Abstract

Present paper shows the results of numerical solution Reynolds equation for laminar, steady oil flow in slide plane bearing gap. Lubrication oil is fluid with micropolar structure. Properties of oil lubrication as of liquid with micropolar structure in comparison with Newtonian liquid, characterized are in respect of dynamic viscosity additionally dynamic couple viscosity and three dynamic rotation viscosity. Under regard of build structural element of liquid characterized is additionally microinertia coefficient. In modeling properties and structures of micropolar liquid one introduced dimensionless parameter with in terminal chance conversion micropolar liquid to Newtonian liquid. The results shown on diagrams of hydrodynamic pressure in dimensionless form in dependence on coupling number $N_2$ and characteristic dimensionless length of micropolar fluid $\Lambda_1$. Presented calculations are limited to isothermal models of bearing with infinite breadth.

Keywords: micropolar lubrication, journal plane bearing, hydrodynamic pressure

1. Introduction

Presented article take into consideration the laminar, steady flow in the crosswise cylindrical slide plane bearing gap. Non-Newtonian fluid with the micropolar structure is a lubricating factor. Materials engineering and tribology development helps to introduce oils with the compound structure (together with micropolar structure) as a lubricating factors. Exploitation requirements incline designers to use special oil refining additives, to change viscosity properties. As a experimental studies shows, most of the refining lubricating fluids, can be included as fluids of non-Newtonian properties with microstructure [4-7]. They belong to a class of fluids with symmetric stress tensor that we shall call polar fluids, and include, as a special case, the well known Navier-Stokes model. Physically, the micropolar fluids may represent fluids consisting rigid randomly orientated spherical particles suspended in a viscous medium, where the deformation of fluid particles is neglected [4]. Presented work dynamic viscosity of isotropic micropolar fluid is characterized by five viscosities: shearing viscosity $\eta$ (known at the Newtonian fluids), micropolar coupling viscosity $\kappa$ and by three rotational viscosities bounded with rotation around the coordinate axes. This kind of micropolar fluid viscosity characteristic is a result of essential compounds discussed in works [4-6]. Regarding of limited article capacity please read
above works. In presented flow, the influence of lubricating fluid inertia force and the external elementary body force field were omit [3-6].

2. Reynolds equation

Basic equation set defining isotropic micropolar fluid flow are describe following equations [2-6]: momentum equation, moment of momentum equation, energy equation, equation of flow continuity. Incompressible fluid flow is taken into consideration with constant density skipping the body force. We assume also, that dynamic viscosity coefficients which characterize micropolar fluid are constant. According to above velocity flow field is independent from temperature field and the momentum equation, moment of momentum equation and equation of flow continuity are part of closed system of motion equations.

Lubricating gap is characterize by following geometric parameters: maximal gap height $h_o$, minimal gap height $h_e$, gap length $L$ and gap breadth $b$ (Fig. 1) model the following assumption were made: lubricating gap dimensions along it’s width of mating surfaces remain identical. Lubricating gap height after gap length was described in cartesian co-ordinate system by the following dimensionless form:

\[
h_1(x_1) = \varepsilon - (\varepsilon - 1)x_1 \quad \text{for} \quad 0 \leq x_1 \leq 1
\]

Dimensionless values [2, 3] that characterize lubricating gap are: length coordinate $x_1$, gap height coordinate $h_1$ and gap convergence coefficient $\varepsilon$:

\[
x_1 = \frac{x}{L}; \quad h_1 = \frac{h}{h_m}; \quad \varepsilon = \frac{h_e}{h_m}
\]

The constant viscosity of micropolar oil, independent from thermal and pressure condition in the bearing. Quantity of viscosity coefficient depend on shearing dynamic viscosity $\eta$, which is decisive viscosity in case of Newtonian fluids. Reference pressure $p_0$ is also described with this viscosity, in order to compare micropolar oils results with Newtonian oil results. In micropolar oils decisive impact has quantity of dynamic coupling viscosity $\kappa$ [1,4]. In some works concerning bearing lubrication with micropolar oil, it’s possible to find the sum of the viscosities as a micropolar dynamic viscosity efficiency. In presented article coupling viscosity was characterized with coupling number $N^2$, which is equal to zero for Newtonian oil:
Quantity $N^2$ in case of micropolar fluid, define a dynamic viscosity of coupling share in the oil
dynamic viscosity efficiency. From the coupling number $N^2$ we can determine both dynamic
viscosity ratio, which is dimensionless micropolar coupling viscosity:

$$\kappa_i = \frac{\kappa}{\eta} = \frac{N^2}{1 - N^2}, \quad \kappa_i \geq 0$$

(4)

From the dynamic rotational viscosities at the laminar lubrication, individual viscosities are
compared to viscosity $\gamma$, which is known as the most important and it ratio to shearing viscosity $\eta$
is bounded to characteristic flow length $\Lambda$, which in case of Newtonian flow assume the zero
quantity [8]. Dimensionless quantity of micropolar length $\Lambda_1$ and micropolar length $\Lambda$ are defined:

$$\Lambda = \sqrt{\frac{\gamma}{\eta}}, \quad \Lambda I = \varepsilon$$

(5)

Dimensionless micropolar length $\Lambda_1$ in case of Newtonian oil approach infinity.

Reynolds equation for stationary flow of laminar micropolar fluid in the lengthwise, plane
slide bearing gap can be present [1,2,7,8] in dimensional form:

$$\frac{\partial}{\partial x} \left( \frac{h^3}{\eta} \Phi(\Lambda, N, h) \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\eta} \Phi(\Lambda, N, h) \frac{\partial p}{\partial z} \right) = 6 \frac{dh}{dx}$$

(6)

$$\Phi(\Lambda, N, h) = 1 + 12 \frac{\Lambda^2}{h^2} - 6 \frac{N\Lambda}{h} \coth \left( \frac{Nh}{2\Lambda} \right)$$

(7)

Reynolds equation (6) can be presented in dimensionless form [1,7] using the method of changing
into this values:

$$\frac{\partial}{\partial x_1} \left( \Phi_1(\Lambda_1, N, h_1) \frac{\partial p_1}{\partial x_1} \right) + \frac{1}{L_1^2} \frac{\partial}{\partial z_1} \left( \Phi_1(\Lambda_1, N, h_1) \frac{\partial p_1}{\partial z_1} \right) = 6 \frac{dh_1}{dx_1}$$

(8)

for  $0 \leq x_1 \leq 1; \ 0 \leq y_1 \leq h_1; \ -1 \leq z_1 \leq 1$

where:  \[ \Phi_1 = h_1^3 + 12 \frac{h_1}{\Lambda^2} - 6 \frac{Nh_1^2}{\Lambda} \coth \left( \frac{Nh_1}{2\Lambda} \right) \]

(9)

The dimensionless values for pressure $p_1$, bearing breadth $L_1$ and remaining coordinates $y_1$ and $z_1$
are described as follows:

$$p = p_0 p_1, \quad L_1 = \frac{b}{L}, \quad z = b z_1, \quad y = h_0 y_1, \quad$$

(10)

Reference pressure $p_0$ caused by linear velocity $U$ of slide bearing was assumed in (11) taking
into consideration dynamic viscosity of shearing $\eta$ and the lubricating gap height $h_0$ in form:
\[ p_0 = \frac{U_n}{\psi L}, \quad \psi = \frac{h_e}{L} \]  

(11)

where:
\( \psi \) – relative clearance \((10^{-4} \leq \psi \leq 10^{-3})\)

3. Hydrodynamic pressure distribution

Below solutions (8) for infinity breadth bearing is presented. In this solution the Reynolds boundary conditions, applying to zeroing of pressure at the beginning \((x_1=0)\) and at the end \((x_1=1)\) of the oil film ended. The pressure distribution function in case of the micropolar lubrication has a form:

\[ p_1(x_1) = 6 \int_0^{x_1} \frac{h_1 - C_1}{\Phi_1(\Lambda_1, N, h_1)} \, dx_1 \quad ; \quad C_1 = \frac{\int_0^1 \frac{1}{\Phi_1} \, dx_1}{\int_0^1 \frac{1}{\Phi_1} \, dx_1} \]  

(12)

In the boundary case of lubricating Newtonian fluid, pressure distribution function is a pressure \(p_{1N}(x_1)\) and is in form:

\[ \lim_{N \to 0} \Phi_1 = h_1^3 \lim_{N \to 0} C_1 = 1 \quad \lim_{\Lambda_1 \to \infty} p_{1N}(x_1) = 6 \int_0^{x_1} \frac{h_1 - 1}{h_1^3} \, dx_1 \]  

(13)

where: \( p_{1N}(x_1) \) - pressure distribution for Newtonian oil

\[ p_{1N} = \frac{6(\varepsilon - 1)(1-x_1)x_1}{(\varepsilon + 1)(\varepsilon - \varepsilon x_1 + x_1)^2} \]  

(14)

Example numerical calculation were made for the infinity breadth bearing with convergence coefficient \( \varepsilon \): \( \varepsilon_{\text{opt}} = 1 + \sqrt{2} \) and \( \varepsilon = 1,4 \) marked continuous and discontinuous lines.

![Fig.2 The pressure distributions \( p_1 \) in direction \( x_1 \) in dependence on coupling number \( N^2 \) by micropolar \((N^2>0)\) and Newtonian \((N^2=0)\) lubrication for convergence coefficient \( \varepsilon_{\text{opt}} \) and \( \varepsilon = 1,4 \) from characteristic length of micropolar fluid \( \Lambda_1 = 20 \)](image-url)
Analyzing the influence of coupling number $N^2$ and the influence of dimensionless micropolar length $\Lambda_1$ on hydrodynamic pressure distribution in the bearing liner circuital direction. At the Fig.2 pressure distribution for individual coupling numbers at constant micropolar length $\Lambda_1 = 20$. The pressure increase effect is caused by oil dynamic viscosity efficiency increase as a result of coupling viscosity $\kappa$. At $N^2 = 0.5$, coupling viscosity is equal to shearing viscosity. Pressure graph in the Fig.2 for micropolar oil lubrication ($N^2 > 0$) find themselves above the pressure graph at the Newtonian oil lubrication ($N^2 = 0$). Pressure distribution is higher for higher coupling number. It is caused by oil viscosity dynamic efficiency. In the Fig.3 the course of dimensionless pressure $p_1$ for few micropolar length quantity $\Lambda_1$ is shown. Decrease of this parameter determine the increase of micropolar oil rotational dynamic viscosity. Pressure distribution are presented at the constant coupling number $N^2 = 0.4$. Rotational viscosity increase determine the pressure distribution increase and is caused, because both the oil flow and microrotation velocities are coupled. Quantities of coupling number $N^2$ and dimensionless micropolar length where taken from works [1,2].
Based on given hydrodynamic pressure distribution $p_1$ on longitudinal of the bearing $x_1$, the numerical quantities of maximal pressure $p_1m$ and the lengthwise coordinate $x_{1pm}$ (at the maximal position) were obtain. Quantities $p_1m$ are presented in the Fig.4 in the coupling Number $N^2$ function for chosen micropolar length $\Lambda_1$. All lines are coming out from the maximal pressure point in case of Newtonian fluid flow. We observe maximal pressure increase when the coupling number $N^2$ increases (coupling viscosity increases $\kappa$) and the micropolar length decreases $\Lambda_1$ (rotational viscosity increases $\gamma$). Full range of coupling number change, that covers the range [0;1), apply to coupling viscosity $\kappa$ change from small to very high quantities. In most of the works, the hydrodynamic parameters of the bearing graphs are given in the function, which is nonlinear scale for coupling viscosity $\kappa_1$. In the Fig.5, the same graph is given in the dimensionless viscosity $\kappa_1$ function. Change range $N^2$ from the Fig.4 comply to $\kappa_1$ changes in the Fig.5. Maximal pressure courses presented in the Fig.4, can be more suitable for small quantities for parameter $\kappa_1$. In the Fig.6 presented maximal pressure $p_{1m}$ courses in the dimensionless micropolar length function $\Lambda_1$ for a few coupling number $N$ quantities.
Broken line show the maximal pressure in case of Newtonian oil lubrication. All lines approach asymptotically to the broken line when the micropolar length increases (rotational viscosity decreases $\gamma$). Together with coupling number increase, maximal pressure increases (coupling viscosity increases). Change dimensionless coordinate $\Delta x_{1p}$ of maximal pressure position $p_{1m}$ in the square function of coupling number $N^2$ for chosen micropolar length $\Lambda_1$ were show in the Fig.7. and calculated in formula:

$$\Delta x_{1p} = x_{1pm} - x_{1pmN}$$  \hspace{1cm} (14)

where:

- $x_{1pm}$ - dimensionless coordinate $x_1$ maximal pressure $p_{1m}$
- $x_{1pmN}$ - dimensionless coordinate $x_1$ maximal pressure $p_{1m}$ for Newtonian oil flow

![Fig.7 Change coordinate $\Delta x_{1p}$ situated maximal pressure $p_{1m}$ in dependence on coupling number $N^2$ for dimensionless length of micropolar fluid $\Lambda_1$: 1) $\Lambda_1=10$, 2) $\Lambda_1=20$, 3) $\Lambda_1=30$, 4) $\Lambda_1=40$](image)

All lines come out from the maximal pressure position point in case of Newtonian fluid flow. Increase of maximal pressure change dimensionless coordinate position $\Delta x_{1p}$ is observed while the coupling number increases $N^2$ (coupling viscosity increases $\kappa$) and the micropolar length decreases $\Lambda_1$ (rotational viscosity increases $\gamma$). Graphs in the Fig.7 are described with nonlinear scale of dimensionless coupling viscosity $\kappa_1$ change.

### 4. Conclusions

Presented example of the Reynolds equation solutions for steady laminar non-Newtonian lubricating oil flow with micropolar structure, enable the hydrodynamic pressure distribution introductory estimation as a basic exploitation parameter of slide plane bearing. Comparing Newtonian oil to oils with micropolar structure, can be used in order to increase hydrodynamic pressure and also to increase capacity load of bearing friction centre. Micropolar fluid usage has two sources of pressure increase in view of viscosity properties: increase of fluid efficient viscosity (coupling viscosity increase) and the rotational viscosity increase (characteristic length parameter $\Lambda$). Author realize that he made few simplified assumptions in the above bearing centre model and in the constant parameter characterizing oil viscosity properties. Despite this calculation example apply to bearing with infinity breadth, received results can be usable in estimation of pressure distribution and of capacity force at laminar, steady lubrication of slide plane bearing with infinity breadth. Presented results can be usable as a comparison quantities in
case of numerical model laminar, unsteady flow Non-Newtonian fluids in the lubricating gaps of lengthwise slide plane bearings.

References