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METHODS OF ESTIMATING THE AVAILABILITY OF VEHICLES

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Abstract
The article presents methods of estimating the availability of vehicles supported by numerical examples. All the cases concern the vehicles used In air base logistic system. Markov chains and processes hale been used to work out the mathematical model of their usage.

Key words: *availability, usage, service*

1. Introduction

The mathematical model has been built on the basis of analyzing states collection for vehicles used in air base logistic system. Each of the technical objects can be at certain time *t* in one of extinguished states, which make numerable (finite) set of states. The usage is understood as the movement of the vehicle on the extinguished states collection.

The model of vehicle usage as a random process $X(t)$ with finite set of states. $X(t)= S_i$ means that in time *t* the analyzed vehicle is in a state S_i. The realization of the process is meant as a sequence of extinguished states and their durations. The sequence, their durations and frequency are dependent first of all on work organization, the types of vehicles, the structure of subsystems collaborating in usage process [7].

2. The description of usage states for vehicles

Used vehicles can be in different states whose number is limited. Three models of emergency vehicles in air bases A, B and C of different states collection have been analyzed. Model A (Fig.1) does not include the system of vehicle restoration meant as the replacement of used vehicle with the new one (in working order).

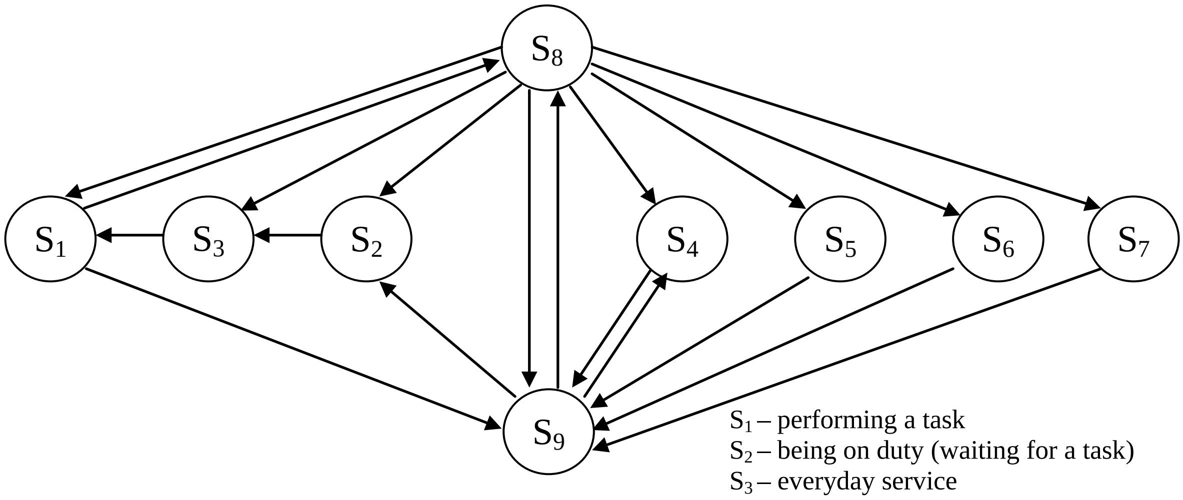


Fig.1. Transition graph of vehicle usage process – model „A”

S_1 – performing a task
S_2 – being on duty (waiting for a task)
S_3 – everyday service
S_4 - renovating
S_5 -1-st level service
S_6 - 2-nd level service
S_7 – periodic service
S_8 - vehicle supply
S_9 - diagnostics

Unknown theoretical probabilities of transition are estimated by empirical probabilities in a year time – transition frequencies for a vehicles as:

$$\omega_{ij} = n_{ij} / n_i \tag{1}$$

where:
$n_i = \sum_j n_{ij}$ - the number of transitions from S_i state;
n_{ij} – the numbers of transitions from state i to j in a period of time.

Stochastic process $X(t)$ in a continuous time is ergodic, if at least one positive boundary probability of finding a vehicle in state S_j for $t \to \infty$ exist, which is called ergodic probability p_j:

$$p_j = \lim_{t\to\infty} P(X(t \in <t; t+\Delta t>) = S_j); \quad p_j \geq 0; \ \sum_j p_j = 1 \tag{2}$$

where:
p_j – boundary probabilities;
$P(X(t \in <t; t+\Delta t>) = S_j)$ - the probability of being in state j in time interval $<t, t+\Delta t>$ for a vehicle.

Boundary (ergodic) probabilities fulfil standardizing conditions which means that at least one of them is positive. In relation to Markov processes it was proved [1, 2, 3] that if boundary probabilities exist, they can be calculated from boundary matrix in n steps $M_n = M_1^n$. In other words the linear equation or equivalent matrix equation must be solved, i.e. coming from continuous time t to discrete time n being of number of further experiment of observing the vehicle in time Δt:

$$\bigwedge_j p_j = \lim_{n\to\infty} p_{ij}(n) = \sum_i p_i p_{ij} \Leftrightarrow M_1^T [p_j] = [p_j], \ \text{przy} \ \sum_j p_j = 1 \tag{3}$$

where:
M_1^T - transposed transition matrix M_1;
$[p_j]$ – vector of ergodic probabilities;
p_{ij} – probability of transition from state i to state j.

Giving consideration to a condition: $M^T \cdot [p_j] = [p_j]$, and $\sum_j p_j = 1$, for A model boundary probabilities must be calculated by doing the following systems of equations shown in this matrix form:

$$\begin{bmatrix} 0 & 0 & p_{31} & 0 & 0 & 0 & 0 & p_{81} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{82} & p_{92} \\ 0 & p_{23} & 0 & 0 & 0 & 0 & 0 & p_{83} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{84} & p_{94} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{85} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{86} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{87} & p_{79} \\ p_{18} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{98} \\ p_{19} & 0 & 0 & p_{49} & p_{59} & p_{69} & p_{79} & p_{89} & 0 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \end{bmatrix} \quad (4)$$

$$\sum_{j=1}^{9} p_j = 1 \quad (5)$$

Model B of a vehicle maintenance description additionally takes into account the replacement state S_{10} (Fig.2), which is the reflection state with the same probability transitions S_{10} $(p_{910} = p_{109})$ and zero $p_{1010}=0$.

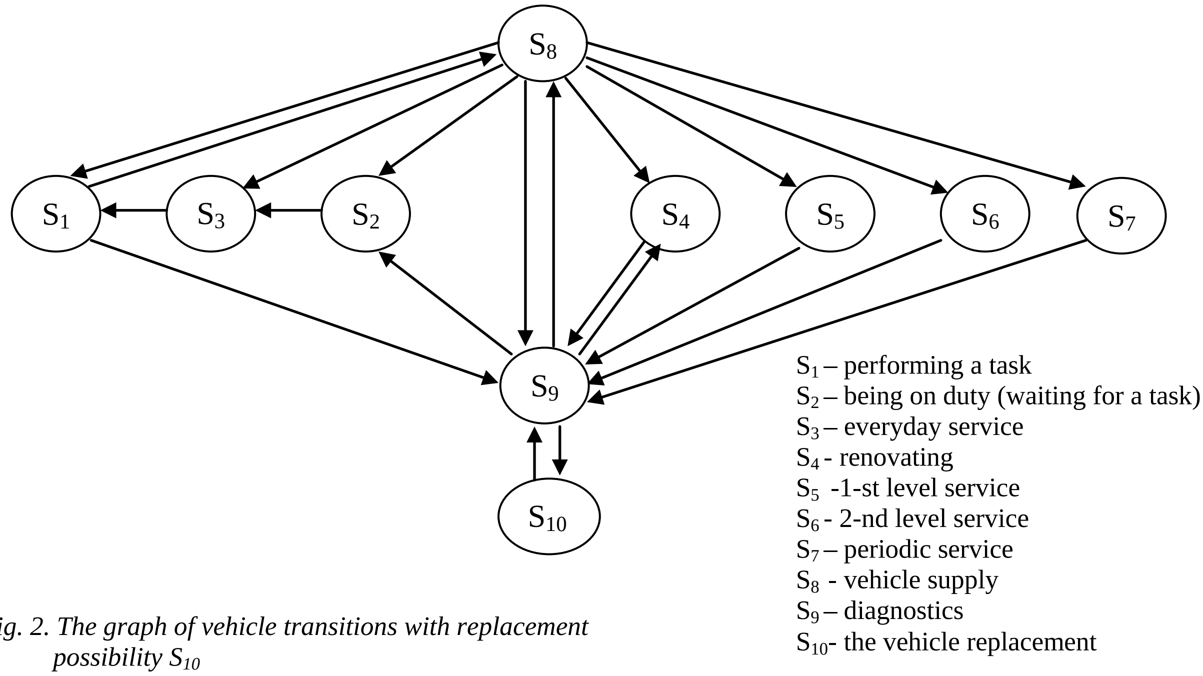


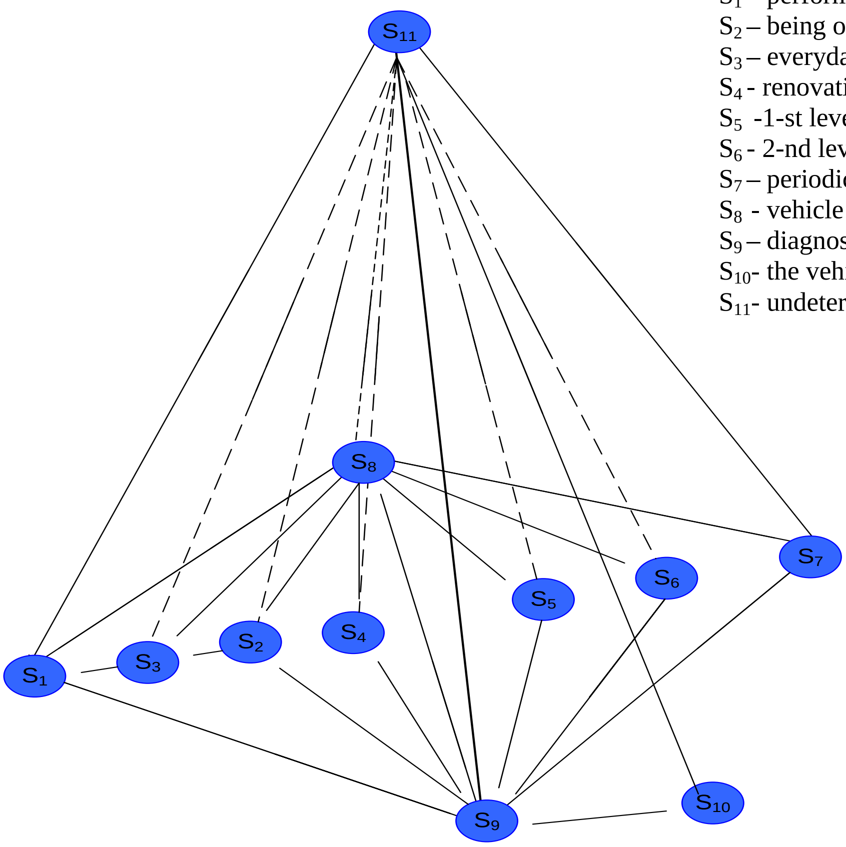
Fig. 2. The graph of vehicle transitions with replacement possibility S_{10}

Also for B model the following systems of equations for ergodic probabilities p_j have been estimated:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{18} & p_{19} & 0 \\ 0 & 0 & p_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{49} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{59} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{69} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{79} & 0 \\ p_{81} & p_{82} & p_{83} & p_{84} & p_{85} & p_{86} & p_{87} & 0 & p_{89} & 0 \\ 0 & p_{92} & 0 & p_{94} & 0 & 0 & 0 & p_{98} & 0 & p_{910} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{109} & 0 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \\ p_{10} \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \\ p_{10} \end{bmatrix} \tag{6}$$

$$\sum_{j=1}^{10} p_j = 1 \tag{7}$$

In model C undetermined (universal) state S_{11}, (Fig. 3) has been taken into account. It separates the states from S_1 to S_{10} and has zero probability of return *(*$p_{11,11}=0$; $p_{11,j}\geq 0$; $p_{i,11}\geq 0$ *for* $i,j\neq 11$*).* S_{11} state is meant as time loses of a vehicle being of each of the remaining states resulting from organizational reasons (technical etc.). in a real time S_{11} is parallel, but in the graph it has a terraced character.



S_1 – performing a task
S_2 – being on duty (waiting for a task)
S_3 – everyday service
S_4 - renovation
S_5 -1-st level service
S_6 - 2-nd level service
S_7 – periodic service
S_8 - vehicle supply
S_9 – diagnostics
S_{10}- the vehicle replacement
S_{11}- undetermined (universal) state

Fig.3. The graph of vehicle transitions. The vehicle renovation and replacement SA well as finite times of transition between states are covered

Ergodic probabilities p_j can be calculated in accordance with (3). Suitable systems of equations for C model have been written in matrix form:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{18} & p_{19} & 0 & p_{1,11} \\ 0 & 0 & p_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{2,11} \\ p_{31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{3,11} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{49} & 0 & p_{4,11} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{59} & 0 & p_{5,11} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{69} & 0 & p_{6,11} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{79} & 0 & p_{7,11} \\ p_{81} & p_{82} & p_{83} & p_{84} & p_{85} & p_{86} & p_{87} & 0 & p_{89} & 0 & p_{8,11} \\ 0 & p_{92} & 0 & p_{94} & 0 & 0 & 0 & p_{98} & 0 & p_{9,10} & p_{9,11} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{10,9} & 0 & p_{10,11} \\ p_{11,1} & p_{11,2} & p_{11,3} & p_{11,4} & p_{11,5} & p_{11,6} & p_{11,7} & p_{11,8} & p_{11,9} & p_{11,10} & 0 \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \\ p_{10} \\ p_{11} \end{bmatrix} = 0 \qquad (8)$$

$$\sum_{j=1}^{11} p_j = 1 \qquad (9)$$

After having solved the system of equations (8) i (9), taking into account the empiric probabilities for presented models A, B and C in discrete time the following results have been achieved (Fig. 4):

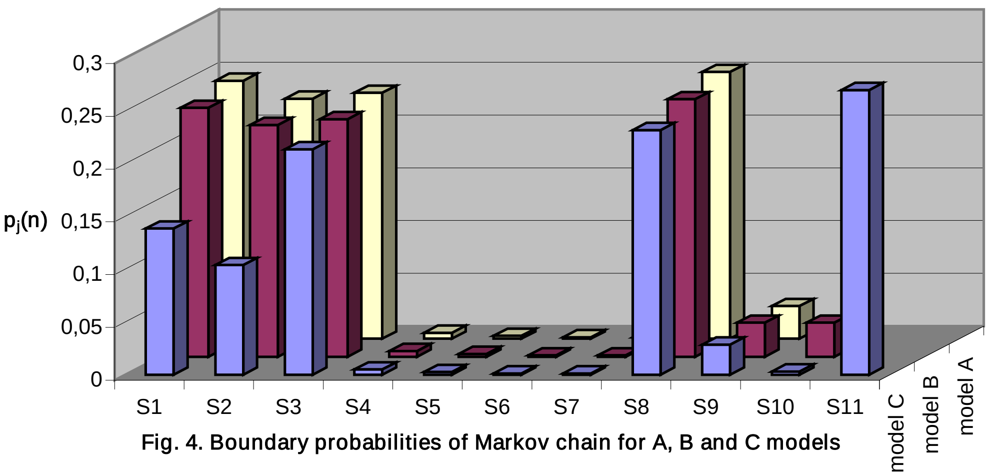


Fig. 4. Boundary probabilities of Markov chain for A, B and C models

Ergodic probabilities $p_j(t=n)$ for S_j state estimated from Markovs chains are the probabilities of input for each state. For example probability of duty $p_3=0,23$ means that the limit of duty state number is 23% of all vehicles states number in a considered period of time. It does not mean that the vehicle was on duty in 23% in average in T_o. However probabilities $p_j(n)$ for Markov chains are ergodic and applicable to state collection process but not a physical time. Therefore they cannot be interpreted in a real time. Only after the standardizing of the transition matrix, which means taking into account the duration time for each states (transitions from discrete times to real one by means of transition intensity λ_{ij}) achieved results will show the real situation of vehicle usage.

3. A method to determine functional availability of vehicles

To estimate ergodic probabilities for continuous time in compliance of Markov processes matrix equations are fulfilled [2]:

$$(\Lambda^T)[p_j] = 0 \tag{10}$$

and

$$\sum_j p_j = 1 \tag{11}$$

where:

$\Lambda=[\lambda_{ij}]$ – intensity matrix with diagonal elements $-\lambda_{ii}$ and λ_{ij}.

For A model the ergodic probabilities satisfy the equation system with its standardization condition (12):

$$\begin{bmatrix} -\lambda_{11} & 0 & \lambda_{31} & 0 & 0 & 0 & 0 & \lambda_{81} & 0 \\ 0 & -\lambda_{22} & 0 & 0 & 0 & 0 & 0 & \lambda_{82} & \lambda_{92} \\ 0 & \lambda_{23} & -\lambda_{33} & 0 & 0 & 0 & 0 & \lambda_{83} & 0 \\ 0 & 0 & 0 & -\lambda_{44} & 0 & 0 & 0 & \lambda_{84} & \lambda_{94} \\ 0 & 0 & 0 & 0 & -\lambda_{55} & 0 & 0 & \lambda_{85} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\lambda_{66} & 0 & \lambda_{86} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{77} & \lambda_{87} & 0 \\ \lambda_{18} & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{88} & \lambda_{98} \\ \lambda_{19} & 0 & 0 & \lambda_{49} & \lambda_{59} & \lambda_{69} & \lambda_{79} & \lambda_{89} & -\lambda_{99} \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \end{bmatrix} = 0$$

$$\sum_{j=1}^{9} p_j = 1 \tag{12}$$

For B model the ergodic probabilities satisfy the equation system with its standardization condition (13):

$$
\begin{bmatrix}
-\lambda_{11} & 0 & \lambda_{31} & 0 & 0 & 0 & 0 & \lambda_{81} & 0 & 0 \\
0 & -\lambda_{22} & 0 & 0 & 0 & 0 & 0 & \lambda_{82} & \lambda_{92} & 0 \\
0 & \lambda_{23} & -\lambda_{33} & 0 & 0 & 0 & 0 & \lambda_{83} & 0 & 0 \\
0 & 0 & 0 & -\lambda_{44} & 0 & 0 & 0 & \lambda_{84} & \lambda_{94} & 0 \\
0 & 0 & 0 & 0 & -\lambda_{55} & 0 & 0 & \lambda_{85} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\lambda_{66} & 0 & \lambda_{86} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{77} & \lambda_{87} & 0 & 0 \\
\lambda_{18} & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{88} & \lambda_{98} & 0 \\
\lambda_{19} & 0 & 0 & \lambda_{49} & \lambda_{59} & \lambda_{69} & \lambda_{79} & \lambda_{89} & -\lambda_{99} & \lambda_{109} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{910} & -\lambda_{1010}
\end{bmatrix}
\cdot
\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \\ p_{10} \end{bmatrix} = 0
$$

$$
\sum_{j=1}^{10} p_j = 1 \qquad (13)
$$

Also for C model equation systems in matrix form with standardized condition are fulfilled (14):

$$
\begin{bmatrix}
-\lambda_{11} & 0 & \lambda_{31} & 0 & 0 & 0 & 0 & \lambda_{81} & 0 & 0 & \lambda_{1,11} \\
0 & -\lambda_{22} & 0 & 0 & 0 & 0 & 0 & \lambda_{82} & \lambda_{92} & 0 & 0 \\
0 & \lambda_{23} & -\lambda_{33} & 0 & 0 & 0 & 0 & \lambda_{83} & 0 & 0 & \lambda_{3,11} \\
0 & 0 & 0 & -\lambda_{44} & & 0 & 0 & \lambda_{84} & \lambda_{94} & 0 & \lambda_{4,11} \\
0 & 0 & 0 & 0 & -\lambda_{55} & 0 & 0 & \lambda_{85} & 0 & 0 & \lambda_{5,11} \\
0 & 0 & 0 & 0 & 0 & -\lambda_{66} & 0 & \lambda_{86} & 0 & 0 & \lambda_{6,11} \\
0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{77} & \lambda_{87} & 0 & 0 & \lambda_{7,11} \\
\lambda_{18} & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_{88} & \lambda_{98} & 0 & \lambda_{8,11} \\
\lambda_{19} & 0 & 0 & \lambda_{49} & \lambda_{59} & \lambda_{69} & \lambda_{79} & \lambda_{89} & -\lambda_{99} & \lambda_{10,9} & \lambda_{9,11} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{9,10} & -\lambda_{10,10} & \lambda_{10,11} \\
\lambda_{11,1} & 0 & \lambda_{11,3} & \lambda_{11,4} & \lambda_{11,5} & \lambda_{11,6} & \lambda_{11,7} & \lambda_{11,8} & \lambda_{11,9} & \lambda_{11,10} & -\lambda_{11,11}
\end{bmatrix}
\cdot
\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \\ p_{10} \\ p_{11} \end{bmatrix} = 0
$$

$$
\sum_{j=1}^{11} p_j = 1 \qquad (14)
$$

After having solved the above-mentioned system of equations the boundary probabilities of the Markov process are obtained $p_j(t)$ for a vehicle (if they exist). If we put the transitions intensities to a system of equations (14) and are not able to solve it that means that the whole process is non-ergodic (not periodic, not regular). One thing we can do is simulation, which shows the irregularity of the process. Moreover it allows to estimate the level of non-ergodicity. Another way [4,5,7] is to calculate the availability according to the relationships (15):

$$K = \frac{\sum_{i=1}^{n} \overline{T}_G}{\sum_{i=1}^{n} \overline{T}_G + \sum_{i=1}^{n} \overline{T}_N} \tag{15}$$

where:

$\sum_{i=1}^{n} \overline{T}_G$ – total sum of medium times of staying a vehicle in availability;

$\sum_{i=1}^{n} \overline{T}_N$ – total sum of medium times of staying a vehicle in unavailability .

The medium time of staying a vehicle in a considered state collection could be estimated by taking into consideration historical events[6]. Empirical results for the vehicles used in logistic air base system have been presented in tables 1-3.

Tab. 1.Collection of data about medium number of entrance, medium transition probabilities, medium time of staying (hrs) and intensities of (1/year) S_1-S_{11} states for a vehicle – C model

State	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}
$\overline{n}_i$	214	161	330	8	4	2	2	358	45,656	2	416
$p_i(n)$	0,138	0,104	0,213	0,005	0,002	0,001	0,001	0,232	0,029	0,001	0,269
$\bar{t}_i$	869,3	5656,3	13,75	252	22	16	240	0,2265	11	240	1439,43
$\hat{\lambda}_i$	10,077	1,5487	637,09	34,762	398,18	547,50	36,50	38675,5	796,4	36,5	6,086

Tab. 2.Collection of data about medium number of entrance, medium transition probabilities, medium time of staying (hrs) and intensities of (1/year) S_1-S_{10} states for a vehicle – B model

State	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
$\overline{n}_i$	173	161	165	4	2	1	1	179	23,65	1
$p_i(n)$	0,243	0,226	0,232	0,005	0,002	0,001	0,001	0,251	0,033	0,001
$\bar{t}_i$	1159,3	5656,3	22	504	144	288	240	4,376	22	720
$\hat{\lambda}_i$	7,556	1,548	398,2	17,38	60,83	30,42	36,50	2001,8	398,2	12,2

Tab. 3.Collection of data about medium number of entrance, medium transition probabilities, medium time of staying (hrs) and intensities of (1/year) S_1-S_9 states for a vehicle – A model

State	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
$\overline{n}_i$	173	161	165	4	2	1	1	179	22
$p_i(n)$	0,244	0,227	0,233	0,005	0,002	0,001	0,001	0,252	0,031
$\bar{t}_i$	1159,33	6376,3	22	504	144	288	240	4,376	22
$\hat{\lambda}_i$	7,5561	1,3738	398,18	17,38	60,83	30,42	36,50	2001,83	398,18

Where:

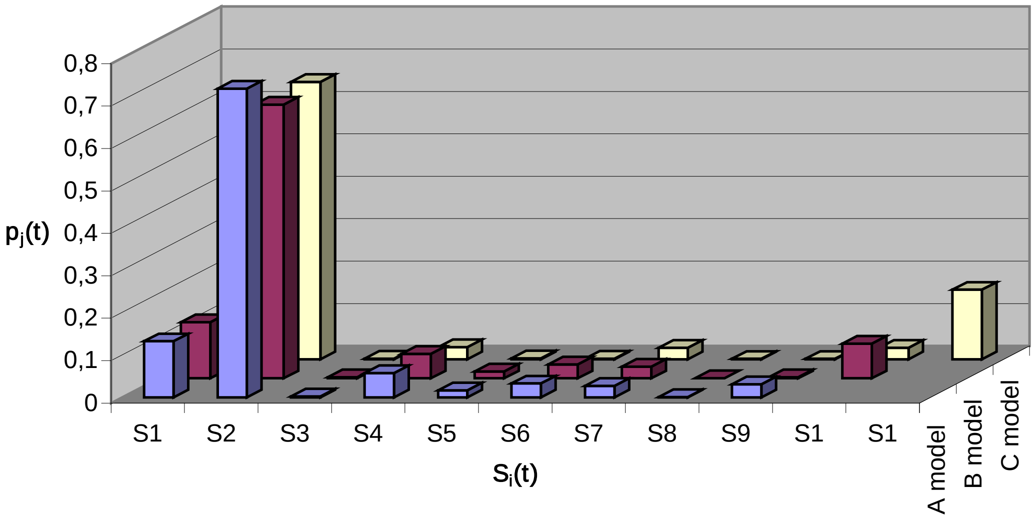
$\overline{n}_i$ – the average transition number from S_i in T_0 period;

$\overline{\varpi}_i$ – frequency S_i state in a considered collection state *1 – s (s = 9; 10 or 11)*;

$\bar{t}_i$ – the average duration time for S_i state before entrance to all S_j states in T_0 period;

$\hat{\lambda}_i$ - intensity of S_i state;

Mathematical dependencies for parameters mentioned above have been presented in [5] thesis.



Rys. 5. Probabilities $p_j(t)$ of staying in availability for vehicles

Fig. 5 describes probabilities staying in availability $p_j(t)$ for vehicles in a considered S_i states, which are calculated as:

$p_i = \frac{\bar{t}_i}{T_0}$, while availability has been calculated according to (15) relationship;

where:

$\bar{T}_G = \bar{t}_1 + \bar{t}_2$

$\bar{T}_N = \sum_{i=3}^{S} \bar{t}_i$ for $S=1\text{-}9$ for A model, $S=1\text{-}10$ for B model, $S=1\text{-}11$ for C model.

4. Conclusions

Estimated annual average sum of tenses $\bar{t}_1 + \bar{t}_2$ for vehicles population in availability is from 6526 hrs (realistic C model) to 7535,63 hrs (optimistic A model – table 3), it means that the availability is from 0,745 to 0,86 in T_0 period.

The availability of vehicle population is high, but there is also time margin in a year $t_{11} = 1439{,}43$ hrs (model C), resulting mainly from time losses during replacement S_{10} and renovating S_4 of the vehicles (tab.1-3). It enables to improve the availability up to 16%. Therefore model C with S_{11} state is justify and realistic. Withdrawal from usage decrease the number of vehicle population on average 7% in a year, but the replacement time is usually longer than one month. S_{10} state cannot be omitted because it significantly changes the duration time balance for all states of the vehicles. Models B i C with S_{10} state reflect the simulation of renovating and replacement of the population in long periods of time.

All presented models are stable and their non-ergodicity in real time is only according the simulation 0,2-1% for availability state after 3 years. The reason for their non-ergodicity can be vast (broad) spectrum of duration times for considered states collection.

However Markov chains for A, B and C models are ergodic, but boundary probabilities p_j concern collection states of the process and they cannot be interpreted in real (physical) time.

It has been illustrated by the proportion of the estimated real time of refuelling max. $p_8(t)= 4,376$ hr (tables 2 and 3) to misinterpreted ergodic time $p_8 * 1\ year \approx 2200$ hours for discrete time estimated from $p_8(n)$.

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