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A MODEL OF MARINE VESSELS MOVEMENT TO ESTIMATE HARMFUL COMPOUNDS IN THE VESSELS EXHAUSTS

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Abstract

Marine transport development and related to it, an increase in emission of harmful compounds forming when burning marine fuels, enforces conducting research on that emission in sea regions located in the vicinity of agglomeration areas. The basis for formulating the models of emission of harmful compounds in marine engines exhausts is formulation and study of the vessels movement models. The models ought to take into consideration the specificity of the processes taking place while a vessel is moving, which influence the formation of harmful compounds and their emission intensity. Emission models can be later used in formulating models of pollutants dispersion and their immission in the areas distant from the source of pollution, e.g. in city agglomeration regions. The paper indicates the possibility of using the Automatic Identification System as a source of statistical data, which can be used to define parameters and characteristics of the vessels movement. Identification of such a model was illustrated by the examples of the Gdansk Bay region.

Key words*: movement models, emission, toxic compounds in exhausts, marine engines*

1. Introduction

Emission of harmful compounds in engines of different applications constitutes a significant ecological problem of the 21st century. For a few decades there have been made many attempts to describe emission and its intensity with some specialised mathematical models [1-5]. When mobile sources of emissions are considered, majority of such models are formulated to conduct research on emission of motorization origin [1-3], both for road vehicles and off-road vehicles (mainly construction equipment and agricultural machinery). There exist also models of emission for aircraft vehicles (planes, helicopters). However, relatively few attempts are made to perform spatial modelling of pollutants emission distribution for the area where the source of these pollutants is the vessels movement. The problem of emission of pollutants in marine engines exhausts is all the more essential, as contrary to observed global land-based emissions decrease, it is projected that international shipping emissions will continuously be growing [6]. Such a situation results from dynamic development of marine transport – it is assumed that over 90 % of the world's total trade is carried by sea [7].

Movement models to estimate emission of pollutants from engines of motorization and aircraft origins, which have been formulated for many years, constitute a significant input into the development of modelling the pollutants emission from combustion engines. However, due to significant differences in topographic, hydrometeorological conditions and the specificity of vessels maintenance, they cannot be directly applied in estimation of emission of these pollutants in marine engines exhausts.

Existing inventories of distribution of a mean number of vessels in a given sea area in given period of time, although they give a general idea on the vessel traffic intensity in a given area, they are often too general and imprecise, especially in the cases when the research do not aim at determining the mean number of vessels in a given sea region, but aim at describing the trajectory of individual vessels operating in a given sea area at given time interval. Modelling should also take into consideration technical parameters of an individual vessel (type, load, power etc.), which would be helpful in modelling the emission and dispersion of toxic compounds in the vessel exhausts.

2. Deterministic model of the marine vessels movement

The length of the segment joining any two points $P_{i-1}, P_i, \quad i = 1,...,N$, of coordinates ($x_i$,$y_i$), between which a vessel moves:

$$|P_{i-1}P_i| = \sqrt{[x(t_i) - x(t_{i-1})]^2 + [y(t_i) - y(t_{i-1})]^2}\,, \tag{1}$$

and after transformations (applying the Lagrange's theorem about average value):

$$|P_{i-1}P_i| = \sqrt{[x'(\theta_i\,)]^2 + [y'(\vartheta_i)]^2}\,\Delta t_i\,, \tag{2}$$

where θ_i , ϑ_i are points from interval $[t_{i-1}, t_i)$, and $\Delta t_i = t_i - t_{i-1}$.

Analysis of emission and its intensity of the compounds in exhausts, should begin with determination of the randomly changing vessel speed at a given time *t*, since the emission intensity depends on this value. Vector stochastic process $\{\mathbf{V}(t) = (V_X(t), V_Y(t)): \quad t \in T\}$ describes the randomly changing vessel speed. At a given time *t*, this value is a two-dimensional random variable indicating a momentary speed vector. It should be noticed that between the random processes of vessel trajectory: $\{\mathbf{S}(t) = (X(t), Y(t)): \quad t \geq 0\}$ and speed $\{\mathbf{V}(t) = (V_X(t), V_Y(t)): \quad t \geq 0\}$ there exist the obvious relations:

$$\frac{d\mathbf{S}(t)}{dt} = \mathbf{V}(t), \quad t \geq 0, \qquad \mathbf{S}(t) = \int_0^t \mathbf{V}(x)dx, \quad t \geq 0 \tag{3}$$

From equation (3) there come the following formulas:

$$V_X(t) = \frac{dX(t)}{dt}, \qquad V_Y(t) = \frac{dY(t)}{dt} \tag{4}$$

Realisation of the process *V(t)* can be thus described by the following formulas:

$$v_X(t) = \frac{dx(t)}{dt}, \qquad v_Y(t) = \frac{dy(t)}{dt} \tag{5}$$

After transformations, the length of the speed vector is determined:

$$|\mathbf{v}(t)| = \sqrt{[v_X(t)]^2 + [v_Y(t)]^2} \tag{6}$$

In modelling the marine vessels movement, there can be assumed that in time intervals $[t_{i-1}, t_i), \quad i = 1,...,N$ the vessel speed is constant: $|\mathbf{v}(t)| = v_i, i = 1,...,N$, and therefore the intensity of their exhausts emission will be constant: $f(x(t), y(t)) = \lambda_i, \quad t \in [t_{i-1}, t_i), \quad i = 1,...,N$.

In reality, due to the yawing, the vessel route from point P_{i-1} to point P_i along a given trajectory, differs from the trajectory determined theoretically by drawing a line from point P_{i-1} to point P_i. The yawing can be written as:

$$\frac{\Delta S}{S} = \pi^2 \left(\frac{\psi_{om}}{v \cdot \tau} \right)^2$$

where: ψ_{om} – yawing amplitude; v – vessel speed; τ – period of dominant harmonic of yawing.

Therefore, due to yawing, the vessel route from point P_{i-1} to point P_i is longer than $\Delta s_i = v_i \Delta t_i$ by a random value $D_i,\quad i = 1,...,N$, and therefore the mass of exhausts emitted at this route interval will increase by a random value $\Delta m_i = \lambda_i D_i,\quad i = 1,...,N$. There can be accepted a preliminary hypothesis that the random variables $D_i,\quad i = 1,...,N$ are mutually independent and have Gaussian distributions $N(d_i, \sigma_i),\quad i = 1,...,N$, where $d_i = \varepsilon \Delta s_i,\quad \sigma_i = \rho \Delta s_i,\quad i = 1,...,N$. Factors $\varepsilon,\ \rho$ are the coefficients of the average route elongation and of the standard error of route elongation.

3. Stochastic model of vessels movement

Number of vessels operating in a given sea area varies in time, which constitutes therefore a stochastic process. Such a process is determined with a symbol $\{X(t): t \in T\}$. The value of this process at time instant t means the number of vessels operating in a given sea area at instant t. Realisations of this process are the functions of constant intervals. Any change of states appears at a moment of entering or leaving an analysed sea area by a vessel. The first approximated model of this process can be assumed the **non-homogenous Markov process** of discrete set of states $S = \{0,1,2,...\}$.

Stochastic process $\{X(t): t \in T\}$ of finite or denumerable set of states is called the *Markov process*, if for any $t_0, t_1, ..., t_n, t_{n+1} \in T$ such as $t_0 < t_1 < ... < t_n < t_{n+1}$ and any $i, j, i_0, i_1, ..., i_{n-1} \in S$,

$$P\{X(t_{n+1}) = j \mid X(t_n) = i,\ X(t_{n-1}) = i_{n-1}, ..., X(t_0) = i_0\} = P\{X(t_{n+1}) = j \mid X(t_n) = i\} \tag{7}$$

If $t_0, t_1, ..., t_{n-1}$ are interpreted as the past instants, t_n as the present instant, and t_{n+1} as the incoming instants (future), then the equation defining the Markov process means that conditional distribution of 'future' states does not depend on the 'past' if the present state of the process is known.

If $T = N_0 = \{0,1,2,...\}$, then the Markov process is called the *Markov chain*, and if $T = R_+ = [0, \infty)$, then it is the Markov process with 'continuous time'.

Conditional probabilities

$$P_{ij}(t_n, t_{n+1}) = P\{X(t_{n+1}) = j \mid X(t_n) = i\},\ i, j \in S \tag{8}$$

are called the probabilities of passing from state i at instant t_n to state j at instant t_{n+1}. Let $t_n = s,\ t_{n+1} = t$, where $0 \le s < t$. From the definition of Markov process there result the following corollaries:

1) $P_{ij}(s,t) \ge 0$ for all $i, j \in S$ and $s, t \in T,\quad s < t$.
2) $\sum_{j \in S} P_{ij}(s,t) = 1$ for all $i \in S$ and $s, t \in T,\quad s < t$.
3) $P_{ij}(s,t) = \sum_{k \in S} P_{ik}(s,u) P_{kj}(u,t)$ for all $i, j \in S$ and $s, u, t \in T,\quad s < u < t$, (Chapman-Kolmogorov equation).

Markov process is called ***homogenous***, if probabilities of passing depend only on the difference of instants (t-s): $P_{ij}(s,t) = P_{ij}(t-s)$. For $s = 0$, $P_{ij}(0,t) = P_{ij}(t)$.

Let $\{X(t): t \in [0,\infty)\}$ be *Markov process* of discrete set of states *S*. Functional matrix, whose elements are the probabilities of transition:

$$P_{ij}(s,t) = P\{X(t) = j \mid X(s) = i\},\ i, j \in S, \quad 0 \le s < t \tag{9}$$

will be designated as:

$$\mathbf{P}(s,t) = [P_{ij}(s,t) : i, j \in S]. \tag{10}$$

The Chapman-Kolmogorov equation in matrix notation will be as follows:

$$\mathbf{P}(s,t) = \mathbf{P}(s,u)\mathbf{P}(u,t) \tag{11}$$

It can be assumed that there exist uniform *t*–relative limits

$$\lambda_i(t) = \lim_{h\to 0} \frac{1 - P_{ii}(t,t+h)}{h}, \quad i \in S, \tag{12}$$

$$\lambda_{ij}(t) = \lim_{h\to 0} \frac{P_{ij}(t,t+h)}{h}, \quad i, j \in S \tag{13}$$

and they are continuous functions of parameter t.

For *homogenous Markov process*, the intensities of transitions are constant $\lambda_{ij}(t) = \lambda_{ij}$.

The Chapman-Kolmogorov equation can be written in the form of:

$$P_{ij}(s,t+h) = \sum_{k\in S} P_{ik}(s,t)P_{kj}(t,t+h), \text{ where } \quad i, j \in S \quad \text{and} \quad 0 \le s < t < t+h. \tag{14}$$

Subtracting from both sides of the equation $P_{ij}(s,t)$ and dividing by h, there is obtained:

$$\frac{P_{ij}(s,t+h) - P_{ij}(s,t)}{h} = \sum_{k\ne j} P_{ik}(s,t)\frac{P_{kj}(t,t+h)}{h} + P_{ij}(s,t)\frac{P_{jj}(t,t+h) - 1}{h}. \tag{15}$$

Passing to the limit at $h \to 0$, assuming that in case of Markov process of countable set of states, the convergence defining the intensities $\lambda_{kj}(t),\ \ k, j \in S$ at every determined j is uniform in relation to *k*, the following system of differential equations is obtained:

$$\frac{\partial P_{ij}(s,t)}{\partial t} = \sum_{k\in S} P_{ik}(s,t)\lambda_{kj}(t), \quad i, j \in S \tag{16}$$

The initial condition constitutes the natural equality:

$$P_{ij}(s,s) = \delta_{ij} = \begin{cases} 1 & \text{dla} \quad j = i \\ 0 & \text{dla} \quad j \ne i \end{cases} \tag{17}$$

In a similar way, the can be derived some differential equations allowing to calculate one-dimensional distribution of the process:

$$P_j(t) = P\{X(t) = j\}, \quad j \in S.$$

From the formula for total probability, it is obtained:

$$P_j(t+h) = \sum_{i\in S} P_i(t)P_{ij}(t,t+h), \quad j \in S \tag{18}$$

Subtracting from both sides $P_j(t)$, dividing by h and passing to the limit at $h \to 0$, the following system of equations is obtained:

$$P_j'(t) = \sum_{i\in S} P_i(t)\lambda_{ij}(t), \quad j \in S \tag{19}$$

The initial condition in this case constitutes the initial distribution of the process $P_i(0) = p_i^0, \quad i \in S$.

For homogenous Markov process, Kolmogorov system of equations (18) is brought to the system of linear differential equations of constant coefficients:

$$P_j'(t) = \sum_{i\in S} P_i(t)\lambda_{ij}, \quad j \in S \tag{20}$$

Such a system of differential equations is the most often solved using operators, applying Laplace's transformation:

$$L[P_i(t)] = \tilde{P}_i(s) = \int_0^{\infty} P_i(t)\, e^{-st}\, dt\,. \tag{21}$$

Making use of the property: $L[P_i'(t)] = s\,\tilde{P}_i(s) - P_i(0)$, the Laplace system of differential equations undergoes transformation. As a result, there is obtained the following system of linear equations, where unknown are the Laplace transforms: $\tilde{P}_i(s),\quad i \in S:\quad s\,\tilde{P}_j(s) - p_i^0 = \sum_{i\in S} \tilde{P}_i(s)\lambda_{ij},\quad j \in S$ [8].

One of limiting theorems for homogenous Markov processes with 'continuous time' at time $t \to \infty$ goes as follows:

Let $\{X(t): t \in [0,\infty)\}$ be a homogenous Markov process of finite set of states S and of the intensity matrix $\mathbf{\Lambda} = [\lambda_{ij} : i, j \in S]$. If there is such a real positive number t_0, that at least one column of the matrix $\mathbf{\Pi} = [\,p_{ij}(t_0) : i, j \in S]$ contains one column consisting of all the positive elements, then there exist limiting probabilities:

$$\lim_{t\to\infty} P_j(t) = \lim_{t\to\infty} P_{ij}(t) = P_j\,, \quad j \in S \tag{22}$$

and they meet the system of linear equations

$$\sum_{i\in S} P_i \lambda_{ij} = 0,\; j \in S, \qquad \sum_{j\in S} P_j = 1\,. \tag{23}$$

Therefore, the limiting distribution is determined by solving the system of linear equations, whose coefficients depend only on constants of the transition intensity.

Using ***non-homogenous Markov process*** as a model of the movement of objects navigating in a given sea area is possible it there is assumed that the Markov process $\{X(t): t \in T\}$, the value of which at instant t constitutes the number of vessels operating in a given sea area at instant t, is a single process, which means that in a short time interval the state can change by one. The matrix of the intensity of transitions [8]:

$$\mathbf{\Lambda}(t) = [\lambda_{ij}(t) : i, j \in S = \{0,1,2,...\}] \tag{24}$$

of this Markov process is as follows [8]:

$$\mathbf{\Lambda}(t) = \begin{bmatrix} -\alpha_0(t) & \alpha_0(t) & 0 & 0 & 0 & ... & 0 \\ \alpha_1(t) & -(\alpha_1(t)+\beta_1(t)) & \beta_1(t) & 0 & 0 & ... & 0 \\ 0 & \alpha_2(t) & -(\alpha_2(t)+\beta_2(t)) & \beta_2(t) & 0 & ... & 0 \\ 0 & 0 & \alpha_3(t) & -(\alpha_3(t)+\beta_3(t)) & \beta_3(t) & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... & ... \end{bmatrix} \tag{25}$$

Analytical solution of the system of the linear heterogeneous differential equations (18) in general form is practically impossible. However, it is possible to find an approximated solution, assuming that the elements of the matrix of the intensity of passages are constant in given time intervals and that the set of states of the process is finite $S = \{0,1,2,...,n\}$. Asymptotic distribution is received when solving the system of equations (23), in which given coefficients λ_{ij} are the elements of the matrix of the intensity of transition:

$$\boldsymbol{\Lambda}(t) = \begin{bmatrix} -\alpha_0 & \alpha_0 & 0 & 0 & 0 & \dots & 0 \\ \alpha_1 & -(\alpha_1+\beta_1) & \beta_1 & 0 & 0 & \dots & 0 \\ 0 & \alpha_2 & -(\alpha_2+\beta_2) & \beta_2 & 0 & \dots & 0 \\ 0 & 0 & \alpha_3 & -(\alpha_3+\beta_3) & \beta_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \alpha_{n-1} & -(\alpha_{n-1}+\beta_{n-1}) & \beta_{n-1} \\ \dots & \dots & \dots & \dots & \dots & \beta_n & -\beta_n \end{bmatrix} \tag{26}$$

The solution of this system of equations, having replaced the constants with the functional coefficients is of the following form [8]:

$$p_0(t) = \frac{1}{1 + \sum_{k=1}^{n} \frac{\alpha_0(t)\alpha_1(t)\dots\,\alpha_{k-1}(t)}{\beta_1(t)\beta_2(t)\dots\,\beta_k(t)}}$$
$$p_k(t) = \frac{\alpha_0(t)\alpha_1(t)\dots\,\alpha_{k-1}(t)}{\beta_1(t)\beta_2(t)\dots\,\beta_k(t)}\,p_0, \quad k = 1,2,\dots,n \tag{27}$$

The number n is the maximal number of vessels in the sea area.

If in a given time interval $[\alpha, \beta]$, the distribution of the number of vessels in the sea area is constant, then the number K can constitute the integer part of the expected value:

$$E[X(t)] = \sum_{k=0}^{n} k\, p_k \tag{28}$$

In order to realize the described above stochastic model of movement of the vessels operating in Gdansk Bay region, it is necessary to know the number of vessels operating in the analysed area, their distributions in relation to the vessels types, their size, speed, power and kind of main engines etc.

3. Estimation of some parameters of a model of emission of harmful compounds in marine vessels exhausts

Estimation of the intensity of movement of the vessels operating in a given sea area at particular time interval, and on its basis – determination of emission of harmful compounds in exhausts of the vessels engines, is possible with the use of data transmitted in the *Automatic Identification System* (AIS), registered by the shore installation, which traces the vessels movement in a given sea area. For the marine vessels passing through certain test segments (fig. 1), situated perpendicularly to shipping lanes approaching the ports of Gdynia and Gdansk, as well as the shipping lane splitting into those two shipping lanes (in the proximity of Hel port), there were analysed the movement parameters of the vessels operating in a given sea area at given time interval.

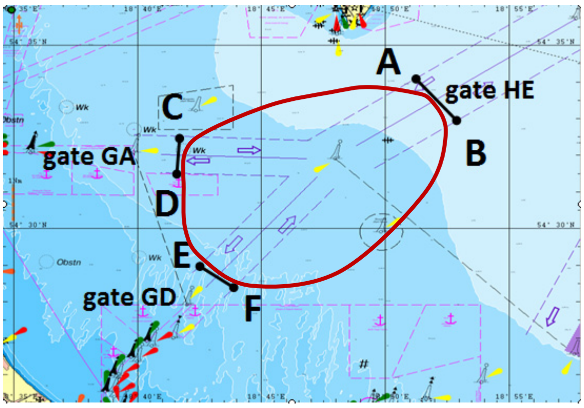

Fig. 1. Map of Gdansk Bay area where there are marked the test intervals [9] of geographic coordinates:
$\overline{AB}$ = (54,58325°N; 18,85362°E) (54,45917°N; 18,88191°E) – gate GD; $\overline{CD}$ = (54,54200°N; 18,69511°E) (54,52508°N; 18,69326°E) – gate GA; $\overline{EF}$ = (54,48666°N; 18,70878°E) (54,47326°N; 18,73178°E) – gate HE.

There were examined the vessels passing in the period of one month (from 1st to 30th June 2006) through the test intervals (called later: 'gates') shown in fig.1. In the analysed Gdansk Bay region (fig.1), the total number of vessels operating in June 2006 amounted to 627 (fig.2). On average, during 24 hours, to the analysed area through the gates there were entering from 3 to 8 vessels (in case of vessels entering the analysed area through the GD gate and GA gate), and from 9 to 14 vessels (in case of vessels entering the analysed area through the HE gate) (rys.3). Majority of these vessels were entering the analysed area in the afternoon (through gates GD and GA) and at time interval 00.00-12.00 (through gate HE) (fig.4).

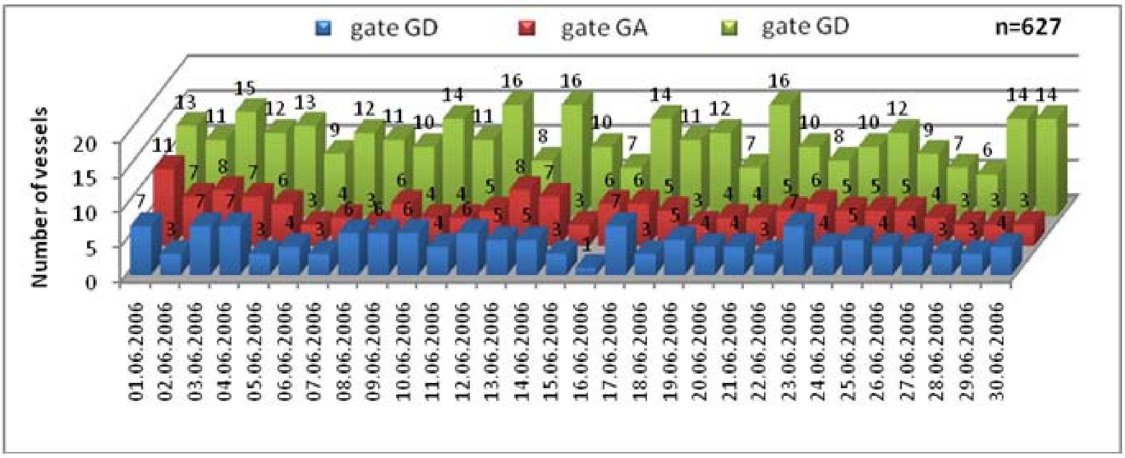

Fig. 2. A 24-hour distribution of the number of vessels: a) outward bound passing through 'gate GD'; b) outward bound passing through 'gate GA'; and c) passing through 'gate HE' and going towards of Gdansk port or Gdynia port (in the period of 01-30.06.2006)

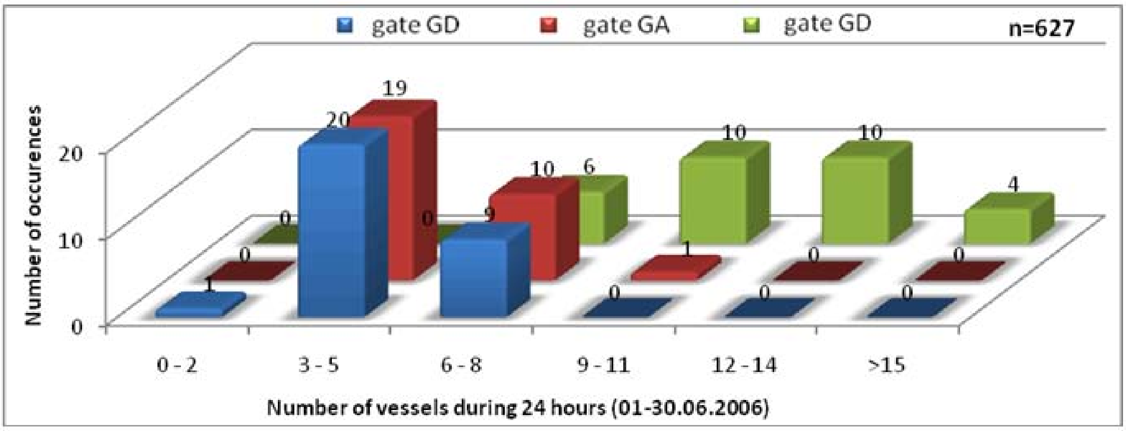

Fig. 3. A 24-hour distribution of the number of vessels: a) outward bound passing through 'gate GD'; b) outward bound passing through 'gate GA'; and c) passing through 'gate HE' and going towards of Gdansk port or Gdynia port (in the period of 01-30.06.2006)

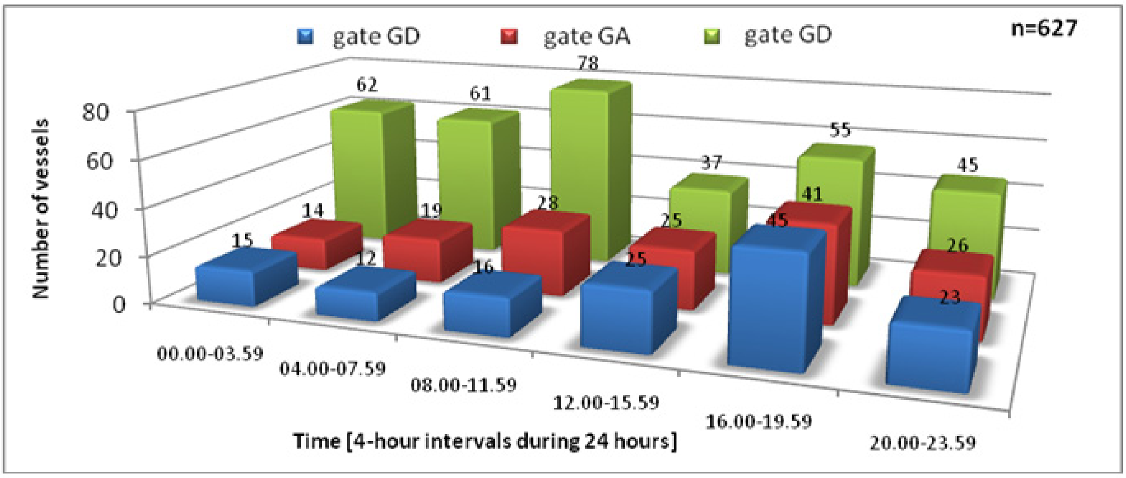

Fig. 4. Distribution of the number of vessels: a) outward bound passing through 'gate GD'; b) outward bound passing through 'gate GA'; and c) passing through 'gate HE' and going towards of Gdansk port or Gdynia port, depending on time intervals during 24 hours (in the period of 01-30.06.2006)

Analysis of the available statistical data allowed to determine values of momentary speed ***v**** of the vessels entering the analysed area in analysed period of time. As it can be noticed, the most frequent speed of the vessels entering the area through individual gates, was from 8 to 18 knots (fig.5).

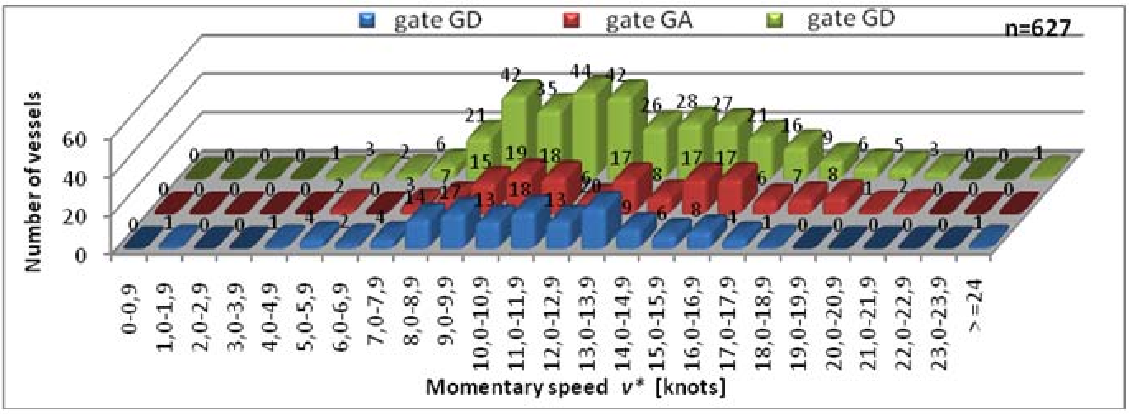

*Fig. 5. Distribution of the momentary speed **v*** (velocity over ground) [in knots] for vessels a) outward bound passing through 'gate GD'; b) outward bound passing through 'gate GA'; and c) passing through 'gate HE' and going towards of Gdansk port or Gdynia port (in the period of 01-30.06.2006)*

The volume of the underwater part of a hull ***V*** was calculated for the analysed group of vessels on the basis of assumed simplified shape of a hull [10]. On the basis of the obtained results, it can be stated that most of the analysed vessels were the ones relatively small – of underwater hull volume up to 25000 m^3 (fig. 6).

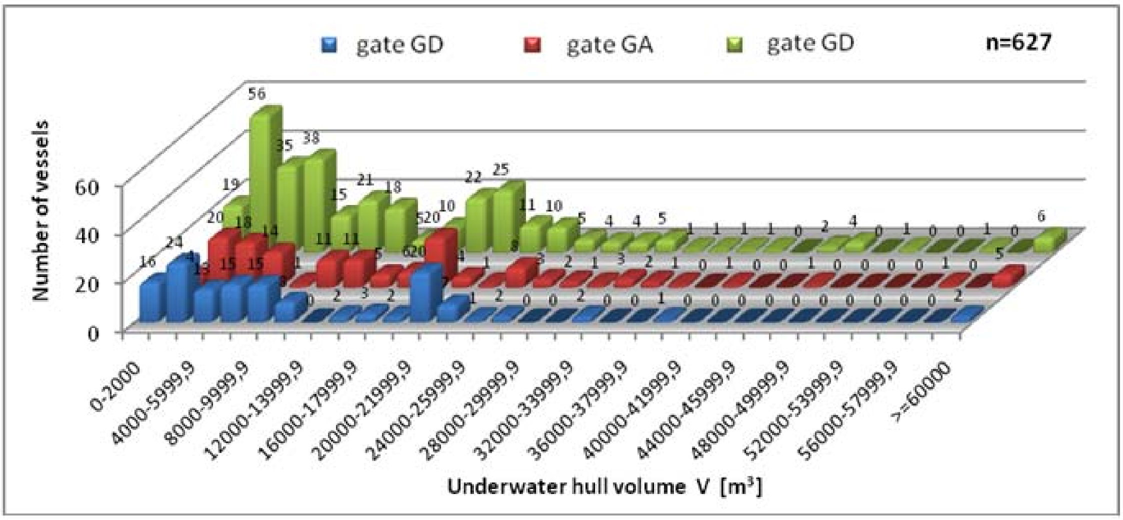

Fig. 6. Distribution of the underwater hull volume of the vessels a) outward bound passing through 'gate GD'; b) outward bound passing through 'gate GA'; and c) passing through 'gate HE' and going towards of Gdansk port or Gdynia port (in the period of 01-30.06.2006)

From the analysed group of vessels there were chosen the ones, which in the period of 01-30.06.2006 were passing through gates GD and GA and going towards gate HE (in total there were 289 vessels). On the basis of analysed change in momentary speed $\boldsymbol{v}^*$ of individual vessels, read when they were passing consecutive two gates, there was calculated the probability of the occurrence of the speed change. There were taken into account the changes of the range of 0.1 knots. In figures 7 and 8, it is shown the probability of occurrence of an increase and decrease in speed (by x-knots) on the route between individual gates. Function $\boldsymbol{y}$, described by a 4th order polynomial, means that probability, and $\boldsymbol{R^2}$ means the adjustment coefficient.

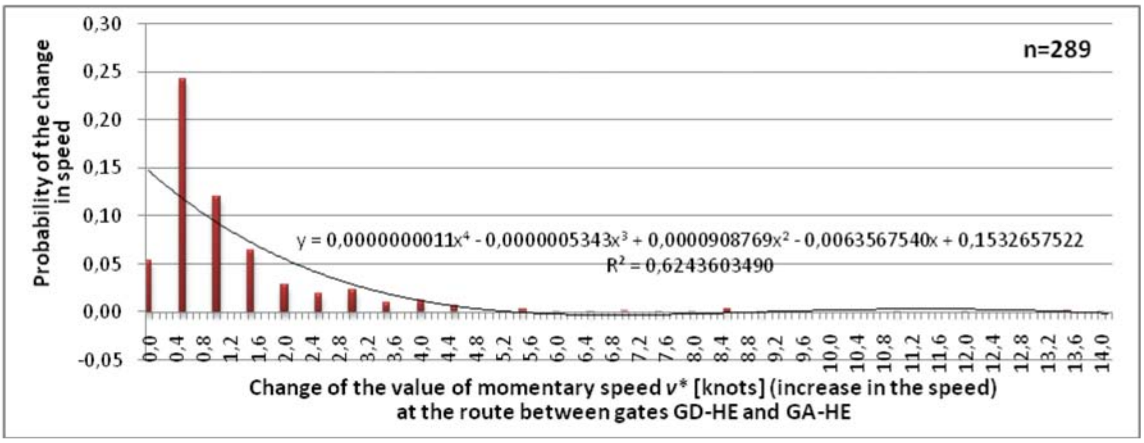

Fig. 7. Probability of occurrence of an increase in momentary speed (by x-knots) for the analysed group of vessels operating at the route between gates GD-HE and GA-HE (in the period of 01-30.06.2006)

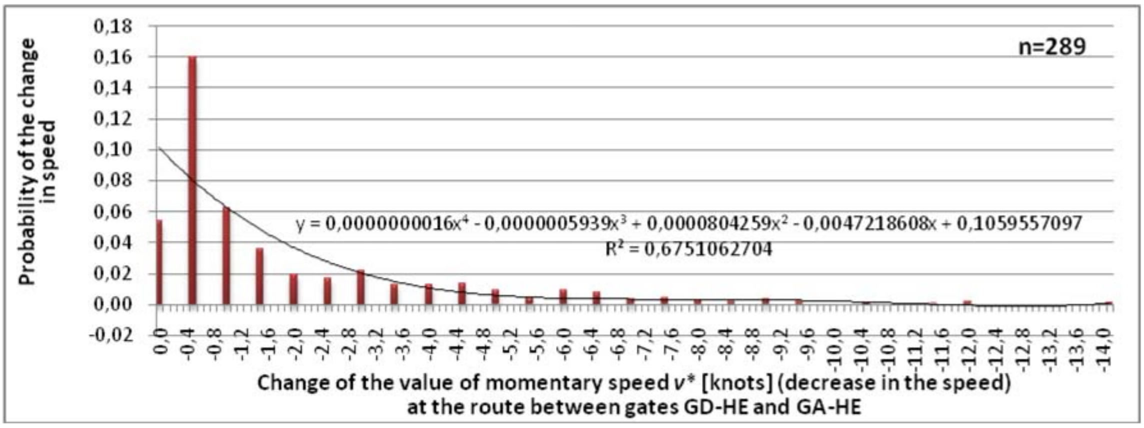

Fig. 7. Probability of occurrence of a decrease in momentary speed (by x-knots) for the analysed group of vessels operating at the route between gates GD-HE and GA-HE (in the period of 01-30.06.2006)

From the analysis of the charts shown in fig. 7 and fig. 8, it results that the changes in the vessels speed on the route from gate GD to gate HE and from gate GA to gate HE are slight, and the probability of occurrence of the changes in speed by more than one knot is very low (below 0.12).

4. Conclusion

In the paper, there were presented formulation and estimation of parameters of models of movement of the vessels operating in the Gdansk Bay region in a particular time period. The analysis was possible with the use of statistical data obtained via *Automatic Identification System* (AIS), which appeared to be a tool that can be effectively used in such purposes. The proposed method is possible to be applied in the process of determination of an approximated level of the toxic compounds emission in marine engines exhausts in given se region at particular time.

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