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NECESSITY FOR AND POSSIBILITY OF APPLICATION OF THE THEORY OF SEMI-MARKOV PROCESSES TO DETERMINE RELIABILITY OF DIAGNISING SYSTEMS

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Abstract

The paper provides justification for the necessity to define reliability of diagnosing systems (SDG) in order to develop a diagnosis on state of any technical mechanism being a diagnosed system (SDN). It has been shown that the knowledge of SDG reliability enables defining diagnosis reliability. It has been assumed that the diagnosis reliability can be defined as a diagnosis property which specifies the degree of recognizing by a diagnosing system (SDG) the actual state of the diagnosed system (SDN) which may be any mechanism, and the conditional probability $p(S^*/\boldsymbol{K}^*)$ of occurrence (existence) of state S^* of the mechanism (SDN) as a diagnosis measure provided that at a specified reliability of SDG, the vector $\boldsymbol{K}^*$ of values of diagnostic parameters implied by the state, is observed. The probability that SDG is in the state of ability during diagnostic tests and the following diagnostic inferences leading to development of a diagnosis about the SDN state, has been accepted as a measure of SDG reliability. The attention has been paid that in order to make an operating decision not only the knowledge of a diagnosis reliability is required, but also the knowledge of consequences (c) of making a given decision that belongs to a set of decisions possible to be made in a given operating situation. The Bayesian statistical decision theory has been proposed to apply for making operating decisions. Herein, it has been used the simplest decision model which assumes that there can only be made one from among two possible operating decisions: 1) perform, first of all, a suitable preventive service for the mechanism (SDN) under operation, in order to renew its functional properties and then start executing the task, 2) start executing the ordered task without prior performance of a preventive maintenance of the mechanism. The theory of semi-Markov processes has been used for defining the SDG reliability, that enabled to develop a SDG reliability model in the form of a seven-state (continuous-time discrete-state) semi-Markov process of changes of SDG states.

Keywords: *decision, diagnostics, probability, reliability, diagnosing system*

1. Introduction

A reasonable operation of mechanisms requires making the right decisions that are appropriate for the current operating situation. An opportunity to make right decisions exists when the reliability of the diagnosis on the technical state of the operated mechanisms is known and it is possible to identify the consequences (*c*) of making the given decision [1, 2, 3, 4]. An assessment of the reliability of the diagnosis on the technical condition of each mechanism (as *SDN* - diagnosed system) is possible only if the reliability of the appropriate diagnosing system (*SDG*) is known, which is necessary to develop a diagnosis on the technical state of *SDN*. The probability of the system's correct work during tests and diagnostic inference that results in a diagnosis, can be taken as a measure of *SDG* reliability. This probability can be determined by applying the theory of semi-Markov processes. The requirement to determine the mentioned probability can be justified wider by describing the importance of the *SDG* reliability for making operating decisions.

2. An importance of reliability of diagnosing systems for making operating decisions during operation of the mechanisms

Knowledge of the reliability of a diagnosing system (*SDG*) is indispensable to define the reliability of the diagnosis on technical state of a diagnosed system (mechanism) (*SDN*), which can be understood differently [2, 4, 5]. However, diagnosis reliability can be regarded:

- in descriptive sense, as a diagnosis property that defines the degree of recognizing by a diagnosing system (*SDG*) an actual technical state of the diagnosed system (*SDN*), which may be any mechanism, and
- in evaluative sense, as a diagnosis property determined by a conditional probability $p(S^*/\mathbf{K}^*)$ of occurrence (existence) of state S^* of the mechanism (*SDN*), provided that the vector $\mathbf{K}^*$ of values of diagnostic parameters being implied by the state, is observed.

Knowing reliability of the diagnosis on technical state of the mechanism (*SDN*), during the phase of its operation, one of the following decisions can be made:

- decision d_1 – perform, first of all, suitable preventive service for the *SDN*, in order to renew its functional properties which are indispensable to execute the task Z_D, and then start executing the task at the time defined by the orderer,
- decision d_2 – start executing the ordered task Z_D without prior performance of the preventive maintenance of the mechanism, which in a formal term can be defined as follows:

$$Z_D = \langle \Phi, W \rangle, t \rangle \qquad (1)$$

where: Φ – correct operation (work) of *SDN*, W – conditions under which *SDN* should properly operate (work), t – time of performing the task Z_D.

However, making a rational operating decision in the phase of operation of the mechanism, requires the knowledge of not only the diagnosis reliability but also the consequences (c) of making the decision. In this situation, for making decisions it is convenient to apply the Bayesian statistical decision theory [1, 3, 4].

Execution of Z_D is possible when *SDN* (mechanism) is in state of ability (s_1^*). The task cannot be performed if *SDN* is in state of disability (s_2^*). However, the mentioned states (s_1^* and s_2^*) that belong to the set of states $S^* = \{s_1^*, s_2^*\}$should be such defined that their occurrences are the mutually exclusive events, so such events whose the probabilities of occurrence satisfy the equations: $P(s_1^* \cup s_2^*) = P(s_1^*) + P(s_2^*)$ and $P(s_1^* \cap s_2^*) = 0$.

Selection of the best decision from among these two (d_1 or d_2) when during performance of the task Z_D a possibility of occurring the states $s_i^*(i = 1, 2)$ exists, requires to take into account the following decision criteria:

- expected value of consequences $E(c|d_1)$ corresponding to decision d_1;
- expected value of consequences $E(c|d_2)$ corresponding to decision d_2.

After estimation of the expected values $E(c|d_k)$, where $k = 1, 2$; the following logic (rule) of making decisions should be applied [1]: *from among decisions* $d_k(k = 1, 2)$ *this one should be selected which the highest value of* $E(c|d_k)$ *is assigned to*.

Estimation of these expected values is possible when reliability or rightness of the diagnosis is identified [4, 5]. In order to avoid misunderstanding it should be assumed that the diagnosis reliability is determined only when the probability that *SDG* is in state of ability is less than unity. Whereas, rightness of diagnosis is identified only when the probability that *SDG* is in state of ability is equal to unity [4, 5].

Usefulness of the diagnosis reliability $p(S^*/\mathbf{K}^*)$ can be presented on the example of the decision situation of which the dendrite is demonstrated in Fig. 1.

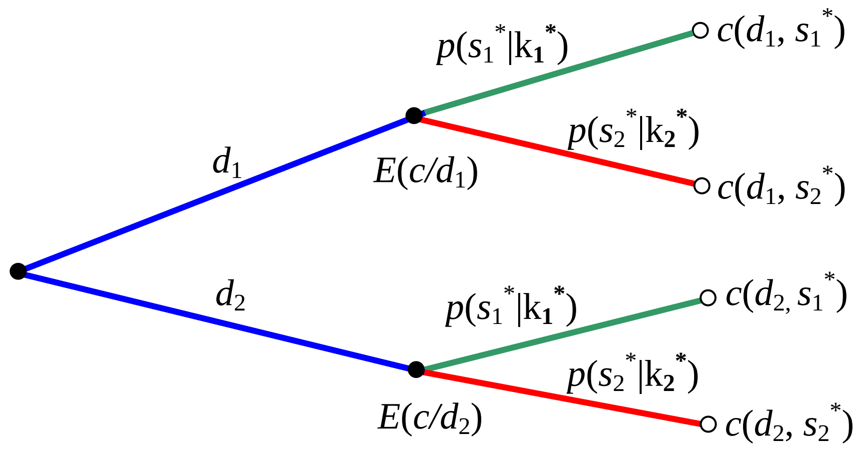

Fig. 1. Exemplary dendrite of the operating decisions for any mechanism: $p(s_1^*/\mathbf{k}_1^*)$ – probability of existence of state s_1 under the condition that the vector $\mathbf{k}_1^*$ of diagnostic parameters is observed, $p(s_2^*/\mathbf{k}_2^*)$ – probability of existence of state s_2 under the condition that the vector $\mathbf{k}_2^*$ of diagnostic parameters is observed, s_1^* – state of ability of the mechanism, s_2^* – state of disability of the mechanism, $c(d_1, s_1^*)$ – consequence resulting from decision d_1 for state s_1^* of the mechanism, $c(d_1, s_2^*)$ – consequence resulting from decision d_1 for state s_2^* of the mechanism, $c(d_2, s_1^*)$ – consequence resulting from decision d_2 for state s_1^* of the mechanism, $c(d_2, s_2^*)$ – consequence resulting from decision d_2 for state s_2^* of the mechanism.

The decision dendrite in Fig. 1 shows that the expected values $E(c|d_1)$ and $E(c|d_2)$ can be derived from the relationships:

$$\left.\begin{aligned} E(c|d_1) &= p(s_1^*|\mathbf{k_1}^*)c(d_1, s_1^*) + p(s_2^*|\mathbf{k_2}^*)c(d_1, s_2^*) \\ E(c|d_2) &= p(s_1^*|\mathbf{k_1}^*)c(d_2, s_1^*) + p(s_2^*|\mathbf{k_2}^*)c(d_2, s_2^*) \end{aligned}\right\} \quad (2)$$

Therefore, in compliance with the presented rule for making decisions, if $E(c|d_1) > E(c|d_2)$ the decision which should be made is d_1 and inversely – if $E(c|d_1) < E(c|d_2)$ the decision which should be made is d_2.

From the considerations it follows that for making operating decisions we need among others the knowledge of diagnosis reliability $p(S^*/\mathbf{K}^*)$ about the technical state of the mechanism whose the measure can be the probability of occurrence of the state s_1^* or s_2^*.

In extreme cases:

- ⇒ reliable (fully reliable) diagnosis can be assigned with a digit 1, which should be considered that the diagnosis is entirely reliable, i.e. right;
- ⇒ unreliable diagnosis can be assigned with a digit 0, which should be considered that the diagnosis is entirely unreliable, i.e. wrong.

In the operating practice, such an alternative assessment of diagnosis reliability, consisting in assigning it with a digit 1 or 0, is insufficient. Therefore, the papers [4, 5] present a proposal to determine diagnosis reliability in the form of the probability $P(S^*/\mathbf{K}^*)$, where S^* – state of mechanism, $\mathbf{K}^*$ – vector of values of diagnostic parameters characteristic for state S^*. The papers provide an assumption that in the evaluative sense the diagnosis reliability as a diagnosis property, can be determined by the values of important in certain cases indexes characterizing the degree of recognizing by SDG the state of SDN (mechanism), so this may be understood as the conditional probability $P(S^*/\mathbf{K}^*)$, logical probability $P_L(S^*)$ or statistical probability $P_S(S^*)$ [9].

The paper [4, 5] submits a proposal of formulas as the measures of the diagnosis reliability and accuracy. For deriving the formulas there were applied: a conditional probability formula for the events A_1, S^*, $\mathbf{K}^*$ and a continuous-time discrete-state semi-Markov model of the process of using *SDG* $\{W(t): t \geq 0\}$ [4, 5], where:

- A_1 – event representing a proper operation of *SDG* during development of the diagnosis,
- S^* – event representing an occurrence of state S^* of the mechanism (*SDN* – diagnosed system),
- $\mathbf{K}^*$ – event indicating an occurrence of a particular vector $\mathbf{K}^*$ of values of diagnostic parameters as a result of occurring the state S^* of *SDN*.

In consequence, the formula defining probability $P(S^*/\mathbf{K}^*)$ as a measure of diagnosis reliability, is obtained in the form [4, 5] as follows:

$$P(S^* / \mathbf{K}^*) = \frac{P(A_1)P(S^*)P(\mathbf{K}^* / S^*)}{P(\mathbf{K}^*)P(A_1 / \mathbf{K}^* \cap S^*)} \tag{3}$$

For a reliable *SDG* (so such *SDG* for which $P(A_1) = 1$ exists) the formula (3) takes the form as follows:

$$P(S^* / \mathbf{K}^*) = \frac{P(S^*)P(\mathbf{K}^* / S^*)}{P(\mathbf{K}^*)}, \tag{4}$$

which is the measure of the diagnosis accuracy [4, 5].

From the formula (4) it follows that even when *SDG* works reliably during diagnostic tests and then during diagnostic inference until the diagnosis on *SDN* technical state is developed, the diagnosis cannot be certain. This results from the fact that while developing a diagnosis (inference) about the state S^* of *SDN*, the statement $\mathbf{K}^*$, so the statement that just this and not any other vector ($\mathbf{K}^*$, in this case) of values of diagnostic parameters was recorded by *SDG*, is regarded to be completely certain premise. While the sentence S^*, saying that just this and not any other state (S^*, in this case) of *SDN*, is an inference developed on the basis of the statement $\mathbf{K}^*$, which is the result of completed non-deductive inference. Therefore, identification of the technical state of *SDN* consists in developing the hypothesis: *SDN is in state S^* because vector $\mathbf{K}^*$ of the values of diagnostic parameters is observed.* In this case this inference is a reductive inference, so proceeds according to the following scheme [9]:

$$\mathbf{K}^*, S^* \Rightarrow \mathbf{K}^* \vdash S^* \tag{5}$$

where:

- $\mathbf{K}^*$ – completely certain premise,
- S^* – inference developed on basis of the sentence $\mathbf{K}^*$.

In the case, when the sentence S^* is an inference being developed on the basis of the sentence $\mathbf{K}^*$ (regarded as a completely certain premise) in the process of inferring, it can be assumed that the sentence S^* is made probable by the sentence $\mathbf{K}^*$. The measure for this can be a conditional probability defined by the formula (3) if there is no certainty that *SDG* works reliably or by the formula (4) if *SDG* works reliably during diagnostic tests and inference.

In general, there is no certainty that SDG works reliably during tests and diagnostic inference [8, 10] and therefore, it is important to determine the probability of its correct operation. For this reason, it is necessary to identify the reliability of *SDG*. This requires consideration of at least a two-state reliability model for *SDG*, thus an assumption that this can find in only two mutually exclusive states, i.e. state of ability (s_0) and state of disability ($\bar{s}_0$). However, due to the fact that in diagnostics of mechanisms (*SDN*) there are applied different methods of testing their states (and hence different diagnosing devices), it becomes necessary to distinguish instead of one state $\bar{s}_0$, a number of states of disability s_j (j = 1, 2, …., n) of *SDG*, which occur in consequence of failures in diagnosing devices that belong to *SDG* used along with the taken methods for testing the *SDN* state. The further considerations on the reliability of *SDG* include the states of disability s_j, j = 1, 2, …, 6 ($s_j \in \bar{s}_0$), where j – type of *SDG* disability resulting from a failure of its subsystem SDG_j, where the indexes $j = \overline{1,6}$ denote e.g.: 1 – subsystem for testing the acoustic emission, 2 – subsystem for visual (endoscopic) testing, 3 – subsystem for thermal (temperature, pressure) testing, 4 – subsystem for vibration testing (for NVH tests), 5 – subsystem for wear testing by employing the method of radionuclide X-ray fluorescence (*XRF*), 6 – subsystem for thermographic testing (for infrared thermal image analysis).

To determine the *SDG* reliability, at such approach to testing the reliability of *SDG*, we can apply the theory of semi-Markov processes (of continuous-time discrete-state type) and develop a seven-state semi-Markov model of the process of changes of *SDG* states (state of ability s_0 and states of disability s_j, $j = \overline{1,6}$), which is necessary to derive the formula for the probability of staying the *SDG* in state of ability (s_0), so in a state which is indispensable to develop a reliable diagnosis.

3. A semi-Markov model of the process of changes of states for diagnosing systems

Application of the semi-Markov model of the process of changes of states for a diagnosing system (*SDG*) enables consideration of its preventive maintenance service [2, 8, 10] and, therefore, consideration of *SDG* reliability state s_0, i.e. state of ability and states of disability s_j ($j = \overline{1,6}$) of its particular subsystems SDG_j, j = 1, 2, …, 6.

The semi-Markov model of the process of changes of reliability states for a the diagnosing system (*SDG*) can therefore, be considered as a semi-Markov process $\{W(t): t \geq 0\}$ with the set of states $S = s_i$; i = 0, 1, ... ,6. The interpretation of the states $s_i \in S$(i = 0, 1, ... ,6) is as follows: s_0 – state of ability of *SDG* and simultaneously of all its subsystems SDG_j, ($j = \overline{1,6}$), s_1 – state of disability of the subsystem SDG_1 for acoustic emission testing, s_2 – state of disability of the subsystem SDG_2 for visual (endoscopic) testing, s_3 – state of disability of the subsystem SDG_3 for thermal (temperature, pressure) testing, s_4 – state of disability of the subsystem SDG_4 for vibration (NVH) testing, s_5 – state of disability of the subsystem SDG_5 for wear testing by using X-ray radionuclide fluorescence method (*XRF*), s_6 – state of disability of the subsystem SDG_6 for thermographic testing (infrared thermal image analysis). Changes of the listed states s_i (i = 0, 1, ..., 6) proceed at subsequent times t_n ($n \in N$), where at time t_0 = 0 a diagnosing system (*SDG*) is in state s_0. The state s_0 lasts until any of the distinguished subsystems SDG_j ($j = \overline{1,6}$) fails. The states s_j(i = 1, ..., 6) last as long as the failed subsystem SDG_j is renovated or replaced by another one in case the renovation is found unprofitable. It can be assumed that the state of *SDG* at time t_{n+1} and the time interval of duration of the state achieved at time t_n do not depend on the states occurred at times t_0, t_1, ..., t_{n-1} or the time intervals of their duration. Thus, the process $\{W(t): t \geq 0\}$ of changes of states s_i; i = 0, 1, ... ,6 is a semi-Markov process [4, 6]. The graph of changes of the reliability states s_j of *SDG* ($i = \overline{0,6}$) is shown in Fig. 2.

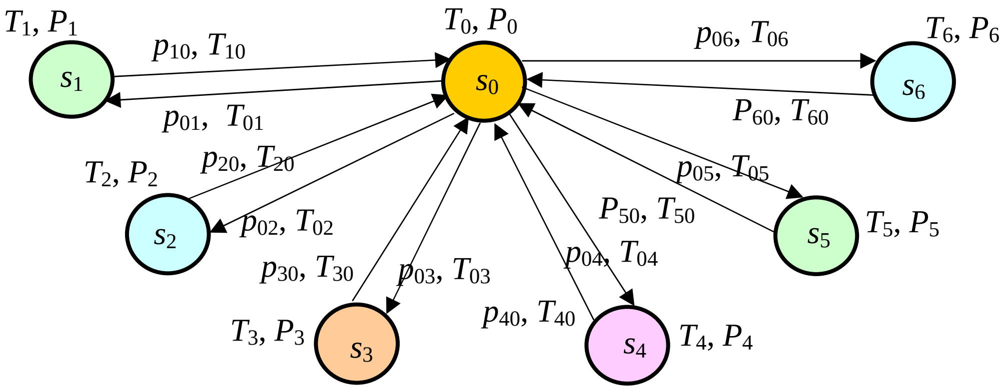

Fig.2. Graph of changes of reliability states s_i; i = 0, 1, ... ,6 for a diagnosing system SDG for a crank-piston mechanism of crosshead main engine:
s_0 – state of full ability of SDG, s_1 – state of disability of the subsystem SDG for acoustic emission testing, s_2 – state of disability of the subsystem SDG for visual (endoscopic) testing, s_3 – state of disability of the subsystem SDG for thermal (temperature, pressure) testing, s_4 – state of disability of the subsystem SDG for vibration (NVH) testing; s_5 – state of disability of the subsystem SDG for wear testing by using X-ray radionuclide fluorescence method (XRF); s_6 – state of disability of the subsystem SDG for thermographic testing (infrared thermal image analysis); T_0 – time of duration of the state of ability s_0; T_1 – time of duration of the state of disability s_1, T_2 – time of duration of the state of disability s_2; T_3 – time of duration of the state of disability s_3; T_4 – time of duration of the state of disability s_4; T_5 – time of duration of the state of disability s_5; T_6 – time of duration of the state of disability s_6;
P_0 – probability of staying SDG in state s_0, P_1 – probability of staying SDG in state s_1, P_2 – probability of staying SDG in state s_2, P_3 – probability of staying SDG in state s_3, P_4 – probability of staying SDG in state s_4, P_5 – probability of staying SDG in state s_5,
P_6 – probability of staying SDG in state s_6, p_{ij} – probability of transition from state s_i to state s_j; T_{ij} – time of duration of state s_i providing that the subsequent state is s_j; i,j = 0,1,2,…, 6; $i \neq j$.

The initial distribution of the process $\{W(t): t \geq 0\}$ is as follows:

$$P\{W(0) = s_i\} = \begin{cases} 1 & dla \quad i = 0 \\ 0 & dla \quad i = 1,\ 2,\ \ldots,\ 6 \end{cases} \tag{6}$$

whereas its matrix function (in accordance with the graph shown in Fig. 2) is of the following form:

$$\mathbf{Q(t)} = \begin{bmatrix} 0 & Q_{01}(t) & Q_{02}(t) & Q_{03}(t) & Q_{04}(t) & Q_{05}(t) & Q_{06}(t) \\ Q_{10}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{20}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{30}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{40}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{50}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{60}(t) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

The matrix function **Q(t)** is a model of changes of the reliability states of *SDG*.

Non-zero elements $Q_{ij}(t)$ of the matrix **Q(t)** depend on the distributions of random variables which are the time intervals of staying the process $\{W(t): t \geq 0\}$ in states $s_i \in S(i = 0, 1, ..., 6)$. The elements are the probabilities of transition of the mentioned process from state s_i to state s_j ($s_i, s_j \in S$) at time not longer than t, defined as follows:

$$Q_{ij}(t) = P\{W(\tau_{n+1}) = s_j,\ \tau_{n+1} - \tau_n < t | W(\tau_n) = s_i\} = p_{ij}F_{ij}(t) \quad (8)$$

where:

p_{ij} – probability of transition at one step in homogeneous Markov chain;

$p_{ij} = P\{Y(\tau_{n+1}) = s_j | Y(\tau_n) = s_i = \lim_{t\to\infty} Q_{ij}(t)$;

$F_{ij}(t)$- distribution function for the random variable T_{ij}, denoting the time of duration of state s_i of the process $\{W(t): t \geq 0\}$, providing that the subsequent state of the process is s_j.

Due to the matrix (7) of the process $\{W(t): t \geq 0\}$ is a stochastic matrix, the matrix of the probability of transition of the Markov chain embedded in this process is as follows [6]:

$$\mathbf{P} = \begin{bmatrix} 0 & p_{01} & p_{02} & p_{03} & p_{04} & p_{05} & p_{06} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

The process $\{W(t): t \geq 0\}$ is irreducible [6, 7] and random variables T_{ij} have finite positive expected values. Therefore, its limiting distribution

$$P_j = \lim_{t\to\infty} P_{ij}(t) = \lim_{t\to\infty} P\{W(t) = s_j\},\ s_j \in S(j = 0, 1, ..., 6) \quad (10)$$

is of the following form [6]:

$$P_j = \frac{\pi_j E(T_j)}{\sum_{k=0}^{6} \pi_k E(T_k)} \quad (11)$$

Probabilities $\pi_j (j = 0, 1, ..., 6)$ in the formula (11) are the limiting probabilities of the embedded Markov chain.

Determination of the limiting distribution (11) requires solution of the following system of equations:

$$\left.\begin{aligned} &[\pi_0, \pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6] = [\pi_0, \pi_1, \pi_2, \pi_3\ \pi_4, \pi_5, \pi_6]\cdot\mathbf{P} \\ &\sum_{k=0}^{6} \pi_k = 1 \end{aligned}\right\} \qquad (12)$$

As a result of solving the system of equations (12) by using the formula (11) the following relationships can be obtained:

$$\left.\begin{aligned} &P_0 = \frac{E(T_0)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)},\ P_1 = \frac{p_{01} E(T_1)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)},\ P_2 = \frac{p_{02} E(T_2)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)}, \\ &P_3 = \frac{p_{03} E(T_3)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)},\ P_4 = \frac{p_{04} E(T_4)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)},\ P_5 = \frac{p_{05} E(T_5)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)}, \\ &P_6 = \frac{p_{06} E(T_6)}{E(T_0) + \sum_{k=0}^{6} p_{0k} E(T_k)}. \end{aligned}\right\} \qquad (13)$$

The probability P_0 is a limiting probability that in longer period of operation (in theory at $t \to \infty$) *SDG* is in state s_0. This probability is therefore a coefficient of the system's technical readiness for diagnosing. However, the probabilities $P_j (j = 1, 2, ..., 6)$ are limiting probabilities of existing states $s_j \in$ S of the system at $t \to \infty$, i.e. the probabilities of being its subsystems SDG_j (j = 1, 2, …, 6) in states of disability, so also the whole *SDG*, due to its serial reliability structure.

An exemplary realization of the process $\{W(t)\colon t \geq 0\}$, showing an occurrence of the reliability states of *SDG* during operation, is presented in Fig. 3.

For the operating practice of *SDG* adopted to identify the states of the diagnosed systems (*SDN*), important is also one-dimensional distribution of the process $\{W(t)\colon t \geq 0\}$, whose elements are the functions $P_k(t)$ denoting the probability that at (any) time t the process is in state $s_k \in S (k = 0, 1, ..., 6)$. This momentary distribution can be calculated by using the initial distribution (6) of the process $\{W(t)\colon t \geq 0\}$ and the functions $P_{ij}(t)$ being the probabilities of transition of the process from state s_i to state s_j ($s_i \in$ S, $s_j \in S$, $i \neq j$; i, j = 0, 1, ..., 6). Calculation of the transition probabilities requires the knowledge of the functions $F_{ij}(t)$, i.e. distribution functions of random variables $T_{ij} (i = j; i, j = 0, 1, ..., 6)$, which are also needed to determine the functions Q_{ij}(t) (with interpretation (8)), which are the elements of the matrix **Q(t)** defined by the relationship (7). Therefore, there are needed the proper reliability tests of *SDG*.

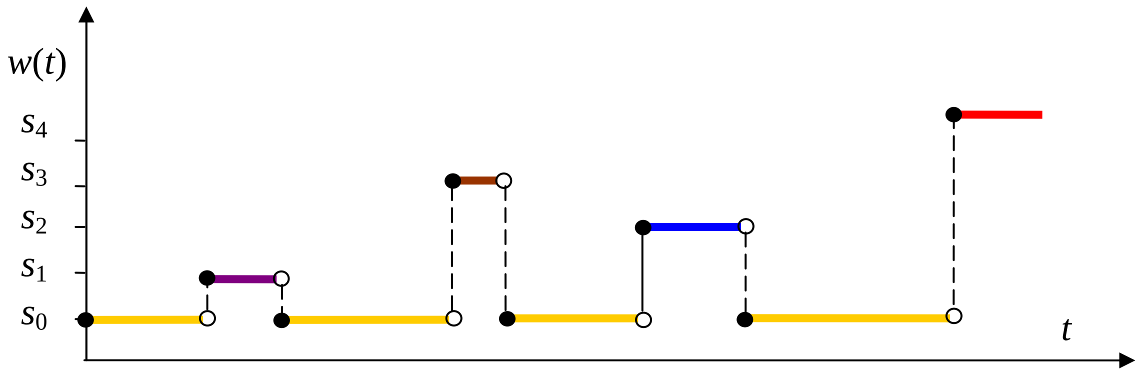

Fig. 3. Exemplary realization of the process {W(t): t ≥ 0} for a diagnosing system (SDG):
s_0 – state of full ability of SDG, s_1 – state of disability of the subsystem SDG for acoustic emission testing, s_2 – state of disability of the subsystem SDG for visual (endoscopic) testing, s_3 – state of disability of the subsystem SDG for thermal (temperature, pressure) testing, s_4 – state of disability of the subsystem SDG for vibration testing (NVH testing)

The presented reliability description of diagnosing systems (*SDG*) can, of course, be developed by specifying as many reliability states as they are essential for the operating practice of the systems, i.e. needed by a user of a given type of systems to ensure their rational operation.

4. Remarks and conclusions

Application of the theory of semi-Markov processes for testing the reliability of diagnosing systems (*SDG*) enables to define not only the probability of staying the systems of this type in state of ability (s_0) and in particular states of disability $s_j(j=\overline{1,6})$, but also the reliability of the diagnosis on the technical state of the diagnosed systems (*SDN*), which can be any mechanisms.

Semi-Markov processes are more and more often used for solving various problems in the field of reliability, mass service and diagnostics of mechanisms.

Application of the processes in the practice requires to satisfy the two conditions:

– collection of the relevant mathematical statistics;

– development of a semi-Markov model of changes of reliability states of a system with a small number of states and a simple (in the mathematical sense) matrix function **Q(t)**.

The second condition is particularly important for calculation of the momentary distribution $P_{ij}(t)$, ($i \neq j$; $i, j = 0, 1, ..., 6$) for the process of changes of reliability states $\{W(t): t \geq 0\}$ for a diagnosing system (*SDG*). As known, this distribution can be calculated when we know the initial distribution of the process $\{W(t): t \geq 0\}$ and the functions $Q_{ij}(t)$ of the matrix **Q(t)**, which are the conditional probabilities of transition of the process from one reliability state to another. Calculation of the transition probabilities $P_{ij}(t)$ consists in solving a system of Volterra integral equations of the second kind (system of equations of convolution type) [6], in which the known quantities are the elements $Q_{ij}(t)$ of the matrix function **Q(t)** for the studied process $\{W(t): t \geq 0\}$. When the number of states of the process is small and/or its matrix function is simple, the system can be solved by applying a Laplace–Stieltjes transform. However, if the number of states of this process is high or when at small number of states its matrix function is very complex, it is possible to obtain only an approximate solution for the system of equations. The solution (numeric) does not provide possibility to determine the probabilities of occurring the particular states of the process, when time of its duration has a large value (in theory, when t $\rightarrow \infty$). From the theory of semi-Markov processes it results that in case of ergodic semi-Markov processes, the probabilities tend over time to strictly defined (constant) numbers. The numbers are called limiting probabilities of the states, and the sequence of the numbers makes a limiting distribution (13).

The limiting distribution enables to define a coefficient of a diagnosing system readiness for proper operation and developing a reliable diagnosis at any time [2, 6].

5. References

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