set defining isotropic micropolar fluid flow are describe following equations [2],[3],[4]: momentum equation, moment of momentum equation, energy equation, equation of flow continuity. Incompressible fluid flow is taken into consideration with constant density skipping the body force. We assume also, that dynamic viscosity coefficients which characterize micropolar fluid are constant. According to above velocity flow field is independent from temperature field and the momentum equation, moment of momentum equation and equation of flow continuity are part of closed system of motion equations. Equation of conservation momentum for above assumptions is:

$$\rho \frac{d\vec{V}}{dt} = -\text{grad } p + \kappa \text{rot}\vec{\Omega} + (\eta + \kappa) \text{rot}(\text{rot}\vec{V}) \quad (1)$$

Angular momentum equation in:

$$\rho J \frac{d\vec{\Omega}}{dt} = -2\kappa\vec{\Omega} + \kappa \text{rot}\vec{V} - \gamma \text{rot}(\text{rot}\vec{\Omega}) + (\alpha + \beta + \gamma) \text{grad}(\text{div}\vec{\Omega}) \quad (2)$$

Equation of flow continuity for incompressible fluid with constant density:

$$\text{div}\vec{V} = 0 \quad (3)$$

Above equation are derive from and described in details in [2],[3]. Further equation analysis were taken in rolling co-ordinate system, where the wrapping coordinate  $\varphi$  describes the wrapping angle of the bearing, the coordinate  $r$  describes radial direction from the journal to the bearing, the coordinate  $z$  describes longitudinal direction of crosswise bearing. In order to make the analysis of basic equations in dimensionless form [6], we input dimensionless quantities characterizing individual physical quantities. Oil velocity vector components are:

$$V_\varphi = UV_1 \quad V_r = \psi UV_2 \quad V_z = \frac{U}{L_1} V_3 \quad (4)$$

Reference pressure  $p_0$  caused by journal rotation with the angular velocity  $\omega$  was assumed in (7) taking into consideration dynamic viscosity of shearing  $\eta$  and the lubricating gap height  $h_1$  at the wrapping angle  $\varphi$  was taken in relative eccentricity function  $\lambda$  :

$$p_0 = \frac{\omega\eta}{\psi^2} \quad ; \quad h_1(\varphi, \lambda) = 1 + \lambda \cos\varphi \quad (5)$$

The constant viscosity of micropolar oil, independent from thermal and pressure condition in the bearing. Quantity of viscosity coefficient depend on shearing dynamic viscosity  $\eta$ , which is decisive viscosity in case of Newtonian fluids. Reference pressure  $p_0$  is also described with this viscosity, in order to compare micropolar oils results with Newtonian oil results. In micropolar oils decisive impact has quantity of dynamic coupling viscosity  $\kappa$  [1],[3]. In some works concerning bearing lubrication with micropolar oil, it's possible to find the sum of the viscosities as a micropolar dynamic viscosity efficiency. In presented article coupling viscosity was characterized with coupling number  $N^2$ , which is equal to zero for Newtonian oil:

$$N = \sqrt{\frac{\kappa}{\eta + \kappa}} \quad 0 \leq N < 1 \quad (6)$$

Quantity  $N^2$  in case of micropolar fluid, define a dynamic viscosity of coupling share in the oil dynamic viscosity efficiency. From the coupling number  $N^2$  we can determine both dynamic viscosity ratio, which is dimensionless micropolar coupling viscosity:

$$\kappa_1 = \frac{\kappa}{\eta} = \frac{N^2}{1 - N^2} \quad \kappa_1 \geq 0 \quad (7)$$

From the dynamic rotational viscosities  $\alpha$ ,  $\beta$ ,  $\gamma$  at the laminar lubrication, individual viscosities are compared to viscosity  $\gamma$ , which is known as the most important and its ratio to shearing viscosity  $\eta$  is bounded to characteristic flow length  $\Lambda$ , which in case of Newtonian flow assume the zero quantity. Dimensionless quantity of micropolar length  $\Lambda_1$  and micropolar length  $\Lambda$  are defined:

$$\Lambda = \sqrt{\frac{\gamma}{\eta}}; \quad \Lambda \Lambda_1 = \varepsilon \quad (8)$$

Dimensionless micropolar length  $\Lambda_1$  in case of Newtonian oil approach infinity. Equations (1),(2) and (3) after writing out in cylindrical coordinates are presented in article [2], where individual phases leading to Reynolds equation for laminar, stationary lubricating process in dimensionless form are mentioned.

### 3. Reynolds equation and hydrodynamic pressure

Reynolds equation for stationary flow of laminar micropolar fluid in the crosswise, cylindrical, slide bearing gap can be present [1],[2],[7] in dimensional form:

$$\frac{\partial}{\partial \varphi} \left( \frac{h^3}{\eta} \Phi(\Lambda, N, h) \frac{\partial p}{\partial \varphi} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\eta} \Phi(\Lambda, N, h) \frac{\partial p}{\partial z} \right) = 6 \frac{dh}{d\varphi} \quad (9)$$

$\Phi(\Lambda, N, h)$  function in form (10) when in case of the Newtonian fluid it has a value 1 and the Reynolds equation (9) change into a non-Newtonian fluid equation.

$$\Phi(\Lambda, N, h) = 1 + 12 \frac{\Lambda^2}{h^2} - 6 \frac{N\Lambda}{h} \coth \left( \frac{Nh}{2\Lambda} \right) \quad (10)$$

Reynolds equation (9) can be presented in dimensionless form [1],[7] using the method of changing into this values:

$$\frac{\partial}{\partial \varphi} \left( \Phi_1(\Lambda_1, N, h_1) \frac{\partial p_1}{\partial \varphi} \right) + \frac{1}{L_1^2} \frac{\partial}{\partial z_1} \left( \Phi_1(\Lambda_1, N, h_1) \frac{\partial p_1}{\partial z_1} \right) = 6 \frac{dh_1}{d\varphi} \quad (11)$$

$$\text{for} \quad 0 \leq \varphi \leq \varphi_k; \quad 0 \leq r_1 \leq h_1; \quad -1 \leq z_1 \leq 1$$

$$\text{where:} \quad \Phi_1 = h_1^3 + 12 \frac{h_1}{\Lambda_1^2} - 6 \frac{Nh_1^2}{\Lambda_1} \coth \left( \frac{h_1 N \Lambda_1}{2} \right) \quad (12)$$

Below solutions (11) for infinity length bearing is presented. In this solution the Reynolds boundary conditions, applying to zeroing of pressure at the beginning ( $\varphi=0$ ) and at the end ( $\varphi=\varphi_k$ ) of the oil film and zeroing of the pressure derivative on the wrapping angle at the end of the film where fulfill. The pressure distribution function in case of the micropolar lubrication has a form:

$$p_1(\varphi) = 6 \int_0^\varphi \frac{h_1 - h_{1k}}{\Phi_1(\Lambda_1, N, h_1)} d\varphi; \quad p_{1N}(\varphi) = 6 \int_0^\varphi \frac{h_1 - h_{1k}}{h_1^3} d\varphi \quad (13)$$

where:  $h_{1k} = h_1(\varphi_k)$  lubricating gap height at the end of the oil film.

In the boundary case of lubricating Newtonian fluid, pressure distribution function is a pressure  $p_{1N}(\varphi)$ . Example numerical calculation were made for the infinity length bearing with the relative eccentricity  $\lambda=0,6$ .

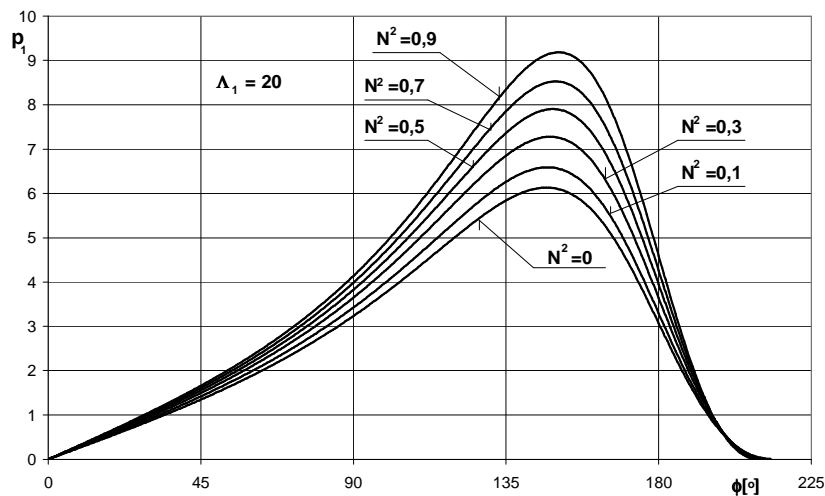


Fig.1 The dimensionless pressure distributions  $p_1$  in direction  $\varphi$  in dependence on coupling number  $N^2$  by micropolar ( $N^2 > 0$ ) and Newtonian ( $N^2 = 0$ ) lubrication for dimensionless eccentricity ratio  $\lambda = 0,6$  and characteristic dimensionless length of micropolar fluid  $\Lambda_1 = 20$

Analyzing the influence of coupling number  $N^2$  and the influence of dimensionless micropolar length  $\Lambda_1$  on hydrodynamic pressure distribution in the bearing liner circuital direction. At the Fig.1 pressure distribution for individual coupling numbers at constant micropolar length  $\Lambda_1 = 20$ . The pressure increase effect is caused by oil dynamic viscosity efficiency increase as a result of coupling viscosity  $\kappa$ . At  $N^2 = 0,5$ , coupling viscosity is equal to shearing viscosity. Pressure graph in the Fig.1 for micropolar oil lubrication ( $N^2 > 0$ ) find themselves above the pressure graph at the

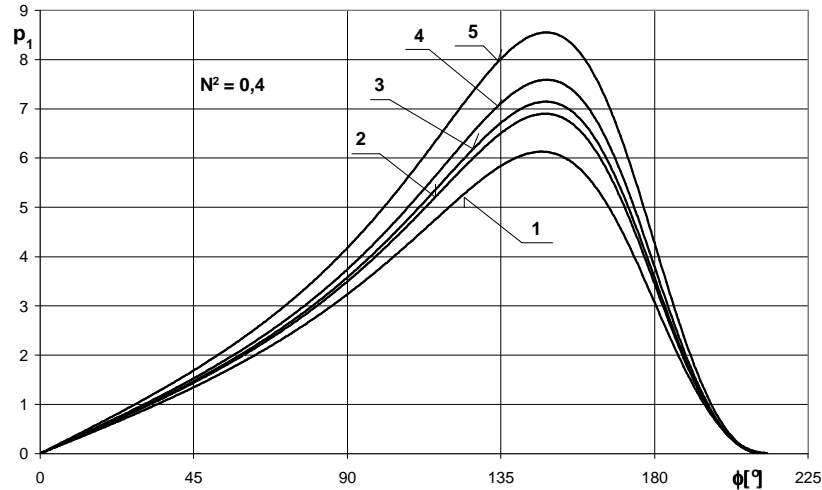


Fig.2 The dimensionless pressure distributions  $p_1$  in direction  $\varphi$  in dependence on characteristic dimensionless length of micropolar fluid  $\Lambda_1$ : 1) Newtonian oil, 2)  $\Lambda_1=40$ , 3)  $\Lambda_1=30$ , 4)  $\Lambda_1=20$ , 5)  $\Lambda_1=10$ , for dimensionless eccentricity ratio  $\lambda=0,6$  and coupling number  $N^2=0,4$

Newtonian oil lubrication ( $N^2=0$ ). Pressure distribution is higher for higher coupling number. It is caused by oil viscosity dynamic efficiency. In the Fig.2 the course of dimensionless pressure  $p_1$  for few micropolar length quantity  $\Lambda_1$  is shown: Decrease of this parameter determine the increase of micropolar oil rotational dynamic viscosity. Pressure distribution are presented at the constant coupling number  $N^2=0,4$ . Newtonian oil pressure in the course 1. Rotational viscosity increase determine the pressure distribution increase and is caused, because both the oil flow and microrotation velocities are coupled. Quantities of coupling number  $N^2$  and dimensionless micropolar length where taken from works [1],[2].

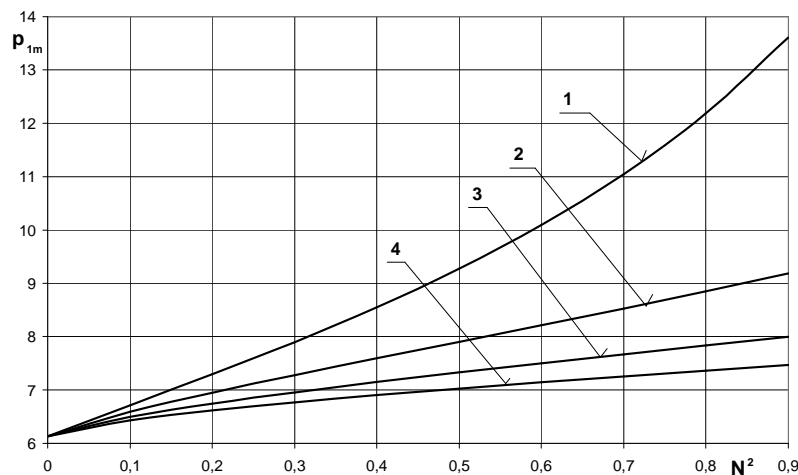


Fig.3 The dimensionless maximal pressure  $p_{1m}$  in dependence on coupling number  $N^2$  for characteristic dimensionless length of micropolar fluid  $\Lambda_1$ : 1)  $\Lambda_1=10$ , 2)  $\Lambda_1=20$ , 3)  $\Lambda_1=30$ , 4)  $\Lambda_1=40$

Based on given hydrodynamic pressure distribution  $p_1$  on wrapping angle of the bearing  $\varphi$ , the numerical quantities of maximal pressure  $p_{1m}$  and the angular coordinate  $\varphi_m$  (at the maximal position) were obtain. Quantities  $p_{1m}$  are presented in the Fig.3 in the coupling Number  $N^2$  function for chosen micropolar length  $\Lambda_1$ . All lines are coming out from the maximal pressure point in case of Newtonian fluid flow. We observe maximal pressure increase when the coupling number  $N^2$  increases ( coupling viscosity increases  $\kappa$ ) and the micropolar length decreases  $\Lambda_1$  ( rotational viscosity increases  $\gamma$ ) . Full range of coupling number change, that covers the range  $[0;1)$ , apply to coupling viscosity  $\kappa$  change from small to very high quantities. In most of the works, the hydrodynamic parameters of the bearing graphs are given in the function, which is

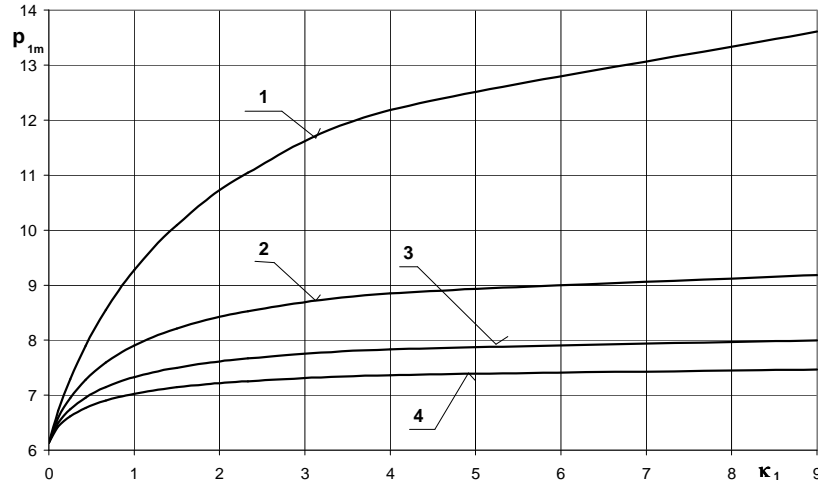


Fig.4 The dimensionless maximal pressure  $p_{1m}$  in dependence on dimensionless coupling viscosity  $\kappa_1$  for characteristic dimensionless length of micropolar fluid  $A_1$ : 1)  $A_1=10$ , 2)  $A_1=20$ , 3)  $A_1=30$ , 4)  $A_1=40$

nonlinear scale for coupling viscosity  $\kappa_1$ . In the Fig.4. the same graph is given in the dimensionless viscosity  $\kappa_1$  function. Change range  $N^2$  from the Fig.3 comply to  $\kappa_1$  changes in the Fig.4. Maximal pressure courses presented in the Fig.4, can be more suitable for small quantities for parameter  $\kappa_1$ . In the Fig.5 presented maximal pressure  $p_{1m}$  courses in the dimensionless micropolar length function  $\Lambda_1$  for a few coupling number  $N$  quantities.

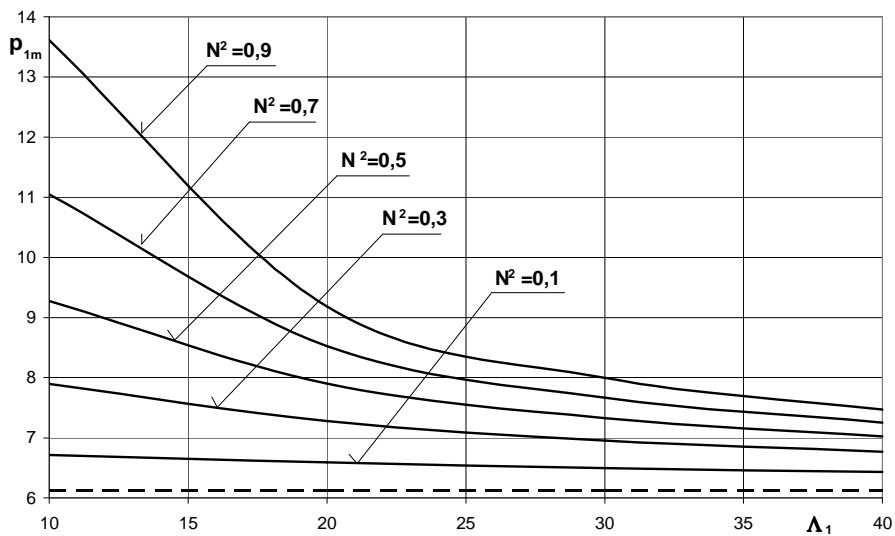


Fig.5 The dimensionless maximal pressure  $p_{1m}$  in dependence on characteristic dimensionless length of micropolar fluid  $\Lambda_1$  for coupling number  $N^2$  (---- Newtonian oil)

Broken line show the maximal pressure in case of Newtonian oil lubrication. All lines approach asymptotically to the broken line when the micropolar length increases (rotational viscosity decreases  $\gamma$ ). Together with coupling number increase, maximal pressure increases (coupling viscosity increases). Angular coordinate  $\varphi_m$  of maximal dimensionless pressure position  $p_{1m}$  in the square function of coupling number  $N^2$  for chosen micropolar length  $\Lambda_1$  were show in the Fig.6. All lines come out from the maximal pressure position point in case of Newtonian fluid flow. Increase of maximal pressure position angle  $\varphi_m$  is observed while the coupling number increases  $N^2$  (coupling viscosity increases  $\kappa$ ) and the micropolar length decreases  $\Lambda_1$  (rotational viscosity increases  $\gamma$ ). Graphs in the Fig.6 are described with nonlinear scale of dimensionless coupling viscosity  $\kappa_1$  change.

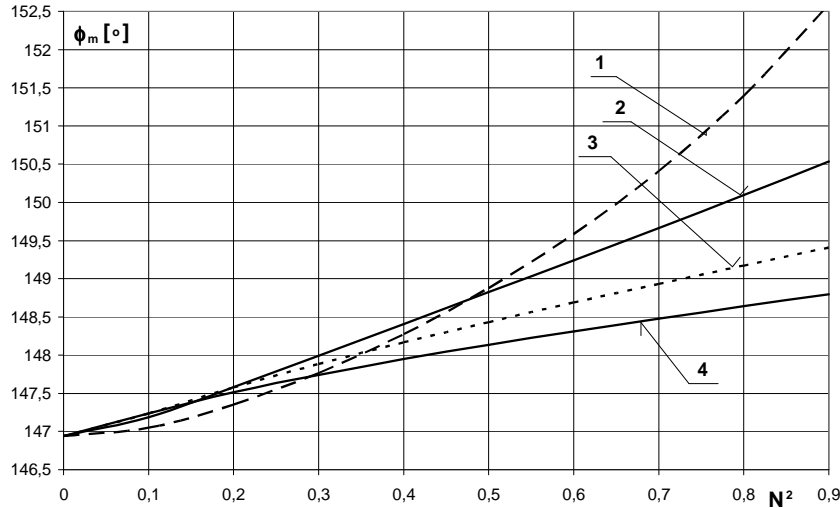


Fig.6 Angle  $\varphi_m$  situated maximal pressure  $p_{1m}$  in dependence on coupling number  $N^2$  for characteristic dimensionless length of micropolar fluid  $\Lambda_1$ : 1)  $\Lambda_1=10$ , 2)  $\Lambda_1=20$ , 3)  $\Lambda_1=30$ , 4)  $\Lambda_1=40$

In the Fig.7 the same graph is presented in the dimensionless coupling viscosity  $\kappa_1$  ( linear viscosity scale ).  $N^2$  change range from Fig.6 comply to change of  $\kappa_1$  in the Fig.7.

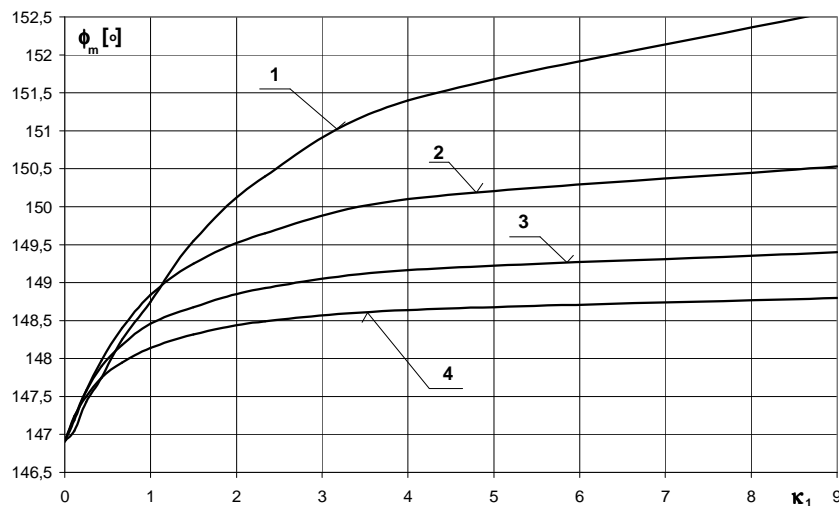


Fig.7 Angle  $\varphi_m$  situated maximal pressure  $p_{1m}$  in dependence on dimensionless coupling viscosity  $\kappa_1$  for characteristic dimensionless length of micropolar fluid  $\Lambda_1$ : 1)  $\Lambda_1=10$ , 2)  $\Lambda_1=20$ , 3)  $\Lambda_1=30$ , 4)  $\Lambda_1=40$

Author claims that angular coordinate  $\varphi_m$  courses of maximal pressure position  $p_{1m}$  presented in the Fig.7 are more suitable for smaller quantities of parameter  $\kappa_1$ .

#### 4. Conclusions

Presented example of the Reynolds equation solutions for steady laminar non-Newtonian lubricating oil flow with micropolar structure, enable the hydrodynamic pressure distribution introductory estimation as a basic exploitation parameter of slide bearing. Comparing Newtonian oil to oils with micropolar structure, can be used in order to increase hydrodynamic pressure and also to increase capacity load of bearing friction centre. Micropolar fluid usage has two sources of pressure increase in view of viscosity properties: increase of fluid efficient viscosity (coupling viscosity increase) and the rotational viscosity increase (characteristic length parameter  $\Lambda$ ). Author realize that he made few simplified assumptions in the above bearing centre model and in

the constant parameter characterizing oil viscosity properties. Despite this calculation example apply to bearing with infinity length, received results can be usable in estimation of pressure distribution and of capacity force at laminar, steady lubrication of cylindrical slide bearing with infinity length. Presented results can be usable as a comparison quantities in case of numerical model laminar, unsteady flow Non-Newtonian fluids in the lubricating gaps of crosswise cylindrical slide bearings.

## References

- [1] Das S., Guha S.K., Chattopadhyay A.K., *Linear stability analysis of hydrodynamic journal bearings under micropolar lubrication* - Tribology International 38 (2005), pp.500-507
- [2] Krasowski P., *Stacjonarny, laminarny przepływ mikropolarnego czynnika smarującego w szczelinie smarnej poprzecznego łożyska ślizgowego* - Zeszyty Naukowe nr 49, pp. 72-90, Akademia Morska, Gdynia 2003
- [3] Łukaszewicz G., *Micropolar Fluids. Theory and Applications* – Birkhäuser Boston 1999
- [4] Walicka A., *Reodynamika przepływu płynów nienewtonowskich w kanałach prostych i zakrzywionych* – Uniwersytet Zielonogórski, Zielona Góra 2002
- [5] Walicka A., *Inertia effects in the flow of a micropolar fluid in a slot between rotating sufrages of revolution* – International Journal of Mechanics and Engineering, 2001, vol.6, No. 3, pp. 731-790
- [6] Wierzcholski K., *Mathematical methods in hydrodynamic theory of lubrication*- Technical University Press, Szczecin 1993.
- [7] Xiao-Li Wang, Ke-Qin Zhu, *A study of the lubricating effectiveness of micropolar fluids in a dynamically loaded journal bearing* – Tribology International 37 (2004), pp.481-490

## Notation

- $L_1$  dimensionless bearing length  $L_1=b/R$
- $J$  microinertia constant ( $m^2$ )
- $N$  coupling number
- $R$  radius of the journal (m)
- $U$  peripheral journal velocity (m/s)  $U = \omega R$
- $V_i$  components of oil velocity in co-ordinate  $i = \varphi, r, z$  (m/s)
- $V_i$   $i=1,2,3$  dimensionless components of oil velocity in co-ordinate  $\varphi, r, z$
- $\Lambda$  characteristic length of micropolar fluid (m)
- $\Lambda_1$  dimensionless characteristic length of micropolar fluid
- $\Omega_i$  components of oil microrotation velocity in co-ordinate  $i = \varphi, r, z$  (1/s)
- $\Omega_i$   $i=1,2,3$  dimensionless components of oil microrotation velocity in co-ordinate  $\varphi, r, z$
- $b$  length of the journal (m)
- $h$  gap height (m)
- $h_1$  dimensionless gap height  $h = \epsilon h_1$
- $p$  hydrodynamic pressure (Pa)
- $p_0$  characteristic value of pressure (Pa)
- $p_1$  dimensionless hydrodynamic pressure  $p_1 = p/p_0$
- $r$  co-ordinate in radial of the journal (m)
- $t$  time (s)
- $z$  co-ordinate in length of the journal (m)
- $z_1$  dimensionless co-ordinate in length of the journal  $z_1 = z/b$
- $\alpha, \beta, \gamma$  micropolar rotational viscosities in co-ordinate  $\varphi, r, z$  ( $Pa \cdot s \cdot m^2$ )
- $\epsilon$  radial clearance (m)
- $\eta$  dynamic oil viscosity ( $Pa \cdot s$ )



- $\kappa$  micropolar coupling viscosity (Pa s)
- $\kappa_1$  dimensionless micropolar coupling viscosity
- $\lambda$  dimensionless eccentricity ratio
- $\rho$  oil density ( $\text{kg/m}^3$ )
- $\varphi$  the angular co-ordinate
- $\varphi_e$  the angular co-ordinate for the film end
- $\psi$  dimensionless radial clearance ( $10^{-4} \leq \psi \leq 10^{-3}$ )  $\psi = \varepsilon/R$
- $\omega$  angular journal velocity (1/s)