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USEFULNESS OF SEMI-MARKOV PROCESSES AS MODELS OF THE OPERATION PROCESSES FOR MARINE MAIN ENGINES AND OTHER MACHINES OF SHIP POWER PLANTS

Jerzy GIRTLER

Gdańsk University of Technology, Faculty of Ocean Engineering and Ship Technology
***Department** of Marine and Land Power Plants*
80-951 Gdańsk, ul. G. Narutowicza 11/12, fax/tel. (058) 347 1981,
e-mail: jgirtl@pg.gda.pl

Abstract:

The paper describes the properties of semi-Markov processes and the opportunities and benefits from their use as models of the operation processes for marine combustion engines and other machines of ship power plants. The emphasis is put on the importance of the theory of semi-Markov processes for development of the theory of marine combustion engines and other machines of ship power plants, as well as for development of the operational progress of the machines. The usefulness of the theory of semi-Markov processes in the theory and operational practice of ship main engines and other machines of ship power plants is exposed, to define operational reliability and safety indexes for the machines. There are defined the conditions with regard to the properties of semi-Markov processes, which should be satisfied to enable the use of them as models of the real processes running during operation of marine main engines and other ship's machinery. The suitability of decision (controlled) semi-Markov processes for making operating decisions is also proved herein, as their aim is to obtain a rational process of operation of the mentioned engines and machines.

Keywords: load, semi-Markov process, diesel engine, wear

1. Introduction

During operation of marine main engines and other machines of ship power plants, there is a need to apply the models of their operation processes. This is due to the fact that the operation phase of engines and machines desires rational control over realization of the operation processes of the machines. The models of the processes, which are indispensable for this purpose, allow to plan actions and make rational operating decisions.

The studies on various machines show that building models of operation processes for the machinery, including marine main engines and other machines of ship power plants, in the form of stochastic processes, requires application of the theory of semi-Markov processes and the theory of decision (controlled) semi-Markov processes [3, 4-11, 13, 14, 15, 20-22, 26]. The mentioned publications, which expose the properties and significance of the theories, argue in favour of their application. However, the application of these theories is not easy due to the nature of semi-Markov processes. Ignorance of the nature or not sufficient consideration of it, may lead to development in the form of the semi-Markov processes, inadequate models of the real processes of operation for marine main engines and other machines of ship power plants. Therefore, the theory of semi-Markov processes should be skillfully used for modeling the operation processes for the machines, including the specificity of the theories. This specificity shows in which cases the semi-Markov processes can be used as models of the real processes of operation of the engines and other machines. This indicates the necessity of having knowledge on physical aspects of applying semi-Markov processes as models of the processes of changes in technical and operational states of marine main engines and other machines of ship power plants, and also the processes of simultaneous changes in the mentioned types of states, which are called the processes of machine operation [4-12].

2. Extent of application of the theory of semi-Markov processes during operation of marine main engines and other machines of ship power plants.

The theory of semi-Markov processes is increasingly used in studies on reliability, wear and safe operation of machines, with regard to decision making, in order to optimize the operation processes of the machines, including marine main engines and other machines of ship power plants. It allows to build models of different real processes, including the models of machine operation processes [5–13, 15, 20, 22, 26]. The models are created in the form of specific stochastic processes, which are semi-Markov processes and recently also decision (control) semi-Markov processes, i.e. whose realizations depend on decisions made at time of changes in their states [9, 10, 13, 15, 16, 21]. The processes are not easy to apply due to their nature. Ignorance of this nature is the cause of building models of real processes as semi-Markov processes which are not able to provide new information on the modeled process, e.g. on life and reliability of the machines, their load spectra, etc. Accordingly, it is necessary to draw attention to the physical aspects of application of semi-Markov processes as the models of real operation processes for marine main engines and other machines of ship power plants. This requires identification of the specific nature of this type of stochastic processes.

A model of any real process in the form of a semi-Markov process, can be built only when the states of the process can be such determined that the duration of a state existing at the moment τ_n and the state possible to get at the moment τ_{n+1} do not depend stochastically from the earlier states and their durations [6, 7, 9, 11-15]. Development of such a model of the real operation process of marine main engines and other machinery of ship power plants is a necessary condition for applying the theory of semi-Markov processes. This theory allows to determine probabilistic characteristics of any random process (if such a model was developed for it in the form of a semi-Markov process), which may have an important practical meaning.

For the operation process of marine main engines and other machinery of ship power plants, mainly the following characteristics are of the important practical meaning [7, 8, 11]:

- one-dimensional distribution of the process (momentary distribution), whose elements are the functions $P_k(t)$ constituting the probability that at time t the process reaches the state *k*.
- limiting distribution of the process $P_j = \lim_{t\to\infty} P\{X(t) = j\}$,
- conditional probabilities called probabilities of the process transition from state „*i*” to „*j*”, $P_{ij}(t) = P\{X(t) = j / X(0) = i\}$ (transition probabilities)
- expected value $E(T_i)$ of the duration time T_i of the i-th state of the process regardless which state it transits at the moment τ_{n+1} to,
- variance $D^2(T_i)$ of the duration time T_i of the i-th state,

- expected value $E(T_{ij})$ of the duration time T_{ij} of the i-th state of the process, providing that the j-th state is the successive state.

States i and $j(i, j = 1, 2, n; i \neq j)$ can be interpreted differently. The publications [5, 6, 8, 9, 11-15] provide interpretations of technical, operational and other states.

Obtaining numerical values for the listed characteristics is possible if the following two conditions are satisfied [7, 9, 10, 12, 13, 15, 26]:

- there are collected the adequate statistics, whose values constitute estimations of the transition probability p_{ij}, of the expected value $E(T_i)$, etc.,
- there is developed a model of the machine operation process in the form of a semi-Markov process, but with a small number of its states and, simultaneously, an uncomplicated (in the mathematical sense) matrix function **Q(t)**.

The second condition is essential to calculate the momentary distribution of states of the process $P_k(t)$, $k = 1, 2, \ldots, m$. As known, the distribution can be calculated from the initial distribution of the process (3) and the functions $P_{ij}(t)$ defined by the formula (9). Calculation of the probabilities $P_{ij}(t)$ consists in solving the system of Volterra equations of the second kind, where the functions $Q_{ij}(t)$, that are the elements of the matrix function **Q(t)** of the process, are known values (2). When the number of states of the process is small, and when thereby the matrix function **Q(t)** of the process is uncomplicated, this system can be solved by applying the Laplace transform [9, 13]. However, when the number of states of the process is high, and the matrix **Q(t)** (kernel of the process) is highly complex, only an approximate solution can be obtained to the system. This solution (numeric one) does not allow to determine the probability values of occurrence of the particular states of the process, when t takes a large value (theoretically, for $t \to \infty$). A numerical solution does not answer the question which is very important for operational practice: how do the probabilities of states of the semi-Markov process change, when t is high? The theory of semi-Markov processes shows that for ergodic semi-Markov processes the probabilities tend over time to the precisely defined constants [13]. The constants are called the limiting probabilities of states and the sequence of the constants forms a limiting distribution of the process. This distribution allows to define the coefficient of the machine's technical readiness and assess the economic income or working costs per unit of the operating time [9, 13]. The obtained quantities are the criterion functions for solving the problems of optimization of machinery operation. This distribution can also be calculated much easier than the momentary distribution.

3. Interpretation of a semi-Markov process

Semi-Markov processes are stochastic processes with special properties. The publications concerning the semi-Markov processes provide different definitions of a semi-Markov process, which are of different scale of generality and accuracy. For the need of modeling of the process of machinery operation, a semi-Markov process (family of random variables) $\{X(t): t \geq 0\}$, can be defined with the so-called homogeneous Markov renewal process. Such a definition provided by F. Grabski [13, 15] is similar to the definitions of W.S. Jewell [17], V.S. Koroluk and A. F. Turbin [19, 20], R. Pyke [25], D.S. Silvestrov [26], and also E. Cinlar [1].

The definition of a semi-Markov process is based on the assumption that the consecutive moments $\{\tau_n\}_{n=0,1,2...}$ of time t create an increasing sequence of non-negative random variables, such that $\tau_0 = 0$. The stochastic process $\{X(t): t \geq 0\}$, that in the random intervals $[\tau_n, \tau_{n+1})$

takes the constants from the finite or countable set S, is called a semi-Markov process (the SM process), if for any $i, j \in S$ and any $t \geq 0$ the following equation is true [13, 15]

$$P\{X(\tau_{n+1}) = j, \tau_{n+1} - \tau_n \leq t \mid X(\tau_n) = i, X(\tau_{n-1}),..., X(\tau_0), \tau_n - \tau_{n-1},..., \tau_1, \tau_0\} =$$

$$= P\{X(\tau_{n+1}) = j, \tau_{n+1} - \tau_n \leq t \mid X(\tau_n) = i\} = Q_{ij}(t). \qquad (1)$$

This condition means that a state of the semi-Markov process and its duration depends only on the directly preceding state, and not on the earlier states and their duration.

The matrix function of the process $\{X(t) : t \geq 0\}$, called the kernel of the semi-Markov process,

$$\mathbf{Q(t)} = \lfloor Q_{ij}(t) \rfloor, \qquad (2)$$

where

$$Q_{ij}(t) = P\{X(\tau_{n+1}) = j, \tau_{n+1} - \tau_n \leq t \mid X(\tau_n) = i\}$$

and its initial distribution

$$p_i = P\{X(\tau_0) = i\}, \quad i \in S \qquad (3)$$

fully define the semi-Markov process [13, 15, 20, 26].

The definition of the SM process indicates that the sequence $\{X(\tau_n) : n = 0, 1, 2,...\}$ is a Markov chain with the matrix of transition probabilities

$$P = \lfloor p_{ij} \rfloor, \qquad (4)$$

where

$$p_{ij} = \lim_{t \to \infty} Q_{ij}(t).$$

The sequence of the random variables $\{X(\tau_n) : n = 0, 1, 2,...\}$ ***is called a Markov chain embedded in the SM process*** $\{X(t) : t \geq 0\}$.

The function

$$F_{ij}(t) = P\{\tau_{n+1} - \tau_n \leq t \mid X(\tau_n) = i, X(\tau_{n+1}) = j\} = \frac{Q_{ij}(t)}{p_{ij}} \qquad (5)$$

is a distribution function of the random variable T_{ij} defining duration of the state i, where the successive state is the state j. The below formula is derived from the previous formula (5):

$$Q_{ij}(t) = p_{ij} F_{ij}(t). \qquad (6)$$

The function

$$G_i(t) = P\{\tau_{n+1} - \tau_n \leq t \mid X(\tau_n) = i\} = \sum_{j \in S} Q_{ij}(t) \qquad (7)$$

is a distribution function of duration of the state i. The random variable with the distribution defined by the distribution function is denoted with the symbol T_i.

In the models of machine life and reliability, the parameters and characteristics of the *SM* process are reflected in the reliability parameters and features of the machinery (marine combustion engine or other devices of ship power plants). The transition probabilities defined as conditional probabilities, belong to the significant characteristics of the process

$$P_{ij}(t) = P\{X(t) = j \mid X(0) = i\}, \; i, j \in S \tag{8}$$

The probabilities (8) satisfy the Feller equation [13]

$$P_{ij}(t) = \delta_{ij}[1 - G_i(t)] + \sum_{k \in S} \int_0^t P_{kj}(t-x)\, dQ_{ik}(x), \qquad i, j \in S \tag{9}$$

In many cases, the conditional probabilities $P_{ij}(t)$ defined by the formula (8) and the probabilities of states

$$P_j(t) = P\{X(t) = j\}, \; j \in S \tag{10}$$

after some time, stabilize and their values tend to precisely defined constants. These probabilities can be replaced by the limiting probabilities

$$P_{ij} = \lim_{t \to \infty} P_{ij}(t), \qquad P_j = \lim_{t \to \infty} P_j(t), \tag{11}$$

The works by Koroluk and Turbin [19], as well as the work by Grabski [13, 15], provide the theorems comprising sufficient conditions for existence of the limiting probabilities. From the theorems, there are limiting probabilities that describe semi-Markov processes as models of the operation process of combustion engines and other machines of ship power plants [5, 7, 9, 13, 15]. They are expressed by the formula

$$P_{ij} = \lim_{t \to \infty} P_{ij}(t) = P_j = \lim_{t \to \infty} P_j(t) = \frac{\pi_j E(T_j)}{\sum_{i \in S} \pi_j E(T_j)}, \tag{12}$$

where the probabilities $\pi_j, \; j \in S$ constitute a stationary distribution of the embedded Markov chain in the *SM* process, so the distribution which satisfies the system of linear equations [13]:

$$\sum_{i \in S} \pi_i p_{ij} = \pi_j, \quad j \in S, \qquad \sum_{i \in S} \pi_i = 1. \tag{13}$$

4. Physical aspects of application of semi-Markov processes as models of operation processes for marine diesel engines and other machinery of ship power plants

The presented considerations indicate that models in the form of semi-Markov processes are characterized by the following [9, 15]:

1) It is satisfied the Markov condition, that future evolution of the operation process of a given machine (marine combustion engine or other machinery of ship power plant) for which the model is built as a semi-Markov process, depends only on its state at a given time *t* of its operation, and not on the operation (functioning) of the machine in

the past, thus that the *future* of the process does depend not on its *past*, but on its *present*,

2) random variables T_i and T_{ij} have distributions other than exponential.

Therefore, the modeling, whose the aim is to develop a model of the operation process, in the form of a semi-Markov process, for any ship's propulsion machinery, including main engine, should include an analysis of state changes in its real process of operation.

For each marine combustion engine (and any other ship propulsion machine), its operation process can be interpreted as a process of simultaneous changes in its technical and operational states [4, 6, 7, 9, 11]. The duration intervals of each of the process states are random variables. Particular realizations of the random variables depend on many factors, including wear of the machine. In the case of diesel engines and many other machines of ship power plants, it was stated that wear of their sliding tribological systems (e.g. bearings) is weakly correlated with time [2, 20, 22, 28, 29, 30]. This observation allows to predict the technical states of the machines by considering only their current states and ignoring the states that existed earlier. An explanation of this fact enabled to develop (as a result of application of the theory of Markov processes or the theory of semi-Markov processes) more adequate probabilistic mathematical models needed to predict the technical states of particular diesel engines and other machines of ship power plants [5, 9, 11]. The mentioned explanation can be formulated as the following hypothesis (H): *state of any sliding tribological system of each machine installed in a ship power plant, and its duration depend significantly on the directly preceding state and not on the earlier states nor their duration periods, because the load of the system as well as the wear rate and the wear increments implied by the load, are the processes with asymptotically independent values*.

The point in the hypothesis that the load as well as the wear rate and wear increments implied by the load, are the processes with asymptotically independent values, results from two facts recognized during empirical research:

1) there is a close relationship between load of sliding tribological systems and their wear [22, 24, 25, 28, 29];
2) there are no monotonic changes in load of the tribological systems in machines in the longer time of their operation, and therefore it can be assumed that the load of the systems is stationary [22, 24, 27, 28].

Stationarity of the load (in a broader sense) means that all the multi-dimensional probability density functions depend only on the mutual distance between the moments τ_1, τ_2, ..., τ_n,, but do not depend on them themselves [2, 8, 11]. Thus, the one-dimensional probability density function of the load value does not depend on the moment, which this value corresponds to, and the two-dimensional probability density function depends only on the difference between the moments, at which the observed values of load appeared. In turn, in the narrower sense, the stationary load (stationary completely) is understood as the load (being a process) whose all possible higher order statistical moments and the combined moments are time independent. If the process is completely stationary (in the narrow sense): the expected value of load $m(t) = m = \text{const}$, and also the variance $V(t) = \sigma^2 = \text{const}$, the autocorrelation $A(\tau_1, \tau_2) = A^*(\tau_2 - \tau_1) = A^*(r)$ and the autocovariance $K(\tau_1, \tau_2) = K^*(\tau_2 - \tau_1) = K^*(r)$. The stationary process in a broader sense is characterized by the following: $m(t) = m = \text{const}$ and $A(\tau_1, \tau_2) = A^*(\tau_2 - \tau_1) = A^*(r)$. In practice, stationarity of load *in the broader sense* is of significant meaning. However, study of load on tribological systems in order to determine the mentioned properties is not necessary in this case. From research so far on different machines it is known that load on their tribological systems changes in a continuous manner so that the individual values measured after very small time intervals are strongly correlated with each other. However, when the time space (distance) between the measurements of the load increases, the correlation between the loads decreases. Thus, the load values measured at time intervals (or at moments) significantly distant from each other, can be considered as independent. This property is called the asymptotic independence of the load value measured at the moment, e.g. τ_{i+1}, from the value measured at the moment τ_i when the gap $\Delta\tau = \tau_{i+1} - \tau_i$ ($\tau_{i+1} > \tau_i$) is large enough. Such understood the asymptotic independence between the load values measured (or calculated) at the moments τ_i and τ_{i+1} reflects the fact that with the increase of the gap $\Delta\tau$ the dependence between these values decreases. Nevertheless, it is known from the operation principles of each machine that in the long time of proper operation, the load considered over time does not undergo (and cannot undergo) any changes, neither monotonically increasing nor decreasing. Thus, it can be assumed that the maximum (as well as minimum) values of load occur randomly at the designated moments, always with a defined probability. This lack of monotonicity of load is called the load stationarity [2, 22, 28].

Verification of the hypothesis (*H*) requires to determine (foresee) the consequences, of which occurrence can be established in empirical studies, if the hypothesis is true. The consequences (*K*), which can be deduced (inferred) from this hypothesis (with regard to the mentioned properties of loads on the machines of ship power plants and their sliding tribological systems) are as follows [2, 11, 22, 28]:

- K_1 – irregular run of the wear process of particular sliding tribological systems,
- K_2 – interweaving realizations of the wear processes of specific sliding tribological systems,
- K_3 – flow of the autocorrelation function for a specific sliding tribological system, that with increase in the gap $\Delta\tau$, decreases rapidly from the beginning and then oscillates around zero, with a relatively small and still getting smaller amplitude,
- K_4 – almost normal distribution of wear increments of sliding tribological systems for a sufficiently long time interval (Δt) of their correct work,
- K_5 – linear dependence of the variance of the wear process of sliding tribological systems, from their operating time.

The consequences can be justified by that when the load characteristics of ship's propulsion machines and their tribological systems are as mentioned, thus the wear process of the systems should be of irregular flow. This, in turn, gives a base to recognize that the wear increments recorded at time intervals significantly distant from each other, are asymptotically independent, and that with the time increase (gap θ, where e.g. $\theta = h\Delta\tau$, $h = 1, 2, ..., n$) between the intervals, the dependence between the mentioned wear increments, decreases. Hence, the wear processes of the systems can be considered as the processes with asymptotically independent increments [2].

One of the most important characteristics of the wear process, like of each stochastic one, is the autocorrelation function $r(\theta)$. This function constitutes the dependence of the autocorrelation coefficient r from the gap θ, so $r = f(\theta)$ is the coefficient value of autocorrelation of two wear increments for the given tribological system, between which the time θ passed. If the function $r(\theta)$ decreases, it can be assumed that the wear process of the tested tribological system is the process with asymptotically independent increments. An assumption that such a process is a wear process of tribological systems, is equal to that a normal distribution of wear increments of this process over the sufficiently long period of time should be expected in approximation. It is also known that if a wear process has asymptotically independent increments, the variance of the process $V(t)$ increases linearly over the time of correct work [2].

The consequences K_i($i = 1, 2, ..., 5$) disclose the probabilistic logic of wear of sliding tribological systems. They are not mutually contradictory, and their logical truthfulness raises no doubts. Thus, the non-contradiction condition for the consequences is satisfied. This means that nothing prevents from using the listed consequences as a combined, i.e. one, consequence *K*, to test empirically the hypothesis (*H*), so to verify the hypothesis for accepting or falsifying. Such verification consists in experimental testing of wear of sliding tribological systems and in checking the veracity of the consequences $K_i(i = 1, 2, ..., 5)$, thus $K_i \in K$, which is synonymous with finding whether the consequences K_i (as facts) occur or not. Verification of the hypothesis *H* requires recognition of the following syntactic implications as true [9]:

$$H \Rightarrow K \tag{14}$$

Then, the non-deductive (inductive) inference can be applied, which proceeds according to the following scheme [9, 23]:

$$(K, H \Rightarrow K) \vdash H \tag{15}$$

where

$$K = \{K_i, i = 1, 2, ..., 5\}$$

The logical interpretation of this inference scheme is as follows: *If the experimental verification of the consequences* $K_i \in K(i = 1, 2, \ldots, 5)$ *proves its rightness, and if the implication* (14) *is true, also the hypothesis H is true and can be accepted.* The inductive inference according to the given scheme (15) is called the reductive inference [23]. The inference, like any other of the same type, does not lead to reliable conclusions, but only probable.

5. Semi-Markov decision processes

Semi-Markov decision (control) processes are a convenient mathematical tool tor formulating and solving decision problems related to operation of marine combustion engines and other machines of ship power plants [9, 10, 13, 16, 21]. The literature on semi-Markov decision processes is extensive. The key publications include works by W.S. Jewell, [18] R.A. Howard [16] H. Main and S. Osaki [21] I. B. Gercbach [3] and F. Grabski [13, 15]. Semi-Markov decision (control) processes were also investigated in the publications [5, 9, 10].

Semi-Markov decision process is the *SM* process $\{X(t): t \geq 0\}$, whose realization depends on the decisions made at the initial moment τ_0 and the moments of state changes $\tau_1, ..., \tau_n, ...$. The decision at the moment τ_n, when $X(\tau_n) = i$, is denoted with the symbol $d_i(\tau_n) \equiv k$. It is assumed that the set of decisions D_i for each state *i* is finite. Decision making means selection of the *i*-th row of the functional matrix (process kernel), which defines the probabilistic mechanism of the process evolution for the interval $[\tau_n, \tau_{n+1})$. The selection is made from the set

$$\left\{Q_{ij}^{(k)}(t): t \geq 0, k \in D_i, i, j \in S\right\}, \tag{16}$$

where

$$Q_{ij}^{(k)}(t) = p_{ij}^{(k)} F_{ij}^{(k)}(t),$$

whereas the set of states *S* is finite. Denoting the particular states of the process as 1, 2, 3, …, N, a set of states $Z = \{1, 2, ..., r\}$ can be considered. The states, however, can be denoted as: $z_1, z_2, z_3, \ldots, z_r$.

The decision $d_i(\tau_n) \equiv k \in D_i$ means that the semi-Markov process evolves in a way that the process state *j* is drawn in accordance with the distribution $(p_{il}^{(k)} : l \in S)$, to which the transition takes place at the moment τ_{n+1}, whereas the length of the interval $[\tau_n, \tau_{n+1})$ is drawn according to the distribution defined by the distribution function $F_{ij}^{(k)}(t)$.
The sequence

$$d = \{(d_1(\tau_1),...,d_N(\tau_n)) : n = 0,\ 1,\ 2,\ ...\} \tag{17}$$

is called *the decision strategy* [13, 16, 21]. The elements of the sequence are vectors whose components are the decisions made at the defined states, in the moments of changes of these states. During operation of combustion engines and other machines of ship power plants, the Markov strategies are of particular significance.

The strategy *d* is called the *Markov strategy* if, for each state $i \in S$, and each moment of change in state τ_n, $n = 1,\ 2,\ ...$, the decision $d_i(\tau_n) \in D_i$ does not depend on realization of the process till the moment τ_n. When the decision does not also depend on *n*, i.e. $d_i(\tau_n) = d_i$, the strategy is called *the stationary decision strategy.* Then the semi-Markov decision process is a homogeneous process. Optimization of the semi-Markov decision process consists in selection of such a strategy for which the function constituting the optimization criterion takes the extreme value (maximum or minimum).

The model of operation process for combustion engines, as a semi-Markov process is presented in the publications [4, 5, 9]. It allows to determine reliability and safety indexes necessary to plan and ensure the rational operation of marine combustion engines. It can also be used to determine the reliability and safety indexes for other machinery of ship power plants. In operational practice, the decision control over the operation process of this type of machines requires, however, a model in the form of a semi-Markov decision (controlled) process. Such a model allows optimization of the operation process of marine combustion engines and other machines of ship power plants. Examples of such optimization are provided in the publications [9, 10, 13, 16, 21].

6. Remarks and conclusions

In research, the semi-Markov processes are convenient models of the real processes that run during machine operation. This is due to the fact that construction of a semi-Markov model for any process, particularly the operation process of a machine of any kind, allows (due to the existing theory of semi-Markov processes) to determine easily probabilistic characteristics of the process.

Semi-Markov processes as models of real processes of machine operation are more useful in practice than Markov processes. This is because the finite-state continuous-time semi-Markov processes are characterized by that the time intervals of staying the processes in particular states are random variables with arbitrary distributions focused on the set $R_+ = [0, \infty)$. This fact differs them from Markov processes, whose time intervals are random variables with exponential distributions.

A semi-Markov model of the machine operation process is a finite-state continuous-time process.

An additional advantage of applying the semi-Markov processes (just like in the case of Markov processes) is that there are available professional computer tools which allow to solve various systems of equations for states of this type of models of real processes.

A semi-Markov model of the operation process can be applied for such technical devices like machines, even when using diagnosing systems (SDG) [9]. This is due to the fact that when formulating a diagnosis (inference) on the state of a given technical device (combustion engine, other machines of ship power plants), considered as a diagnosed system (SDN), the expression that *there is just this and no other vector of values of diagnostic parameters*, is recognized as an absolutely reliable premise. This expression can be denotes as K_d. However, the expression that *there is just this and no other state of SDN* can be denoted as S_{SDN}. This expression is an inference formulated on the basis of the expression K_d,, in the process of non-deductive inference [9, 22, 29]. Such inference proceeds in accordance with the following scheme [9]:

$$(K_d, S_{SDN} \Rightarrow K_d) \vdash S_{SDN}$$

where:

K_d absolutely reliable premise

S_{SDN} inference formulated on the basis of the expression K_d.

The output of this inference is therefore the following hypothesis: *SDN is in the state* S_{SDN},*, because the vector* K_d *of values of diagnostic parameters is observed.* Of course, this hypothesis (hypothesis about the state of *SDN*) may also be formulated in other (equivalent) term, that is: *The vector* K_d *of values of diagnostic parameters is observed because SDN is in the state* S_{SDN}.

Such an inference is reductive one which (as mentioned above) does not allow to develop a reliable inference (in this case, the expression S_{SDN}), only probable inference. Therefore, it is not possible to determine exactly the technical state of *SDN* and thus to control the process of its operation in a way to make the future state dependable on a number of states that occurred earlier.

References

1. Cinlar E.: Markov reneval theory: a survey. Mag. Sci. Nr 7, 1975, p. 727-752.
2. Gercbach I. B., Kordonski CH.B.: Modele niezawodnościowe obiektów technicznych. WNT, Warszawa 1968.
3. Gercbach J.B. Modeli profilaktiki. Sovetskoe Radio, Moskva 1969.
4. Girtler J.: Model procesu eksploatacji okrętowego silnika spalinowego. ZEM, z. 4/1988, s.121-130.
5. Girtler J.: Stochastyczny model procesu eksploatacji okrętowego silnika spalinowego. Zagadnienia Eksploatacji Maszyn, z. 2/1989, s. 79-88.
6. Girtler J.: Semi-Markov model of changes of sea-ships' and aircrafts' moving. Archives of Transport, vol. 11, iss.1-2/99 pp.
7. Girtler J.: Procesy semimarkowskie jako modele procesów eksploatacji urządzeń siłowni okrętowych. Materiały XIV Międzynarodowego Sympozjum Siłowni Okrętowych. Politechnika Szczecińska, Wydział Techniki Morskiej, 1992, s.201-208.
8. Girtler J.: Możliwości zastosowania i przydatność procesów semimarkowskich jako modeli procesów eksploatacji maszyn. Zagadnienia Eksploatacji Maszyn, z. 3/1996, s. 419-428.
9. Girtler J.: Diagnostyka jako warunek sterowania eksploatacją okrętowych silników spalinowych. Studia Nr 28, WSM, Szczeci 1997.
10. Girtler J.: Koncepcja decyzyjnego sterowania procesem eksploatacji urządzeń technicznych z uwzględnieniem diagnostyki. Problemy Eksploatacji, Nr 3/58 (2005) s. 69-80.
11. Girtler J.: Physical aspect of application and usefulness of semi-Markovian processes for modeling the processes occurring in operational phase of technical objects. Polish

Maritime Research 2004 nr 3(41), vol. 11, s. 25-30.
12. Girtler J.: Teoria decyzyjnych procesów semi-Markowa i jej zastosowanie w projektowaniu i eksploatacji okrętowych silników głównych i innych urządzeń siłowni okrętowych. Materiały VII Międzynarodowej Konferencji Naukowo-Technicznej EXPLO-DIESEL & GAS TURBINE'14 „Utrzymanie w ruchu silników tłokowych i turbinowych z uwzględnieniem ochrony środowiska naturalnego", Gdańsk-Gdynia-Karskrona, 23-27.05.2014. Płyta CD, 15 str.
13. Grabski F.: Teoria semi-markowskich procesów eksploatacji obiektów technicznych. Zeszyty Naukowe AMW, nr 75 A, Gdynia 1982.
14. Grabski F.: Czas pierwszego przejścia procesu semi-Markowa do podzbioru stanów. Zeszyty naukowe AMW, Nr 3, Gdynia 1981.
15. Grabski F.: Semi-markowskie modele niezawodności i eksploatacji. Polska Akademia Nauk, IBS.Warszawa 2002. Seria: Badania Systemowe, tom 30.
16. Howard R.A.: Research in semi -Markovian decision structures. J. Oper. Res. Soc. Japan, 1964, 6, nr 4, p.163-199.
17. Jaźwiński J., Borgoń J.: Niezawodność eksploatacyjna i bezpieczeństwo lotów WKiŁ, Warszawa 1989.
18. Jewell W.S. Markov-renewal programming. Operation Research 11, 1963, p.938-971.
19. Koroluk V.S., Turbin A.F.: Fazovoje ukrupnenia sloznych sistem. Wyssaja Skola, Kijev 1978.
20. Koroluk V.S., Turbin A.F: Polumarkovskije processy i ich priłożenija. Naukova Dumka, Kijev 1976.
21. Mine H., Osaki S.: Markovian decision processes. AEPCI, New York 1970.
22. Niewczas A.: Podstawy stochastycznego modelu zużywania poprzez tarcie w zagadnieniach trwałości elementów maszyn. Zeszyty Naukowe Politechniki Radomskiej, nr 19, Rdom 1989.
23. Pabis S.: Metodologia i metody nauk empirycznych. PWN, Warszawa 1985.
24. Piotrowski I., Witkowski K.: Eksploatacja okrętowych silników spalinowych. AM, Gdynia 2002.
25. Pyke R.: Markov reneval processes with finitely many states. Ann. Math. Statist. 1961, 32, nr 4, p. 1243-1259.
26. Silvestrov D.S.: Polumarkovskije processy s diskretnym mnozestwom sostojanij. Sovetskoe Radio, Moskva 1980.
27. Wajand J.A.: Silniki o zapłonie samoczynnym. WNT, Warszawa 1988.
28. Wybrane zagadnienia zużywania się materiałów w ślizgowych węzłach maszyn. Praca zbiorowa pod redakcją W. Zwierzyckiego. PWN, Warszawa-Poznań 1990.
29. Zwierzycki W.: Prognozowanie niezawodności zużywających się elementów maszyn. ITE, Radom 2000.
30. Żółtowski B.: Podstawy diagnostyki maszyn. ATR, Bydgoszcz 1996.