PREDICTION OF THE INFLUENCE OF EMERGENCE OF PROPELLER ON THE PROPELLER THRUST AND SPEED REDUCTION DURING SHIP NAVIGATION ON A GIVEN OCEAN ROUTE

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Abstract

During ship navigation on waves the relative motions i.a. occur which result in propeller emergence, and in consequence they cause the propeller thrust reduction. The article presents the algorithm for calculating the propeller thrust reduction as a result of the ship motions on waves having preset parameters: significant wave height \( H_s \), period \( T \), and geographical direction.

Key words: - ship motions on waves, vertical relative motion, propeller thrust and ship speed reduction

1. Introduction

Direct effect of the ship sailing on waves are the ship motions, occurring in continuous way, like the wave inducing them. Also other dangerous phenomena are associated with ship motions, such as e.g. accelerations or relative motions, which also occur in continuous way, as well as phenomena occurring sporadically, as for example: deck wetness, slamming or emergence of propeller. The latter phenomena result i.a. from the ship’s relative motions and in this case frequency of their occurrence within one hour or per 100 waves is investigated. Emergence of propeller is a dangerous phenomenon for the whole propulsion system and it also causes the propeller thrust reduction which results in effect in the reduction of the ship’s speed on waves (the reduction of the ship’s speed on waves is caused also by other factors). When determining the value of the propeller thrust reduction that shall occur and the ship speed it is not enough to know the frequency of emergences of the propeller e.g. per hour but it is also necessary to know how big the emergences will be along a given navigation route. On the basis of knowledge of the size value of emergences it will be possible to determine values obtainable by the propeller thrust reduction, and then - by the reduction of the ship’s speed on a given ocean route.

2. Relative movement of the ship and emergence of propeller
Using the commonly applied linear theory of ship motions [1], within the scope of which, on regular waves described by equation:

\[ \zeta(t) = \zeta_A \cos(kx - \omega t), \] (1)

the ship motions on these waves are given in the following form:

\[ u(t) = u_A \cos(-\omega_E t + \epsilon_u), \] (2)

where:
- \( \zeta(t) \) – ordinate of regular wave,
- \( \zeta_A \) – amplitude of regular wave,
- \( \omega \) – frequency of regular wave,
- \( t \) – time,
- \( k \) – wave number:

\[ k = \frac{\omega^2}{g}, \] (3)

- \( g \) – acceleration of gravity,
- \( x \) – coordinate on direction of wave propagation,
- \( u(t) \) – ship motion „u“,
- \( u_A \) – amplitude of ship motion „u“,
- \( \epsilon_u \) – angle of phase displacement of ship motion „u“,
- \( \omega_E \) – frequency of encounter of ship motions,

\[ \omega_E = \omega - kV \cos \beta_w, \] (4)

- \( V \) – ship speed,
- \( \beta_w \) – angle of wave effect acting on the ship, \( \beta_w = 0^\circ \) – following waves (from the aft), \( \beta_w = 90^\circ \) – beam wave, \( \beta_w = 180^\circ \) head wave:

\[ \beta_w = \mu - \psi + 180^\circ, \] (5)

- \( \mu \) – direction of waves in geographical direction (\( \mu = 0^\circ \) – northern wave, \( \beta_w = 90^\circ \) – eastern wave),
- \( \psi \) – ship course in geographic co-ordinates (\( \psi = 0^\circ \) – northern course, \( \psi = 90^\circ \) – eastern course),

The random motions of the ship on the irregular waves can be simply determined on the basis of knowledge about amplitude characteristics of the ship motions on linear regular waves and about function of random ship motions energy spectral density. Then variance of ship motions is equal to:
where:

\[ D_{uu} = \text{variance of ship motions } u, \quad u = 1, 2, ..., 6, \]

\[ Y_{uE} = \text{amplitude transfer functions of ship motions } u \text{ on regular waves}, \]

\[ S_{ww}(\omega_E) = \text{function of the random wave energy spectral density, the value of which depends mainly on the significant wave height } H_S \text{ and on period } T_1, \]

During ship motions on waves, its movement (displacement) can be determined, related to wavy water surface. The occurring relative movement (relative displacement) has a decisive influence on the emergence of propeller (Fig. 1).

![Fig. 1](image)

**Fig. 1. Influence of relative movement of the ship on emergence of propeller**

The vertical, relative displacement of the ship resulting from the ship motions is equal to:

\[ R_{zp} = Z_G + y_p \Phi - x_p \Theta - \zeta(t), \]  

(7)

where: \( \zeta(t) \) is the wave profile described by equation (1),

\[ x_1, y_1 \] - coordinates of point coordinates of point, for which relative movement is calculated, in this case the point lying on the propeller blade in its upper position is meant, hence \( y_p = 0 \)

However, the value of relative motion of the ship on irregular waves is calculated from the variance described by equation:

\[ D_{rz}(\beta_w, V) = \int_0^\infty [Y_{rz}(\omega_E / \beta_w, V)]^2 \cdot S_{ww}(\omega_E) d\omega_E, \]

(8)

\[ H_{AZ1/3} = 4 \sqrt{D_{rz}} \]  

(9)

where \( H_{AZ1/3} \) is the significant height of the relative motion of the ship on irregular waves.
3. Thrust of propeller during the ship motions on waves

The separated propeller thrust can be calculated from the formula:

\[ T = K_T \rho D_p^4 n_p^2, \]  

(10)

where:
- \( D_p \) – propeller diameter,
- \( n_p \) – propeller r.p.m.,
- \( K_T \) – thrust coefficient, that for the propeller of the following parameters given:
- propeller pitch ratio, \( \left( \frac{D}{P} \right) \),
- expanded blade area ratio, \( \left( \frac{A_E}{A_0} \right) \),
- number of blades, \( Z \), is approximated by the expression:

\[ K_T = A_0 + A_1 \cdot J + A_2 \cdot J^2 + A_3 \cdot J^3, \]  

(11)

where:
- \( A_0, A_1, A_2, A_3 \) – coefficients of polynomial describing thrust characteristics, dependent on \( \left( \frac{D}{P} \right), \left( \frac{A_E}{A_0} \right), Z \), [5],
- \( J \) – advance coefficient:

\[ J = \frac{V \left[ 1 - w_T(V) \right]}{D_p \cdot n_p}, \]  

(12)

\( w_T(V) \) – wake coefficient, dependent on ship speed \( V \).

The presented expressions for propeller thrust (10) and (11) are correct for the ship that is sailing on calm water or on wavy water but the occurring motions and relative motions are so small that the emergence of propeller does not occur. During ship navigation on waves at high ship motions and the relative motions the propeller works in rather air-locked water or it emerges from water. This causes thrust fluctuation and reduction of the mean effective thrust in relation to the thrust on calm water (even if the ship is sailing with constant speed and the propeller r.p.m is constant).

The thrust reduction is caused, among others, by influence of water particles velocities in the wave motion on the wake (coefficient \( w_T \)), and by propeller emergence as a result of the ship big relative motions on waves. The thrust reduction during ship navigation on waves is presented in various publications in which approximated formulae are included for assessing influence of water relative movements on the propeller parameters.

In the elaboration [3] the formulae are presented for correcting the speed of water reaching the propeller in situation when it is not fully immersed.

The corrected advance coefficient \( J_w \) shall be as follows:
\[ J_w = J \cdot G, \]  

(13)

where:
- \( J \) – advance coefficient according to [3],
- \( G \) – correction factor, dependent on the propeller parameters and its load, which according to [3] has the following form:

\[ G = 1 + 3 \cdot U \left( \frac{T}{\rho_w D_p^2 (1 - w_T)^2 V^2} \right), \]  

(14)

\[ U = \frac{D_p + h_{p0} - T_{aw} - w_n}{D_p}, \]  

(15)

\[ w_n = 0.6 \cdot c_B \cdot B \cdot c_{21}, \]  

(16)

\[ c_{21} = F_n^2 \quad \text{gdyn} \quad F_n < 0.3, \]  

(17)

\[ c_{21} = 0.09 \quad \text{gdyn} \quad F_n \geq 0.3, \]

where:
- \( T \) – thrust of fully immersed propeller,
- \( T_{aw} \) – stern immersion on waves, resulting from relative movements (Fig. 2),
- \( h_{p0} \) – vertical distance from PP to blade tip of the propeller in its lower position (Fig. 2),
- \( c_B \) – block coefficient of ship’s hull,
- \( F_n \) – Froude number, \( F_n = \frac{V}{\sqrt{gL_w}} \).

Emergence of propeller causes not only reduction of thrust but also of the moment which might result is increase of engine speed if it were not for engine speed regulator.

In another publication [4] the thrust reduction coefficient was introduced in the following form:

\[ \beta_T = \frac{K_{pT} \left( \frac{h_p}{R} \right)}{K_T}, \]  

(18)

where:
- \( K_{pT} \left( \frac{h_p}{R} \right) \) – thrust coefficient for the emerging propeller (the quantities \( h_p \) and \( R \) are shown in Fig. 2),
- \( K_T \) – thrust coefficient for fully immersed propeller,

Changes of the \( \beta_T \) coefficient value depending on \( \left( \frac{h_p}{R} \right) \) are shown in Fig. 3[3].
Thrust coefficient $K_{Tw}\left(\frac{h_p}{R}\right)$ occurring in equation (18) can be calculated making the expanded blade area ratio $\left(\frac{A_E}{A_0}\right)$ dependent on relative propeller immersion depth $\left(\frac{h_p}{R}\right)$ and taking into account the corrected advance coefficient $J_w$, equation (13). The instantaneous propeller immersion depth [draught] $h_p(t)$, or $T_{aw}(t)$ occurring in equation (15) shall be calculated on the basis of the ship motions on waves and emergences of propeller, resulting from them.

Fig. 2. Propeller immersion draft $h_p$ [3]

Fig. 3. Thrust reduction [coefficient] during emergence of propeller [4]
The emergence of propeller shall occur when the relative movement (significant amplitude $R_{AZ1/3}$ of the relative movement from equation (9) shall exceed the immersion depth $T_{pw}$ (Fig. 2) of the blade tip of the propeller in its upper position:

$$H_{AZ1/3} > T_{pw}$$

(19)

hence the size of emergence $\Delta Tp$ shall be:

$$\Delta Tp = H_{AZ1/3} - T_{pw}$$

(20)

and the propeller immersion draught $h_p(t)$ in equation (18) shall be:

$$h_p(t) = \frac{1}{2}D_p - \Delta Tp$$

(21)

Calculating, for different wave parameters: $H_s$, $T$, and $\mu$ occurring on the ocean route and the ship courses $\psi$ and speeds $V$ it shall be possible to calculate the value of the emergence of propeller $\Delta Tp$, or propeller immersion draught $h_p(t)$ as well as probability of occurrence of these values. Then the thrust reduction coefficient $\beta_r$ can be calculated, equation (24) and the ship speed reduction on a given ocean route.

5. Ship and weather parameters on an ocean route

The calculations have been performed for the ship M1 (bulk cargo ship, table 1) and for the ocean route no. 2 from the Western Europe to USA (Fig. 4) which runs through water areas for which the average statistical parameters of waves are included in atlas [2].

**Tab.1. Parameters of bulk cargo ship**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars L [m]</td>
<td>138.0</td>
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<tr>
<td>Breadth B [m]</td>
<td>23.0</td>
</tr>
<tr>
<td>Draught T [m]</td>
<td>8.55</td>
</tr>
<tr>
<td>Displacement for T V [m³]</td>
<td>21441</td>
</tr>
<tr>
<td>Contractual speed VK [m/s]</td>
<td>7.15</td>
</tr>
<tr>
<td>Propeller diameter Dp [m]</td>
<td>5.0</td>
</tr>
<tr>
<td>Propeller pitch P [m]</td>
<td>4.2</td>
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<tr>
<td>Nominal power of the propulsion motor $N_a$ [kW]</td>
<td>4710</td>
</tr>
<tr>
<td>Nominal r.p.m. of the propulsion motor $n_n$ [1/s]</td>
<td>1.85</td>
</tr>
<tr>
<td>Ship resistance on calm water for T and $V_k$ R [kN]</td>
<td>405.9</td>
</tr>
<tr>
<td>The sea margin assumed in the ship propulsion $K_z$ [%]</td>
<td>15</td>
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</tbody>
</table>
The atlas [2] includes, for particular water areas (Fig. 4) mean statistical values of the significant wave height $H_s$, the period $T$, and the geographical direction $\mu$ as well as probability of occurrence of these values (exemplary values are presented in table 2).

### Tab. 2. Probability of occurrence of wave height $H_s$ and period $T$ for a given direction $\mu$ at a given water area [2]

<table>
<thead>
<tr>
<th>$H_s$ [m]</th>
<th>$T$ [s]</th>
<th>calm</th>
<th>&lt;5</th>
<th>6-7</th>
<th>8-9</th>
<th>10-11</th>
<th>12-13</th>
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6. Instantaneous ship service speed on waves

The instantaneous ship service speed on a given wave having parameters $H_s$, $T$, and $\mu$ shall be obtained by the ship when the total ship resistance will be balanced by the propeller thrust, taking into account its potential drop caused by the emergence, and the moment on the propeller will be equal to the propulsion engine moment. The aforementioned conditions are written in form of the set of 2 nonlinear equations:
\[ T \cdot \beta_T - \frac{R_C}{1-t} = 0, \]
\[ Q - \frac{N \cdot \eta_S \cdot \eta_R \cdot \eta_G}{2 \pi n} = 0. \]  

(22)

where:
- \( T \) – The separated propeller thrust given in equations (16), (17) and (18) in [3],
- \( \beta_T \) – the ship thrust drop coefficient as a result of the propeller emergence during ship navigation on waves,
- \( R_C \) – the total ship resistance is dependent on the ship speed \( V \), ship course \( \psi \), waves parameters \( H_S, T, \mu \) and wind parameters \( V_A, \gamma_A \),
- \( t \) – thrust deduction factor,
- \( Q \) – torque on the separated propeller,
- \( N \) – power of the propulsion engine (resulting from the motor/engine operation area),
- \( \eta_S \) – shaftline efficiency,
- \( \eta_G \) – efficiency of gear (if it is applied on board, when it is missing then \( \eta_G = 1 \)),
- \( \eta_R \) – rotational “efficiency”,
- \( n \) – nominal r.p.m. of the propulsion motor.

Method of calculating the total ship resistance on waves \( R_C \) and the moment on the propeller \( Q \) as well as the driving engine capacity \( N \) from the engine \( N \) operation area are presented in [3] and [4].

The solution of the non-linear equations (22) for each set of data concerning:
- ship movement. \( V, \psi \),
- waves: \( H_S, T, \mu \),
- wind: \( V_A, \gamma_A \),

gives the instantaneous ship speed \( V_i \).

7. Mean long-term ship service speed on a given ocean route

During the ship voyage on a given ocean route where parameters of wave and wind shall be changing as well as the ship course and preset ship speed, the ship resistance on waves shall be changing, its motions and relative movements and thus propeller emergence and possible thrust drop.

Hence the ship thrust drop and the ship service speed on a given ocean route shall depend on:
- route of navigation and probability of ship’s sailing (staying) at particular water areas.
- statistical parameters of waves \( (H_S, T, \mu) \) wind \( (V_A, \gamma_A) \) and probability of occurrence of these parameters at given water areas.
- the probability of occurrence of the ship movement parameters, i.e. speed \( V \) and course \( \psi \) (the speed \( V \) should first be assumed, so as it could be later calculated and so as its assumed value could be corrected).

Probability of the ship staying in a given situation during navigation on wavy water along the preset navigation route is as follows:
\[ p_w = f_A \cdot f_S \cdot f_\mu \cdot f_{HT} \cdot f_V \cdot f_\psi, \quad (23) \]

where:

- \( f_A \) – frequency (probability) of the ship sailing at a given water area \( A \),
- \( f_S \) – frequency (probability) of the ship sailing in a given season of the year \( S \) at a given water area \( A \),
- \( f_\mu \) – frequency (probability) of occurrence of wave direction \( \mu \) in a given season of the year \( S \) at a given water area \( A \),
- \( f_{HT} \) – frequency (probability) of occurrence of waves having parameters \( H_S \) and \( T_i \) from direction \( \mu \).
- \( f_V, f_\psi \) – frequency (probability) of the ship sailing at speed \( V \) and at course \( \psi \).

Values of additional resistance due to wind and values of the propeller thrust drop due to the ship motions on waves depend on random parameters of waves and wind. Hence the same (identical) values of additional resistance and values of the propeller thrust drop can occur for different values of parameters \( V_A, \gamma_A, H_S, T_i, \mu, V, \psi \). For each value of additional resistance and the propeller thrust drop thus calculated the ship speed is calculated.

Total probability \( P_{TV} \) of the ship reaching the speed \( V \) at occurrence of additional resistance \( \Delta R \) and the propeller thrust drop \( \Delta T \) having specified value is equal to:

\[
P_{TV} = \sum_{A=1}^{n_A} \sum_{S=1}^{n_S} \sum_{\mu=1}^{n_\mu} \sum_{H,T=1}^{n_{HT}} \sum_{V=1}^{n_V} \sum_{\psi=1}^{n_\psi} P_{TV} \left[ V_i(\Delta R_i, \Delta T_i) \right], \quad (24)\]

where:

- \( V_i(\Delta R_i, \Delta T_i) \) – instantaneous ship service speed versus instantaneous additional resistance and instantaneous thrust drop of propeller,
- \( n_A, n_S, n_\mu, n_{HT}, n_V, n_\psi \) – are numbers of water areas through which the ship is sailing, seasons of the year, waves directions (angles), waves parameters, ship speeds and courses.

Calculating the distribution function \( f(V_i) \) of probability of occurrence of instantaneous ship speed \( f(V_i) \) it is possible to calculate the long-term ship service speed for the preset navigation route:

\[
\overline{V} = \frac{\sum_{i=1}^{n_V} P_{TV_i} \cdot V_i(\Delta R_i = \text{const}, \Delta T_i = \cos nt)}{\sum_{i=1}^{n_V} P_{TV_i}}, \quad (25)\]

where \( n_V \) is number of ranges including instantaneous ship service speeds of approximate values.
8. Results of calculations of the propeller thrust drop and the ship speed drop on a given ocean route

The calculations of propeller thrust drop as a result of the propeller emergence and of the reached ship service speed have been performed for the container ship (table 1) and for the ocean route from the Western Europe to USA (Fig. 4). The obtained results have been compared with identical calculations but with the assumption that the propeller does not emerge and there is no thrust drop.

In Fig. 5 the bar chart and probability distribution function of propeller immersion draught are presented (100% means total immersion/draught of propeller). In Fig 6 the propeller bar chart without taking the propeller emergence into account and taking the propeller emergence on a given ocean route into account. After taking into account the emergence of propeller due to the ship motions, on the same ocean route the mean statistical value of the propeller thrust has been reduced by c.a. 2.5%. In Fig. 7 the propeller bar chart of probability of reaching ship service speed with and without taking the propeller thrust drop into account. In this case the drop of the mean long-term ship service speed was equal to 0.64 m/s.

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{Fig5.png}
\caption{The bar chart and probability distribution function of ship propeller immersion draught on the Western Europe – USA ocean route}
\end{figure}
The propeller thrust [kN]

- **Orange** without taking the emergence of propeller into account (average thrust = 532.8 kN)
- **Green** with taking the emergence of propeller into account (average thrust = 519.2 kN)

Fig. 6. The bar chart of the ship propeller thrust on the Western Europe - USA ocean route

The ship service speed [m/s]

- **Green** with reduction of propeller thrust (average speed = 6.23 m/s)
- **Orange** without reduction of propeller thrust (average speed = 6.87 m/s)

Fig. 7. The bar chart of the ship speed on the Western Europe - USA ocean route
References
