ANALYSIS OF RELATIONS BETWEEN THE COMPRESSION RING CHARACTERISTIC PARAMETERS

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Abstract

A proper design of compression ring secures its correct and long term operation. A good ring contact to cylinder wall along the whole circumference with the required distribution of circumferential pressure at the same time are symptoms of this correctness. The analytical methods and more often numerical ones are applied when designing piston rings. A characteristic parameter most often designated as K, which facilitates the comparison of different ring designs and allows for anticipation of its elastic properties is used at the stage of ring design. The following study presents the most significant mathematical relations between the ring geometry and forces that are acting on ring, and shows that results of force operation could differ relative to the point of their application. Relations between the tangential force and the circumferential one have been established as well. For three compression rings verifying tests consisting in definition of selected parameters using analytical and numerical methods have been carried out. The analysis of attained results and trials on explanation of noticed discrepancies are presented in the study as well.

Keywords: marine combustion engine, piston ring, oil film, ring wall pressure

1. Introduction

The compression ring should touch the cylinder wall with all its circumference and move over a layer of lubricating oil in order to serve its purpose, i.e. to keep the combustion chamber tight, to transfer the piston heat and to distribute lubricating oil. According to the hydrodynamic theory of lubrication the formation of oil layer separating working surfaces of ring and cylinder requires a selection of adequate wall pressure as well as micro- and macrogeometry of collaborating surfaces. It is generally considered that thanks to suitable selection of these parameters a period of ring reliable operation could be significantly extended. Other measures helpful in extension of ring life are: application of ring face cover with chromium-ceramic covers or appropriate formation of collaborating surface, e.g. deep honing of cylinder liner or chrome plated ring grooves on piston.

2. Characteristic parameters of compression ring

Characteristic parameters of ring geometry (see Fig. 1) and of ring material (represented by e.g. the modulus of elasticity) are used for definition of ring design. In case of modern
compression rings the range of those values is very wide.

For example, the diameters of contemporary engine piston rings could range from dozen millimeters or so to more than meter while the axial width – from a fraction to a few dozens millimeters.

The efforts on definition of an optimum design of compression ring, in particular on relations between ring geometry and its elastic properties have been carried out for years. The most considerable progress was done in the 40-ties of the last century, when so called ring free form was established using analytical formulas. A parameter that could facilitate a comparison of various rings was searched for as well. It was accepted that the quantity further called a ring characteristic parameter and given by the following formula

\[
K = 12 \frac{p_o}{E g_p} \left( \frac{r_m}{g_p} \right)^3,
\]  

(1)

satisfies these demands. In this formula \(r_m\) denotes a radius of neutral layer while \(p_o\) is a constant circumferential pressure (resulting from ring installation in liner).

The way the radius of neutral layer \(r_m\) was determined needs explanation. If the rectangular ring outer radius was denoted by \(r_z\) (see Fig.1a) and the inner radius by \(r_w\) then the radius of neutral layer is given by the formula:

\[
\ln \frac{r_z}{r_w} = \ln \frac{r_z}{r_w} = \ln \frac{d}{d - 2g_p}.
\]  

(2)

For majority of piston ring calculations the simplified form of the Eq. (2) reduced to

\[
r_m = 0.5 \cdot (d - g_p)
\]  

(3)
is used which means that the neutral layer agrees with the cross-section center of gravity. Eq. (3) could be obtained as a result of the expansion of function \( \ln \left( \frac{r_z}{r_w} \right) \) in an exponent series taking into consideration only the first term of this expansion.

Using a well-known relation between the tangential force \( F_t \) (this force acts at the ring gap tangentially to the neutral layer) and the circumferential wall pressure \( p_o \) ([9])

\[
p_o = \frac{F_t}{r_m \cdot h_p}
\]

one can obtain other forms of the Eq. (1) (more important ones are presented in Table 1).

**Tab. 1. Formulas for calculations of the ring characteristic parameter \( K \)**

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
<th>Column 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K = 12 \frac{p_o \cdot \left( \frac{r_m}{g_p} \right)^3}{E} )</td>
<td>( K = \frac{p_o \cdot h_p \cdot r_m^3}{E \cdot I} )</td>
<td>( K = \frac{F_t \cdot r_m^2}{h_p \cdot g_p^3} )</td>
<td>( K = \frac{3 \cdot (d - g_p) \cdot F_t}{E} )</td>
<td>( K = \frac{m}{3 \cdot \pi \cdot r_m} )</td>
</tr>
</tbody>
</table>

Terms as in Fig. 1, another ones: \( I \) – moment of inertia, given by the formula \( I = h_p \cdot g_p^3 / 12 \) for a ring of rectangular cross-section.

As verifying calculations show for a compression ring the parameter \( K \) takes the value within the range from 0.01 to 0.05, independently on dimensions and material properties.

When trying to define the properties of ring which data are unknown, the formula written in column 5 of the Table 1 linking the \( K \) parameter with the clearance of ring gap might be useful (see dimension \( m \) in Fig. 1a). To define this one should assume that the ring is a curved rod of satisfactorily big radius \( r_m \) relatively to the radial wall thickness \( g_p \). The change in ring gap, i.e. the displacement of ring free ends, could be defined as the derivative of rod potential energy \( V \) relative to the force \( P \) (according to the Castigliano’s theorem):

\[
f_y = \frac{\partial V}{\partial P}.
\]

An increase in potential energy \( dV \) caused by bending moment \( M(\varphi) \) along the increase of angle \( d\varphi \) equals (as very small, other forces and moments acting upon the ring are omitted):

\[
dV = \frac{M^2(\varphi) \cdot r_m}{2 \cdot E \cdot I}.
\]

**Fig. 2. A schematic draw used for determination of a displacement of ring ends loaded by the force \( P \)**
A potential energy contained in ring within the section defined by the angle \(0 - \varphi_1\) equals:

\[
V = \frac{P^2 \cdot r_m^2}{2 \cdot E \cdot I} \int_0^{\varphi_1} (1 + \cos \varphi)^2 d\varphi,
\]

and the total displacement of a point subjected to the force operation searched for is:

\[
f_y = \frac{P \cdot r_m^3 \pi}{E \cdot I} \int_0^{\varphi_1} (1 + \cos \varphi)^2 d\varphi.
\]

Remembering that the total size of clearance at ring gap is \(2f_y\), a formula linking the \(K\) parameter of ring with the total clearance \(m\) has been obtained:

\[
K = \frac{m}{3 \cdot \pi \cdot r_m} = \frac{2 \cdot m}{3 \cdot \pi \cdot (d - g_p)}.
\]

For a known geometry of ring (and using the formulas presented in Table 1) one can estimate a hypothetic value of ring pressure \(p_o\) or tangential force \(F_t\) on purely computational way (however the knowledge on properties of ring material given by the Young modulus \(E\) is indispensable).

2. Relations linking the tangential and radial force

To determine the ring elastic properties the measuring devices of different construction are being used (e.g. those presented in [8]) which allow to measure a value of tangential \(F_t\) or radial \(Q\) force. Knowing one of the forces the another one can be determined according to the formula \(Q = \kappa \cdot F_t\) (literature gives various values of \(\kappa\) within the range from 2 to 3). In order to assume the factor suitable for certain measurements one should take into consideration the force location and the results of force operation as well. During measurements the force \(F_t\) can load the ring evenly (by the clamping band – see Fig. 3a) or on the ring end at the point situated on its neutral layer (Fig. 3b). The value of force should be selected so as to bring about the ring ends as near as gap clearance \(l_z\). On some measuring devices the direction of tangential force is moved from the neutral layer to the ring outer surface (as in Fig. 3a – the force is marked as \(F_{t,d}\)) which affects the measured value. The following relation takes place between the forces mentioned:

\[
F_t = \frac{F_{t,d} \cdot 2d - g_p}{d - g_p}.
\]
On the other hand the radial force should be selected so as to make ring ends come closer to the distance of $l_z$ (the $Q_1$ force in Fig. 3c) or cause a partial ring closure to the value of $l_d = d$ (the $Q_2$ force in Fig. 3d). Moreover, it should be mentioned that ring deformation occurs in all cases presented (except the use of clamping band), which additionally affects the variability of the $\kappa$ factor.

In order to avoid an erroneous selection of the $\kappa$ parameter the author took a trial to verify its value and variability dependently on analyzed case. A verification of the factor $\kappa_1$ value linking the $F_t$ and $Q_1$ forces (for a case presented in Fig. 3c) was carried out at the beginning.

![Figure 4. An auxiliary sketch for determination of the $\kappa_1$ parameter](image)

According to the sketch in Fig. 4 the load of force $Q_1$ brings about a displacement of point $A'$ to $A$ ($f_A$ is the vertical component of this section) and of point $B'$ to $B$ (its vertical component is $f_B$) at the same time. The length of section $s$ does not change (for simplicity of sketch the $l_z$ slit was not marked because is far shorter than the $f_B$ section) because the right hand side of ring is not subjected to load. Remembering that the same displacement $f_B$ should be brought about by the tangential force $F_t$ acting at the ring gap, a relation linking both forces has been established (relation between the force $F_t$ and displacement $f_B$ was given in [3], for example).

$$\frac{\pi \cdot Q_1 \cdot r_m^3}{4 \cdot E \cdot I} + r_m \left[1 - \cos \left(\frac{Q_1 \cdot r_m^2}{E \cdot I}\right) + \sin \left(\frac{Q_1 \cdot r_m^2}{E \cdot I}\right)\right] = \frac{3 \cdot \pi \cdot F_t \cdot r_m^3}{2 \cdot E \cdot I}.$$  \hspace{1cm} (11)

As a solution of (11) the following relation has been obtained:

$$\kappa_1 = \frac{Q_1}{F_t} = \frac{6\pi}{4 + \pi} = 2.639.$$  \hspace{1cm} (12)

Almost identical value, i.e. 2.632 has been given in [4].

The Goetze Company gives other values applied when defining relation between $F_t$ and $Q_2$ [5]. The value of radial force $Q_2$ should be big enough to make the ring ends come to the distance of $l_d = d$ (measured on the diameter of ring in cylinder which corresponds to the case presented in Fig. 3d). The relation describing this situation, determined experimentally, has the following form:

$$\kappa_2 = \frac{Q_2}{F_t} = 2.2667 \left[1 - 0.33 \cdot K - 7.8 \frac{u}{d} - 24 \left(\frac{g_n}{d}\right)^2\right].$$  \hspace{1cm} (13)
while the value of $\kappa_2$ factor should lie within the range from 2.05 to 2.30.

The form of Eq. (13) shows that the initial value of relation $Q_2/F_t$ equal to 2.2667 is consecutively reduced by the individual elements of the formula more the higher is the value of factor $K$, oval deformation $u$ and radial wall thickness $g_p$. Test computations carried out for a group of rings proved that the relative decrease in this value could reach 0.2 which explains reasons why the range of variability for this factor was assumed as it was presented earlier.

The author decided to perform the check tests of this factor (using analytical formulas and mathematical model of ring).

Expanding ring causes the displacement of point P (visible at an angle of $\varphi$ from the start of coordinate system – see Fig. 5) to the point P which corresponds to the $\Delta x$ and $\Delta y$ displacements relative to the axes.

![Fig. 5. Supplement sketch for definition of ring free form](image)

Information on the way the Eq. (15) was obtained, methods of its solution and perspectives of implementation when constructing mathematical models of piston ring one may find in [1, 2].
\[
    r_m \cdot K \cdot \left(1 + \frac{\pi}{4}\right) = \frac{\pi \cdot r_m^3 \cdot Q_2}{4 \cdot E \cdot I},
\]
which leads to
\[
    \kappa_2 = \frac{Q_2}{F_t} \approx \frac{\pi + 4}{\pi} = 2.273
\]
after suitable transformations. It is value close to the one given by the formula (13).

3. Application of numerical method to verification of the \( \kappa \) parameter and description of compression ring shape

A practical implementation of the analytical relations presented in chapter 2 requires a fulfillment of many conditions. For instance, it is being assumed that the ring is installed in an ideally round cylinder and touches to the liner with its entire circumference, and the ring wall pressure is always even. Analytical relations allow to take into consideration changeability of many quantities related to ring geometry and to the material used as well. The numerical methods are free of such limitations. In literature, also in domestic one there are descriptions of numerous methods that facilitate to design the ring of any form and pressure distribution with any accuracy. A method of ring elastic pressure distribution was presented by A. Iskra [1]. The mathematical model developed by the author on the basis of the above method can be found in [6].

A fragment of the model verification process was shown in [9]. This consists in a comparison of computational results accomplished by analytical and numerical methods (implemented to a mathematical model of ring). The comparative analyses embraced among other the magnitude of ring ends displacement resulted from loading forces. The tests concerned rings of three dimensional categories, namely of the automotive engine (the 170A.000 type), of a bulldozer (the DTI-817C type), and the marine one (the L48/60CR type). The ring characteristic parameters were measured by the author or were obtained from related catalogues (see Table 2).

Using a mathematical model of piston ring, the forces extorting the displacement of ring sections (according to the description in Fig. 3) as well as the \( \kappa_1 \) and \( \kappa_2 \) parameters were calculated and the obtained results were compared with the results of analytical calculations (it was assumed that the latter were accurate).

<table>
<thead>
<tr>
<th>Tab. 2. Technical data of exemplary IC engine compression rings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>cylinder diameter ( d ) [m]</td>
</tr>
<tr>
<td>ring neutral radius ( r_m ) [m]</td>
</tr>
<tr>
<td>axial height ( h_p ) [m]</td>
</tr>
<tr>
<td>radial thickness ( g_p ) [m]</td>
</tr>
<tr>
<td>gap clearance ( m ) [mm]</td>
</tr>
<tr>
<td>Young modulus ( E ) [Pa]</td>
</tr>
<tr>
<td>mean pressure ( p_v ) [MPa]</td>
</tr>
<tr>
<td>tangential force ( F_t ) [N]</td>
</tr>
<tr>
<td>stiffness ( EI ) [Nm]</td>
</tr>
<tr>
<td>parameter ( K ) [–]</td>
</tr>
</tbody>
</table>
As it outcomes from the results summarized in Table 3, there is a high agreement between parameters obtained with analytical (A) and numerical (N) methods, because the relative differences do not exceed 2%. The condition valid for analytical calculations (about keeping the round form by ring) was not fulfilled in numerical calculations which seems to be a probable reason for these differences.

4. Evaluation of ring deformation under the load of external forces

The shape of ring loaded with the forces $F_t$, and $Q_1$ and $Q_2$ differs considerably from the circle. Fig. 6 shows the courses of circumferential relative change in ring radius – calculated analytically – for selected load cases (cases summarized in Fig. 3). The ring deformations compared with the case of even load could reach several percent depending on load case and the highest ones are those for the load of $Q_2$ force (line 4). It should be emphasized that despite similar shape the courses obtained for various rings differ one from another (Fig. 7). It means that individual characteristic course should be determined for each ring.

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**Fig. 6. Courses of the circumferential relative change in ring radius for selected load cases on following engines: a – 170A.000 , b – DTI-817C, c – L48/60CR; 1 – constant load, 2 – loaded with the $F_t$ force, 3 – loaded with the $Q_1$ force, 4 – loaded with the $Q_2$ force**

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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
<th>Ring 1 Results</th>
<th>Ring 2 Results</th>
<th>Ring 3 Results</th>
<th>δ [%]</th>
<th>δ [%]</th>
<th>δ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_1$</td>
<td></td>
<td>25.7</td>
<td>49.7</td>
<td>583</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_2$</td>
<td></td>
<td>22.1</td>
<td>42.5</td>
<td>503</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi_1$</td>
<td></td>
<td>2.639</td>
<td>2.672</td>
<td>2.657</td>
<td>1.439</td>
<td>1.251</td>
<td>0.682</td>
</tr>
<tr>
<td>$\chi_2$</td>
<td></td>
<td>2.273</td>
<td>2.284</td>
<td>2.296</td>
<td>1.276</td>
<td>0.484</td>
<td>1.011</td>
</tr>
</tbody>
</table>
Presented analyses and defined relations concern above all the situations relative to the tests of compression rings outside engine but they can be also useful during ring design process and analyses of its behaviour when moving on a running engine.

References
