Journal of POLISH CIMAC
Faculty of Ocean Engineering & Ship Technology
GDAŃSK UNIVERSITY OF TECHNOLOGY

# RANDOMNESS OF ROLLING BEARINGS' ELEMENTS GEOMETRY AS PROBABILISTIC FACTOR OF BEARINGS FATIGUE LIFE

**Michał Styp-Rekowski**

*University of Technology and Life Sciences*
*al. Prof. S.Kaliskiego 7, 85-789 Bydgoszcz, Poland*
*tel.:+48 52 3408623, fax: +48 52 3408245*
*e-mail: msr@utp.edu.pl*

***Abstract***

*The great number of factors generate situations that real dimensions of each machine elements are different from nominal ones. In the case of rolling bearings elements mentioned disagreements are small – their values are rank of micrometers. However, such small differences can generate significant changes of operational features of bearings. The analysis of influence of ball dimensional deviation on load distribution in ball bearings is presented in this paper. The probability of occurrence of ball diameters of the range of acceptable values is statistically determined. Random distribution of this important dimensions results in other than nominal level of internal stresses (in the contact area of balls and raceways) and therefore true values of some operational features of bearings are other – the most often smaller – than calculated ones. In presented analysis sequence of occurrence of different ball diameters is also taken into account.*

***Key words***: *ball bearing, dimensional deviation, operational features of bearing, random character of balls diameter*

## 1. Preface

A lot of factors which determine each manufacturing process cause situation which has random character. In this connection real dimensions of every machine elements, including rolling bearings, are encumbered with machining dimensional deviations. Therefore, real dimension can differ from nominal in some value, however, not greater than boundary value for specific accuracy class of machining. Consequently, in case of rolling bearings the essential factor is dimensional selection of their elements because it determines inner load distribution in contact area of rolling elements and raceways. The more uniform distribution - the achievement of calculated durability of bearings close to the true value is more probable.

The statistical analysis of probability of existence in the bearings the balls' diameters of specified values is presented below. The results of analysis are necessary for exact analytical designation of fatigue life of the rolling bearings, e.g. Applied in combustion engines.

For standard rolling bearings analytical procedures are well-known and experimentally verified, instead for special bearings – this kind is analyzed in this paper – appears necessity of description new ones.

## 2. Load distribution inside bearing

The rolling bearings contain rolling elements (e.g.: balls, rollers, needles) of different diameters (of course in the range of acceptable deviations for specific bearings' accuracy class), just for that reason real load distribution, and thus also contact pressure between rolling elements and raceways, will be distorted in comparison to analytical – Fig. 1.

a) b)

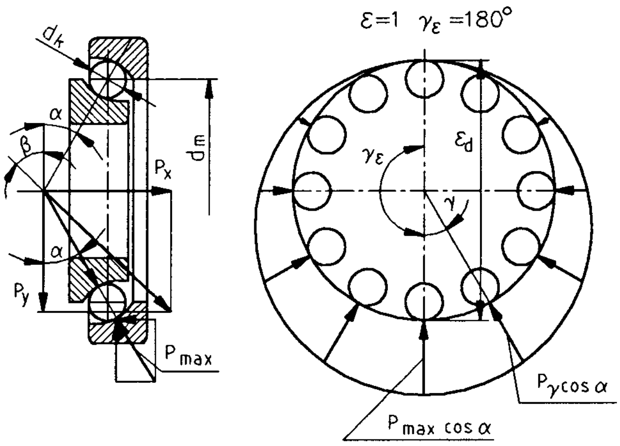


*Fig. 1.Theoretical load distribution in angular ball bearing: a) distribution of forces in contact area of balls and raceways, b) distribution of internal load of bearing circuit for determined load conditions*

The values of internal load existing in the rolling bearings and stresses in the contact area of balls and raceways one can calculate by well-known relationships, e.g. [2, 5]. For inner forces $P_\gamma$ it is the following equation:

$$P_\gamma = P_{\max}\left[1-\frac{1}{2\varepsilon}(1-\cos\gamma)\right]^{\frac{3}{2}}, \tag{1}$$

in which:

$$P_{\max} = \frac{P_y}{J_y(\varepsilon) z \sin\alpha}, \tag{2a}$$

or

$$P_{\max} = \frac{P_x}{J_x(\varepsilon) z \sin\alpha}, \tag{2b}$$

where:

$P_{max}$ - maximum internal force,
ε - load distribution factor,
$\gamma$ - position angle,
$P_{x,y}$ - load components, respectively: axial or radial,
$J_{x,y}$ - load integrals, respectively: axial or radial,
$z$ - number of the balls,
$\alpha$ - contact angle.

For calculation of the maximum contact stress value $\sigma_{max}$ the following relationship is used [5]:

$$\sigma_{\max} = \frac{858}{a* \cdot b*}\left[P_{\max}\left(\sum\rho\right)^2\right]^{\frac{1}{3}}, \tag{3}$$

where:

$a*$, $b*$ - dimensionless axis of contact elipse, respectively: semimajor or semiminor,

$\Sigma\rho$ - curvature sum.

In consideration of the fact that loads values are function among others of rolling bearings dimensions (especially contact area geometric features), calculated values of inner loads existing in the bearing, assigned to specific angles, one should admit as approximate values.

As a result of changed distribution, attitude towards nominal load distribution, and thereby other values of contact pressure, bearings' fatigue life can be different (most often smaller) because amplitude values of stresses in contact area of rolling elements and bearings' rings also are different (usually greater than nominal).

In order to settle quantitative determination of that influence, below the analysis is carried out. To this aim theory of probability is applied [1]. Analysis is conducted on the example of angular ball bearing applied in the front wheel hub of a bicycle.

## 3. Analysis of balls dimensions

The accuracy of rolling bearings is determined conjointly by geometrical constructional features of following bearings' elements:

- rolling elements (balls, rollers, needle rollers, barrels, cones).
- inner ring,
- outer ring,

In manufacturing consideration, conveniently is to assume that bearing element diameter (in analyzed example – ball) is redundant dimension. Its value is assorted to specific, measured values of raceways diameters of rolling bearings – on the outer and inner rings. Thanks to it, the proper value of clearance is obtained in the bearing.

In the all manufacturing processes surface geometric structure (SGS), with random and determined components, is generated on machined elements [7]. Therefore, true dimension is stochastic quantity – different from nominal dimension assumed by designer. For the aim of balls selection, in order to limit range of these differences, in standard [6] are created nine accuracy classes. In each class, values of boundary deviations of: diameter, geometrical tolerance and surfaces' roughness are determined. On account of diameters' spread in the inspection lot, in quoted standard the division of individual accuracy classes to dimensional groups and subgroups is suggested.

The rolling bearings are composed of finite, strictly determined number of rolling elements (balls) $z$. Therefore, the probability of existing of balls with determined diameters should be limited and refer to the lot number $z$. The distribution of diameter deviations in the lot of balls is presented in Fig. 2.

Individual quantities in the mentioned figure are defined in standard [6]. The way of their values delimitation is also described in this reference. In below presented considerations the most fundamental of them is $V_{DwL}$ – scatter of diameters in the lot. It is the difference between the greatest $D_{wmL\ max}$ and the smallest $D_{wmL\ min}$ average balls' diameter in the lot.

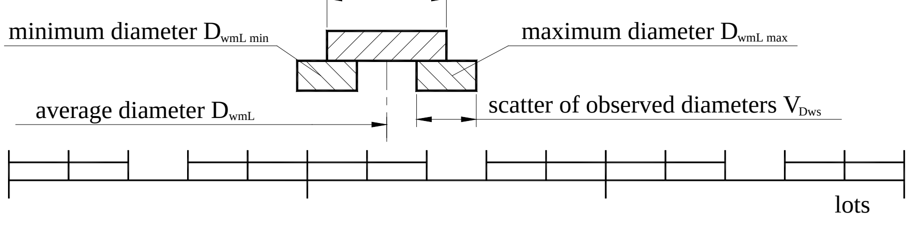


*Fig. 2. The distribution of bearings' balls dimensional deviations in inspection lot*

Particular attention should be paid to the fact that average ball diameter $D_{wm}$ is arithmetic average of the greatest and the smallest individual balls' diameters.

**4. Randomness of balls' diameters - statistical distribution**

Factor $X$, refers to true ball diameter, have Gaussian distribution with average value $m$ and standard deviation $s$. This fact may be written as follows:

$$X \sim N(m, s), \tag{4}$$

In the bearing exist the balls with the diameters in specific accuracy class, therefore new random variable $\tilde{X}$ is introduced. Its features limit the set above mentioned what can be note as follows:

$$\tilde{X} = \begin{cases} X & \text{for } |X - m| \le a \\ 0 & \text{for } |X - m| > a \end{cases}, \tag{5}$$

This variables set has Gaussian distribution, with two-sided section, limited by assumed deviations values $a$ of average balls diameters. This type of distribution is shown in Fig. 3.

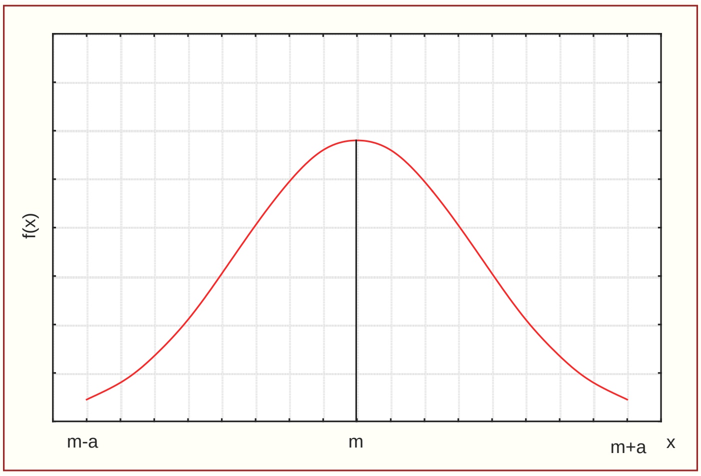


*Fig. 3. The graph of probability density function of random variable* $\tilde{X}$

The probability density function of analyzed factor $\tilde{X}$ has alternate form:

$$f_{\tilde{X}}(x) = \begin{cases} \dfrac{1}{cs\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2s^2}} & \text{for } |x - m| \le a \\ 0 & \text{for } |x - m| > a \end{cases}, \tag{6}$$

where:

$c$ – constant, calculate from equation:

$$c = 2\left[1 - \phi\left(\frac{a}{s}\right)\right], \tag{7}$$

in which

$\phi(x)$, for $x = a/s$ – distribution function of standard Gaussian distribution, which can be described by relationship:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}} du \, , \tag{8}$$

and
$u$ – working variable.

The distribution function of random variable $\tilde{X}$, in individual function intervals, assumes values in accordance with dependences mentioned below:

$$F(x) = \begin{cases} 0 & \text{if} \quad x \le m - a \\ \frac{1}{c}\left\{\phi\left(\frac{x-m}{s}\right) - \phi\left(-\frac{a}{s}\right)\right\} & \text{if} \quad m - a < x \le m + a \, , \\ 1 & \text{if} \quad x > m + a \end{cases} \tag{9}$$

Let $X_{1,n}$, $X_{2,n}$, ..., $X_{n,n}$ are ordinal statistics from random test ($X_1$, $X_2$, ..., $X_n$) arising from distribution with distribution function $F(x)$. The value of average *r-th* ordinal statistics $X_{r,n}$ one can assign from relationship:

$$\mathrm{EX}_{r,n} = n\binom{n-1}{r-1}\int_{m-a}^{m+a} x f(x)[F(x)]^{r-1}[1 - F(x)]^{n-r} dx \, , \tag{10}$$

In the case of analyzed real rolling bearing, parameters' values in equation (10) are following:

- $n = z = 9$,
- $r = 1, 2, ..., 9$.

The statistic assumed in presented analysis:

$$W_n = X_{n,n} - X_{1,n} \tag{11}$$

is called the range of the sample and it is difference between maximum and minimum average diameter in the sample – see Fig. 2. For example, in 40th accuracy class of the bearings, value of this quantity, according to standard [6], is equal ±16 μm. Average value of the range of the sample, for $n$ elements in the sample, can be calculated from relationship:

$$EW_n = \int_{-\infty}^{\infty} \{1 - F^n(x) - [1 - F(x)]^n\} dx \, , \tag{12}$$

Analytical solution of considered problem makes possible to illustrate the dependence of $EW_n$ on standard deviation $s$. This way can be determined which part of the range $\langle m\text{-}a, m\text{+}a \rangle$ is covered by ordinal statistics $X_{1,9}$, $X_{2,9}$, ..., $X_{9,9}$.

In this aim dependence of average value of range of sample $EW_n$ on standard deviation is evaluated for the whole lot. For better interpretation of computation's results, values of ratio $EW_n/2a$ are given on the $Y$-axis – Fig 4.

With the help of above described procedure, values of balls diameters are calculated, in assumption that machined balls are subjected to Gaussian distribution with two-sided section. In the calculations the true constructional features of tested bearings are taken as input data.

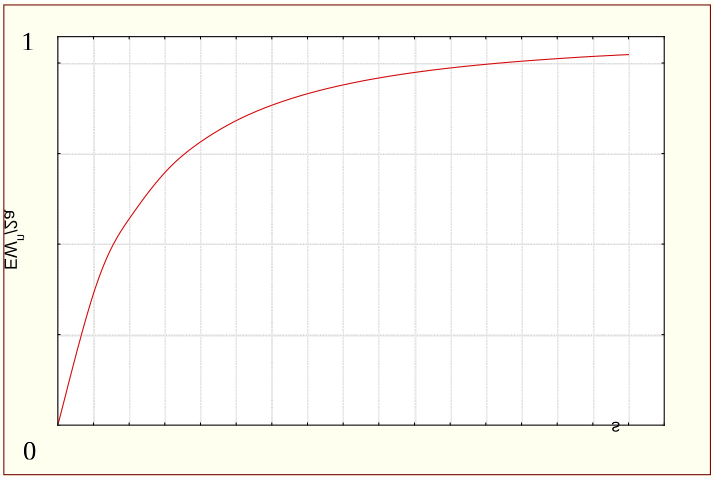


*Fig. 4. Dependence of $EW_n$ versus standard deviations (value and unit of s – adequate to basic size)*

As results of calculation the following values of balls diameters $d_k$ were obtained: 4.754, 4.757, 4.760, 4.762, 4.762, 4.762, 4.764, 4.767, 4.770 mm. They composed input set of quantities in bearings' fatigue life calculations.

## 5. Probabilistic analysis of bearings fatigue life

As earlier was found, differences between nominal and true diameters of balls in the bearings, which are contained inside the range of particular sample (in considered example the lot of analyzed size is $z$ = 9 and it is equivalent to balls number in the bearing), cause that inner loads distribution, shown in Fig. 1, is only theoretical. The true value of amplitude, as the result of such changing load, will be also other than theoretical. Thus, there are good reasons to state that analytical fatigue life will be different from determined on experimental way.

The sequence of the balls of differentiated diameters occuring in the bearing is also random factor. Differences of balls' diameters cause differentiation of contact stress - see equations (2) and (3), therefore, order of balls sequence – Fig. 5, generates changes of stress amplitude – quantity which decides about fatigue life of bearings.

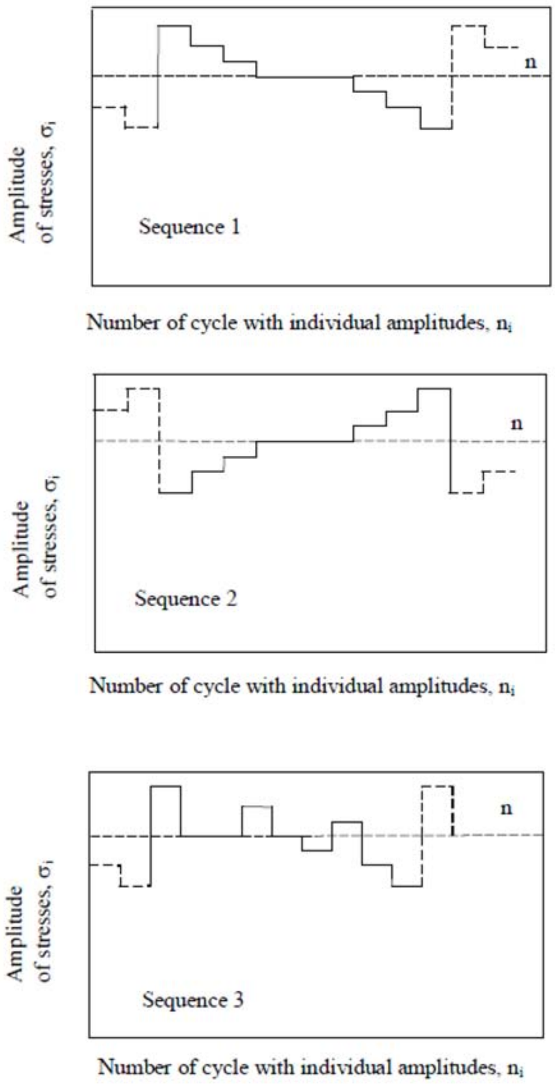


*Fig. 5. The examples of possible cycles of load in the contact ball-raceway caused by nominal dimension deviations of ball diameters: seq.1) decreasing sequence, seq.2) ascending sequence,seq.3) mixed sequence; n – nominal level of stresses' amplitude*

The above figures show that balls' sequence existing in the bearing directly determines run of load cycle, therefore, it can have significant meaning for fatigue life of analyzed rolling bearings [8, 9].

## 6. Closure

In the operational processes of all machines one observed that durability of correctly serviced bearings' pairs is different (usually greater) than determined on computational way. This is caused by analytical methods imperfection. That is why evolution of computational methods is observed, e.g. [3, 4, 10]. It appears in including increasing number of the factors, which determine durability of bearings.

Application of the above presented analysis in constructional practice should increase calculations accuracy and thereby, to induce better closing of analytical and practical results. This way the mathematical models, describing observed changes will be more correlated (congruent) to real run of researched phenomena.

## References


[1] Draper N.R., Smith H., *Applied Regression Analysis*, John Wiley & Sons Inc., New York – London - Sydney 1973.

[2] Harris T.A., *Rolling Bearing Analysis*, 3rd edition, John Wiley & Sons Inc., New York 1991.

[3] Ioannides E., *Life Prediction in Rolling Element Bearings*, in: *New Direction in Tribology*. Proceedings of Ist World Tribology Congress, pp. 281÷290, London 1997.

[4] Krzemiński Freda H., *Modern Methods of Calculation of Rolling Bearings Fatigue Life*, Tribologia No. 2, pp. 223÷232, 1993 (in Polish).

[5] Krzemiński Freda H., *Rolling Bearings*, PWN, Warsaw 1985 (in Polish).

[6] Oczoś K.E., Lubimov V., *Geometrical Structure of Surfaces*, Rzeszów Technical University Publishers, Rzeszów 2003 (in Polish).

[7] Polish standard PN-83/M-86452 – *Rolling Bearings. Balls*.

[8] Styp-Rekowski M., *Significance of Constructional Features for Angular Ball Bearings Life*, Scientific Number of Technical and Agricultural University, series Dissertations, No. 103, Bydgoszcz 2001 (in Polish).

[9] Szala J., *Assessment of Machines Elements Fatigue Life in Random and Programmable Load*, Scientific Number of Technical and Agricultural University, series Mechanics, No. 22, Bydgoszcz 1980 (in Polish).

[10] Warda B., *New Computational Model of Rolling Bearings Fatigue Life*, Proceedings of XVIIth Symposium on Fundamentals of Machine Design, pp. 265÷268, Lublin 1995 (in Polish).