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MODEL OF BUS ELECTRICAL SYSTEM FAILURE PROCESS

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Abstract

The lifetime distribution is important in reliability studies. There are many situations in lifetime testing, where an item (technical object) fails instantaneously and hence the observed lifetime is reported as a small real positive number. Motivated by reliability applications, we derive the branching Poisson process and its property. We prove that the branching Poisson process is adequate model for the failure process of the bus electrical system. The method is illustrated by two numerical examples. In the second example, we derive the times between the failures of a bus electrical system.

Keywords: *mean residual life function, equilibrium distribution, Poisson process, branching Poisson process, primary and secondary failures, early and instantaneous failures.*

1. Introduction

The standard practice in modeling statistical data is either to derive the appropriate model based on physical properties of the system or choose a flexible family of distributions and then find a member of the family that is appropriate to the data. In both situations it would be helpful if we find the model of lifetime that explains the distribution using important measures of indices. For example, in reliability theory and survival analysis, identification of probability models is often achieved through studying the characteristic measures such as failure rate function, mean residual life function, mean time to failure, burn-in time etc.

Occurrence of instantaneous and early failures in lifetime testing is observed in sets of failures of machines. These occurrences may be due to faulty construction or inferior quality. Some failures result from natural damages of the machine while the other failures may by caused by inefficient repair of previous failures resulting from incorrect organization of the repair process. These situations can be modeled by modifying commonly used parametric model such exponential, gamma and Weibull distributions. In the papers [14,15,16] the set of the failures of machines is divided into two subsets, namely into the set of primary failures and secondary failures. This division suggests that the population of the lifetime is heterogeneous. The set of secondary failures is “similar” to the set of instantaneous and early failures. The population of time to failures can be described by using the statistical concept of mixture of distributions. The lifetime distribution as the mixture of exponential and Rayleigh’s distributions is considered in [14]. The mixture of a distribution with distribution function F and one-point distribution is often analyzed in literature. The problem of statistical inference about the set of parameters when F is exponential is analyzed in [2, 11, 12, 13, 18, 19]. Statistical inference when F is a two parameter gamma distribution is investigated in [20] and when F is two parameter Weibull distribution is considered in [21]. In this paper, we consider the stationary branching Poisson process as a model of the failure process of bus electrical system. The idea of application of the branching Poisson process to the failure process was first introduced by Bartlett [4] and Lewis [17].

2. Definitions and background

Let T be a non-negative random variable denoting the life length of a component having the distribution function F(t) with F(0) = 0, the reliability (survival) function R(t) = 1− F(t), and the probability density function f(t). Then the failure rate function is given by $\lambda(t) = f(t) / R(t)$. We also assume that f(t) is continuous and twice differentiable on $(0,\infty)$. In renewal theory and maintenance the equilibrium distribution corresponding to lifetime distribution play an important role. The distribution function of the equilibrium distribution corresponding to the lifetime T is defined as

$$F_E(t) = \int_0^t R(u)du / ET$$

The probability density function of the equilibrium distribution is (see [6])

$$f_E(t) = R(t) / ET$$

and the failure rate function of the equilibrium distribution is

$$\lambda_E(t) = R(t) / (ET\, R_E(t)), \text{ where } R_E(t) = 1 - F_E(t)$$

A key role in this paper will be playing by the mean residual life function. If $ET < \infty$, then the mean residual life function of T is defined by

$$m(t) = E(X - t \mid X > t) = \int_t^\infty R(u)du / R(t) \text{ if } R(t) > 0$$

and m(t) = 0 for t such that R(t) = 0. Then

$$ET = m(0) = \int_0^\infty R(u)du$$

It is well known that the mean residual function uniquely determines by the reliability function through the following inversion formula

$$R(t) = \frac{m(0)}{m(t)} \exp\left(-\int_0^t \frac{du}{m(u)}\right)$$

for all $t \geq 0$ such that and R(t) > 0 (see [8, 10]).

The mean residual life function m(t) can have various shapes of those labeled increasing, decreasing, bathtub and upside-down bathtub (unimodal) are given the most attention. Many authors [7, 9] convincingly argue that the shapes of m(t) and $\lambda(t)$ provide useful information with

regard to, for example, the quality of a product. The shape of the mean residual life function m(t) also provides a good idea about behavior of failure rate function and vice versa, but the relationship between the two is usually very complex. Thus $\lambda(t)$, m(t) and R(t) are equivalent in the sense that, given one of them, the other two can be determined.
The idea of total time on test (TTT – transformation) processes was first defined by Barlow and Campo [3]. The TTT – transform has been found useful to study the ageing properties of the underling distribution. For the distribution function F(t), we define

$$F^{-1}(t) = \inf\{ x: F(x) \geq t \}, \text{ where } p \in \langle 0, 1).$$

The function

$$H^{-1}(t) = \int_0^{F^{-1}(t)} R(x)dx$$

is called the TTT – transform and the function

$$\Phi_F(t) = H^{-1}(t) / ET$$

is the scaled TTT – transform.
It is known that

$$H^{-1}(1) = ET.$$

Note that if F(t) is the exponential distribution function then scaled TTT – transform is given by $\Phi_F(t) = t$ for $0 \leq t \leq 1$.
Now let $t_{(1)} \leq t_{(2)} \leq, \ldots, \leq t_{(n)}$ to be an ordered sample from life distribution, and let

$$Dj = (n - j + 1)(t_{(j)} - t_{(j-1)}), \text{ where } t_{(0)} = 0,$$

then $$S_j = \sum_{k=1}^{j} D_k \quad \text{for } j = 1,2,\ldots, n.$$

denotes the TTT – transform at $t_{(j)}$, where $S_0 = 0$. The value S_j / S_n is an estimate of the scaled TTT – transform. The TTT – plot is obtained by plotting $u_j = S_j / S_n$ against j / n for j = 0, 1, 2, …, n and joining the points by straight lines. Scaled TTT – transform for some families of the lifetime distributions are given by Barlow and Campo in [3].

3. Model for failures process

In this chapter of paper, we will construct the model of failures process of bus electrical system. This process is built up as follows. There is a series of primary failures, separated by the random variables Z_1, Z_2, … and each of these primary failures generates a subsidiary series of failures. In each subsidiary process there are a random number S of failures separated by random variables Y_1, Y_2, …, Y_s although S may have the value zero in which case no subsidiary failure follows the primary failure. The subsidiary process is assumed to be independent of one another and have identical structure. The complete process is then the superposition of primary failures and subsidiary failures processes, the assumption is that two types of events are indistinguishable. This process is called the stationary branching Poisson process. When this process is used as model for bus failures, a primary event is associated with the initial failure of a component. However, repair of the bus may not always be effected and then, because the bus uses all of its components all the time, a subsidiary failure occurs Y_1 later when the failed component is again needed for the correct operation of the bus The failed component is finally located and removed after S + 1 attempts as repairs have been made. Fortunately, this Poisson assumption is reasonable in some applications and from here we assume that Z_i, i = 1, 2, … are independent and identically distributed with the probability density function.

$$f_Z(t) = \lambda \exp(-\lambda t), \text{ where } \lambda > 0 \text{ and } t \geq 0.$$

Then the series of the random variables Z_i generates the stationary Poisson process. By the random variable T, we denote the time between successive failures in stationary branching Poisson process. Then by [5,17] the reliability function of the random variable T is

$$R_T(t) = \frac{1+ES \times R_Y(t)}{1+ES} \exp\{-\lambda t - \lambda \times ES \int_0^t R_Y(u)du\} \quad (1)$$

Then it is possible to derive almost all of the probabilistic properties of the failure process. Now, the failure process is the stationary branching Poisson process. We denote by T the time between successive failures in the stationary branching Poisson process.
Let

$$ET_T(t) = \int_0^t R_T(u)du$$

By (1), we have

$$ET_T(t) = (1 - \exp(-\varphi(t)) / (\lambda (1 + ES))$$

where $\varphi(t) = \lambda t + ES \int_0^t R_Y(u)du$

We observe that

$$ET_T(0) = 0$$

$$ET_T(\infty) = ET = 1/(\lambda (1 + ES))$$

By [5] and [17] the formula for the variance D^2T is given by

$$D^2T = (1 + 2 ES \exp(-\lambda (1 + ES)) EY) / (\lambda (1 + ES))^2$$

The coefficient of variation of random variable T is

$$V(t) = 1 + 2 ES \exp(-\lambda (1 + ES) EY)$$

Lower and upper bound for the variance and the coefficient of variations can be written as

$$1 / (\lambda^2 (1 + ES)^2) \leq D^2T \leq (1 + 2 ES) / (\lambda^2 (1 + ES)^2)$$

$$1 \leq V(t) \leq 1+2 ES$$

This shows that the coefficient of variations of intervals between the failures in stationary branching Poisson process is always greater than or equal to1. The mean residual life function of T can be written as

$$m(t) = (ET - ET(t)) / R_Y(t)$$

and

$$m(t) = 1 / \varphi'(t)$$

From the above, it is clear that the random variable T has the following properties:

Property 1. The mean residual life function m(t) of the life time T is increasing.

Property 2. For the function $\varphi(t)$, we conclude that $\varphi'(t) > 0$ and , $\varphi''(t) \leq 0$.

Property 3. The failure rate function of the lifetime T is

$$\lambda_T(t) = \varphi'(t) - \varphi''(t) / \varphi'(t)$$

Property 4. The distribution function of the equilibrium distribution is given by

$$F_E(t) = 1 - \exp(-\varphi(t)) \text{ for } t \geq 0$$

Property 5. The failure rate function of the equilibrium distribution is given by

$$\lambda_E(t) = \varphi'(t)$$

4. Numerical examples

Example 1. In this example, we assume that the random variable Y has Weibull distribution with the reliability function

$$R_Y(t) = \exp(- a\, t^{b}) \text{ for } t \geq 0,\ a > 0,\ b > 0.$$

Also, we assume that ES = 1, a = 0.5, λ = 1 and $b \in \{1.5, 1.75, 2\}$. For its parameters sets, we obtain unimodal failure rate functions $\lambda_T(t)$ of T. Figure 1 shows the realization of numerical calculation of $\lambda_T(t)$.

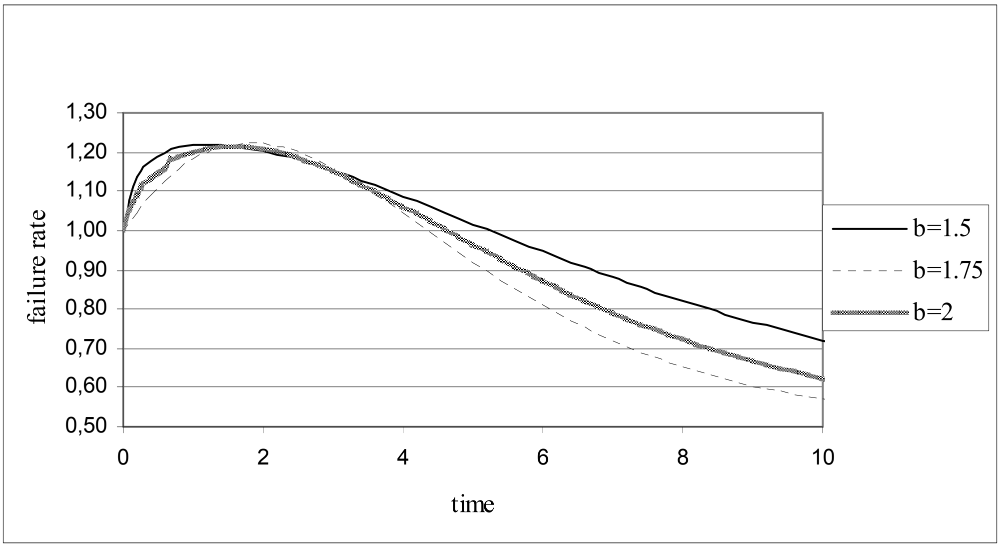

Fig.1. Failure rate function for b∈{1.5, 1.75, 2}

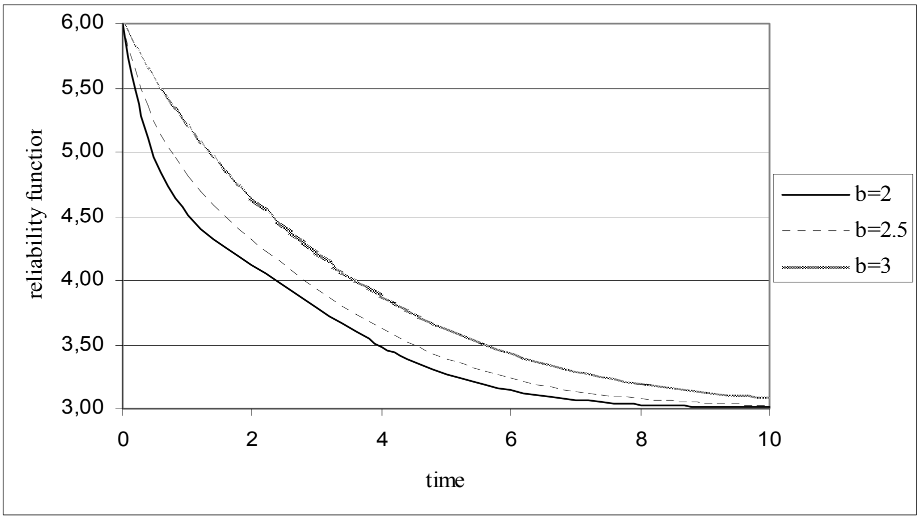

Fig.2. Failure rate function for b∈{2, 2.5, 3}

In the second part of this example, we assume that $a = 0.5$, $\lambda = 1$ and $b \in \{2, 2.5, 3\}$. Realization of these computation shows Figure 2. All the failure rate functions are decreasing.

Example 2. The object of the investigation is a real municipal bus transport system within a large agglomeration. The analyzed system operates 190 municipal buses of various types and makes. In this example, we consider a real time data on failure of the electrical system of a bus. The data set contains n = 2565 times between successive failures of the electrical system of a bus. We apply the maximum likelihood estimates of the parameters a, b and λ. As the initial solution of the likelihood equation, we give b = 1. We calculate the values of the parameters a = 0.8744, b = 0.40729, λ = 0.05457, and ES = 1. For these values of parameters, we prove the Kolmogorov's test of goodness of fit and compute the associated p–value = 0.87. It shows a good conformity of the empirical data with the distributions with reliability function (1). Figure.3 shows the reliability functions for this example. Figure 4 shows the graphs of the function $\varphi_T(t)$ for estimated parameters, $\varphi_E(t)$ for data set and $\varphi_{EXP}(t)$ for the exponential distribution. Figure 5 shows the graphs of the empirical TTT–transformation of empirical distribution (TTTe), estimated distribution (TTTt) obtained by (1), and for the exponential distribution (EXP) (see[1]).

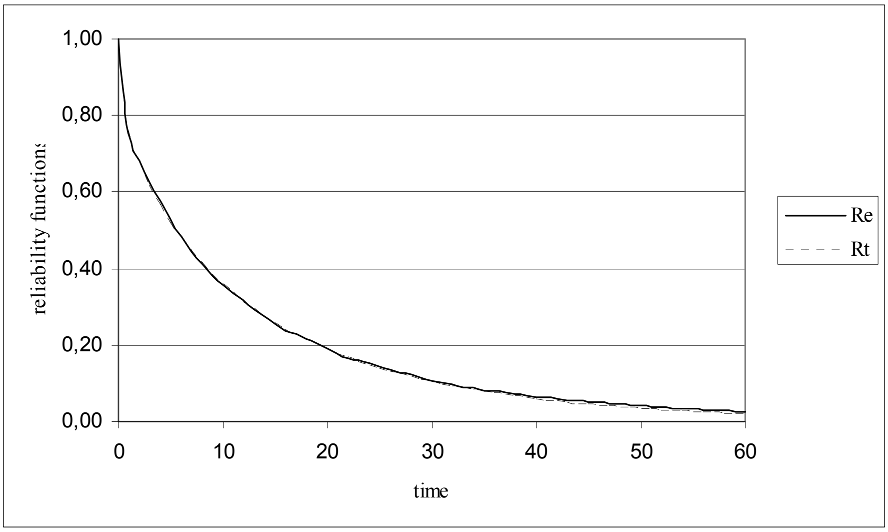

Fig. 3. Reliability functions empirical and theoretical

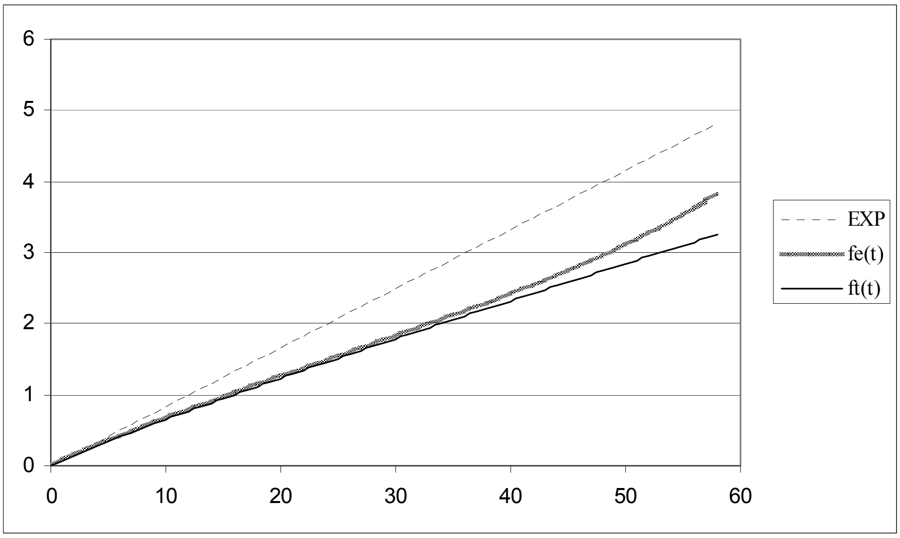

Fig.4. Graphs functions $\varphi_e(t)$, $\varphi_t(t)$ and $\varphi_{EXP}(t)$

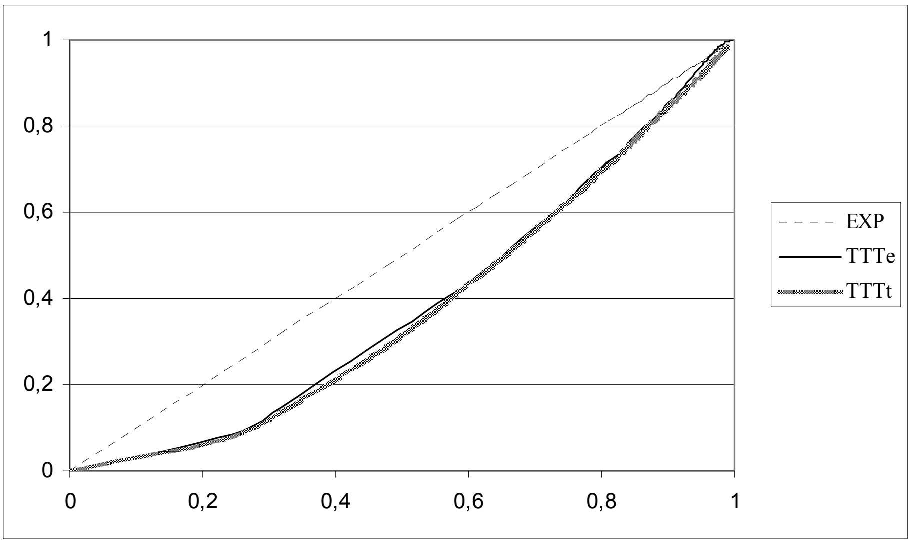

Fig. 5. Plots TTT-transform empirical, theoretical and exponential

5. Conclusions

In this paper, we study the lifetime model for instantaneous and early failures of bus electrical system. When the parameters are estimated, it is possible to apply for further calculations, such that as MTTF (Mean Time to Failure), burn–in time, the failure rate function and the replacement time. The development of efficient parameters estimation methods for this failure process model and their application for times to failure modeling are topics for further study. An application to a real data set shows that this model may be applicable in practice.

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