A novel approach has been proposed in understanding and possible eliminating dangerous bending vibrations of high pressure fuel lines of low speed marine diesel engines. Although the theory of vibration of piping systems is rather developed today, the reasons for significant vibrations occurring in high pressure fuel pipes are not completely understood, and require thorough investigation and study. Both analytical and experimental studies performed in order to analyze dependence of diesel engines operational modes on the level of vibration and its properties. Results of numerical studies of the equation of motion of high pressure fuel pipes were obtained using Mathieu-Hill method. Those results were supplemented with data from full-scaled experiments performed on Sulzer 14RT-flex96C low speed main engine onboard of 15550 TEU container vessel Ebba Maersk. It has been proven that oscillations of the fuel in high pressure line, which take place in-between injections at the partial engine load, cause parametric bending vibration, and in some cases parametric resonance. Author has proposed a few methods in order to avoid that.

**Keywords:** high pressure fuel pipe, bending vibration, parametric resonance, reliability

**1. Introduction**

According to the statistics regularly published by the P&I clubs the reliability of high pressure fuel systems of marine low speed diesel engines remains not sufficiently high. For example, Swedish Club claims that defects of fuel equipment make five the most common failures of low speed diesel engines. Among those defects a special attention is drawn by the significant number of recorded cases of high pressure fuel pipes breakage. Since such failure leads to the complete stop of main engine (or slow down), it can be crucial for the safe navigation while in port area, during canal transit, or in other restricted areas.

A possibility of the fire hazard in case of an oil spill caused by a broken pipe is prevented today by a double wall around all high pressure fuel lines of diesel engine. This measure became mandatory since July, 2003 when rule II-2/4 55 of the SOLAS Convention came into force.

However, this does not eliminate the reason for the pipe breakage. This paper provides results of the research aimed to solve the problem of high pressure fuel systems inadequate reliability. Paper is a part of the complex research in reliability of heavy fuel and other oil products pumping systems.

**2. Methods of analysis**

The research was constructed on the basis of system-analysis technique. There were following stages included in the research process:
- Identification of high pressure fuel system components and their functional properties;
- Determination of the distinctions of high pressure fuel pipes breakage;
- Synthesis the conception of the research;
Performing theoretical and experimental studies following by the final conclusions.

It is known that design of high pressure fuel systems of marine diesel engines has not been changed significantly over the last century. Basically, fuel system consists of high pressure fuel pumps, fuel injection valves, and piping connecting them [9].

Modern systems with an electronic control bring some changes; however those do not change the main principle of the functioning. Fuel has to be compressed, fed and injected into a combustion chamber during an every cycle [3].

Here we determine the principal feature of the high pressure fuel system. An impact created by the high pressure fuel pump plunger is followed by the process of a continuous waves generation and propagation in a fuel. Those induce vibrational modes in pipes itself. It will be shown below that vibration will be our main concern.

In order to understand those reasons, which lead to the breakage of high pressure fuel pipes, the literature survey was performed.

Some authors report that vulnerability of high pressure pipes can be a result of structural defects, low quality steel, excessive operational pressure, and excessive bending due to vibration [2]. If the first three ones are easy to deal with, as damaged pipe can be replaced with a new one of higher quality, then the vibrational aspect brings some challenges.

Since reasons for the critical vibration appearance are not always easy to find and eliminate, pipe breakage can repeat continuously. Therefore, this paper has raised a very actual and important problem.

Crack formation due to a critical vibration of high pressure fuel pipes has one important distinct. It is oriented in the transverse plane with respect to the pipe’s axis, as shown in Fig. 1. Such typical crack will normally occur under the influence of the shear stresses, which appear due to the pipe bending vibration.

![Fig. 1. Typical transversally oriented crack of high pressure fuel pipe of Sulzer RT-flex main engine](image)

However, not always vibration can result in a pipe breakage. We can say vibration is absolutely normal condition of structural elements on a motor vessel. When a certain circumstances are fulfilled the resonant vibration can occur which can lead to the material disintegration. In order to determine the mechanism of dangerous resonance appearance in high pressure fuel pipes there were some theoretical and experimental tasks solved.
3. Theoretical background

As the governing concept of the research can now be formulated as critical bending vibrations of high pressure fuel pipe can take place in case of resonance, it is necessary to get numerical values of both forced and natural frequencies of those vibrations involved in creating a resonance condition. However, in case of high pressure fuel lines we need to consider parametric vibrations. Those occur when external force dynamic parameters are being changed in time. Such vibration can lead to the parametric resonance, which occurs in a mechanical system when a system is parametrically excited and oscillates at one of its resonant frequencies [8, 11].

In order to derive differential equation of high pressure fuel pipe bending vibration due to the fuel pressure oscillation let us consider the geometrical interpretation of the problem. As it can be seen from Fig. 2 element $dx$ with mass $dm = (m / L) dx$ of the rigidly mounted pipe oscillates in direction of the $x$ axis and creates displacement $w$ in direction of the $z$ axis.

Transverse distributed load $q_t$ on a pipe expressed by the formula:

$$q_t = -m(x) \frac{\partial^2 w}{\partial t^2} + P_f \frac{\partial^2 w}{\partial x^2},$$  \hspace{1cm} (1)

where $P_f = P_0 + P_a \cos \omega t$ is the oscillating fuel pressure with its static $P_0$ and dynamic $P_a$ amplitude components, $F_i = \pi r^2$ is the internal area of the radius $r$, $m(x)$ is the mass per unit length, $t$ is the time.

Differential equation of the bended axes $Dx$ can be written as

$$EJ \frac{\partial^4 w}{\partial x^4} = -M.$$  \hspace{1cm} (2)

Then according to d'Alembert principle equation of motion can take form [1, 4, 5]:

$$EJ \frac{\partial^4 w}{\partial x^4} = q_t,$$  \hspace{1cm} (3)

or

$$EJ \frac{\partial^4 w}{\partial x^4} + m(x) \frac{\partial^2 w}{\partial t^2} - F_i \left( P_0 + P_a \cos \omega t \right) \frac{\partial^2 w}{\partial x^2} = 0,$$  \hspace{1cm} (4)

where $E$ is the Young’s modulus and $J$ is the axial moment of inertia of the pipeline cross sectional area.

*Fig. 2. Analytical model of high pressure fuel pipe*
Eq. (4) describes forced bending vibration of a pipe when energy losses, shear stress and steadying effect are considered not to be important. Since external force expressed by the harmonic function of time, bending vibration of a pipe has to be parametric. Parametric vibration is often analyzed by the Mathieu-Hill method. According to this method partial differential equation of motion can be brought to the Mathieu form and then studied for stability [8, 11].

Let us search solution of Eq. (4) in the form of
\[ w(x, t) = X(x) \cdot T(t). \]
Then we obtain:
\[
\frac{1}{X} \left[ \frac{d^4 X}{dx^4} - Q_0^2 \left( 1 + Q_1 \cos \omega t \right) \frac{d^2 X}{dx^2} \right] + \frac{1}{T} a^2 \frac{d^2 T}{dt^2} = 0,
\]
where
\[ a^2 = \frac{m(x)}{EJ}, \quad Q_0^2 = \frac{P_0}{EJ}, \quad Q_1 = \frac{P_1}{P_0}. \]
Eq. 5 splits into two equations:
\[
\frac{1}{X} \left[ \frac{d^4 X}{dx^4} - Q_0^2 \left( 1 + Q_1 \cos \omega t \right) \frac{d^2 X}{dx^2} \right] = \alpha + \beta Q_0^2 \left( 1 + Q_1 \cos \omega t \right),
\]
and
\[
\frac{1}{T} a^2 \frac{d^2 T}{dt^2} = -\alpha - \beta Q_0^2 \left( 1 + Q_1 \cos \omega t \right),
\]
where \( \alpha \) and \( \beta \) are the unknown equation splitting constants.

When integrating eq. (6) we need to consider \( t \) as a constant. Let us search its partial solution in the form of
\[ \phi(x) = e^{i \phi}. \]
By substituting eq. (8) into eq. (6), we obtain
\[
\left[ q^4 - Q_0^2 \left( 1 + Q_1 \cos \omega t \right) q^2 \right] \cdot e^{i \phi} = \left[ \alpha + \beta Q_0^2 \left( 1 + Q_1 \cos \omega t \right) \right] \cdot e^{i \phi},
\]
or
\[
q^4 - Q_0^2 \left( 1 + Q_1 \cos \omega t \right) q^2 = \alpha + \beta Q_0^2 \left( 1 + Q_1 \cos \omega t \right).
\]
From eq. (10) we find
\[
q^2 = \frac{Q_0^2 \left( 1 + Q_1 \cos \omega t \right)}{2} \pm \sqrt{\frac{Q_0^4 \left( 1 + Q_1 \cos \omega t \right)^2}{4} + \left( \alpha + \beta Q_0^2 \left( 1 + Q_1 \cos \omega t \right) \right) = -\delta_1 \pm \delta_2.}
\]
By defining
\[
z_j = \delta_1 + \delta_2; \quad z_k = \delta_1 - \delta_2,
\]
we obtain integral of the eq. (6), which can be expressed in a following form:
\[ X = A \cosh z_j x + B \sinh z_j x + C \cos z_k x + D \sin z_k x. \]

In order to find unknown constants \( A, B, C \) and \( D \) we need apply boundary conditions at the ends of the pipe in coordinates \( x = 0; \ x = L \). Since pipe is rigidly mounted boundary, conditions should be
\[
X(0) = X(L) = 0, \quad \frac{dX(0)}{dx} = \frac{dX(L)}{dx} = 0.
\]
These conditions yield to the set of four equations, which have only trivial solution. By assuming \( z_j = z_k \), we can simplify those equations. Then by equating the determinant of the
system to the zero, frequency equation can be obtained. It is expressed as \[1, 5\]
\[\Delta(z_k L) = 0,\]
or
\[\text{ch}(z_k L) \cos(z_k L) = 1.\] (12)

Transcendental eq. (12) has no analytical solution. Its first four roots are 4.73, 7.853, 10.996 and 14.137. In general form solution of the eq. 12 can be expressed with sufficient approximation as
\[z_k = \frac{\pi(2k + 1)}{2L}; \quad k = 1, 2, 3...n.\] (13)

By excluding constants \(C, D\) and \(B\) from the eq. (11) and assuming \(A=1\), we can simply find solution in the form of
\[X_k = \frac{\text{ch}_k x - \cos x z_k}{\text{ch}_k L - \cos z_k L} + \frac{\text{sh}_k x - \sin x z_k}{\sin z_k L - \text{sh}_k L} X.\] (14)

Then general integral of the eq. (5) can be expressed as a sum:
\[w(x) = \sum_k X_k T_k.\] (15)

In terms of the result (13) solution for biquadratic equation will take now the form as
\[\frac{\pi^4 (2k + 1)^4}{16L^4} - \frac{\pi^2 (2k + 1)^2}{4L^2} (1 + Q_i \cos \omega t) = \alpha + \beta Q_i^2 (1 + Q_i \cos \omega t).\] (16)

From where we define \(\alpha\) and \(\beta:\)
\[\alpha = \frac{\pi^4 (2k + 1)^4}{16L^4}, \quad \beta = -\frac{\pi^2 (2k + 1)^2}{4L^2}.\]

Now let us integrate eq. (7), which takes the following form:
\[
\frac{a^2}{T} \frac{d^2T}{dt^2} = -\frac{\pi^4 (2k + 1)^4}{16L^4} - \frac{\pi^2 (2k + 1)^2}{4L^2} Q_i^2 (1 + Q_i \cos \omega t)
\]
or
\[
\frac{d^2T}{dt^2} + p_k^2 (1 - q_k^2)(1 - \frac{b_k^2}{1 - q_k^2} \cos \omega t) T = 0,
\] (17)

where
\[p_k = \frac{\pi^2 (2k + 1)^2}{a4L^2}, \quad b_k^2 = \frac{4L^2 \cdot P_i \cdot F_i}{\pi^2 (2k + 1)^2 \cdot EJ} = \frac{P_i \cdot F_i}{P_{\text{crit}}}, \quad q_k^2 = \frac{P_i \cdot F_i}{P_{\text{crit}}}.\] (18)

Formula for \(p_k\) defines natural frequency of order \(k\) of unloaded fuel pipe \([1, 4, 5]\). Vectors \(b_k^2\) and \(q_k^2\) give the ratio between the maximum and minimum internal load and the critical load \(P_{\text{crit}}\) (for each \(k\)), correspondingly. It is clear that natural frequency of the loaded pipe can be found as
\[\Omega_k = p_k \sqrt{1 - \frac{P_i \cdot F_i}{P_{\text{crit}}}}.\]

Eq. (18) is the required in our research Mathieu equation. In order to study motion described by the eq. (18) for stability, let us convert this equation to a more suitable form.
By introducing another independent variable as
\[2x = \omega t,\]
we can rewrite eq. (18) as
\[
\frac{d^2 T}{dx^2} + \frac{4}{\omega^2} p^2_k \left(1 - q^2_k\right) \left(1 - \frac{b^2_k}{1 - q^2_k} \cos 2x\right) T = 0.
\]

If we define

\[
\frac{4}{\omega^2} p^2_k \left(1 - q^2_k\right) = \lambda,
\]

and \(\frac{b^2_k}{1 - q^2_k} = 2h^2\),

then eq. (18a) will take well known form [8, 11]:

\[
\frac{d^2 T}{dx^2} + \left(\lambda - 2h^2 \cos 2x\right) T = 0.
\]

Eq. (18b) can be also written in the Ince form:

\[
\frac{d^2 T}{dx^2} + (\alpha - 20 \cos 2x) T = 0.
\]

Parameters \(\lambda\) and \(h^2\) as well as \(\alpha\) and \(\theta\) characterize the stability of motion. Combination of those parameters indicates either vibrations are limited or they do have high intensity and amplitude. In order to determine the stability of motion using parameters \(\lambda\) and \(h^2\) specially built chart is used. It is divided into zones of stable and unstable motion and called Ince-Strutt chart (Fig. 3) [11].

Mathieu equation is the subcase of the Hill’s equation, where a general solution can be given by taking the "determinant" of an infinite matrix. For those who interested see literature [7, 11].

4. Experimental setup

Full-scale experimental study of vibrating processes in the fuel system pressure carried on the ship Ebba Maersk with displacement of 170,794 tons. Her propulsion plant is equipped with two-stroke low speed main engine Sulzer 14RT-flex96C. Engine maximum power is 80080 kW at 102 rev/min. This engine is equipped with a common rail fuel system with electrohydraulic control of the fuel injection [3].

The objective of the study was to get a full picture of the high pressure fuel pipes bending vibration. For this purpose bending vibration was registered during engine operation at different load (nominal and partial). Data obtained were electronically stored and then analyzed by using computer software.

Since piezoelectric vibration sensors are commonly used for vibration detection [10], this was chosen for the experiment. Thin piezoelectric sensor was mounted by means of straps on the high pressure pipe.
During engine operation as shown in Fig. 4. Securely mounted sensor provides a qualitative signal transition. Oscillograms of vibration processes were obtained with the use of portable digital oscilloscope and recorded in its memory.

During measurement the intermediate clamp for fixing the high pressure fuel pipe (Fig. 4) was made loosen. This was made to avoid any influence on a pipe bending vibration, and thus to have an agreement with analytical model. Pipe dimensions were measured to use in the modeling process.

Each engine cylinder has three fuel injectors connected to injection control unit by means of high pressure pipes. Two of them have approximately the same length, and third one is shorter (center one). When performing experiment it was decided to obtain oscillograms for that pipe, which was considerably useful for the research. Pipe with higher vibration intensity was selected. This was one of the long pipes.

![Fig. 4. Vibration sensor mounting method](image)

5. Results and discussion

Some interesting results were received during pipe bending vibration analysis. It was established that the oscillatory process in high pressure fuel pipe has a much longer duration and intensity when engine is running at partial load than at nominal. Bending vibration shown in Fig. 5 taken at 55 rpm has amplitude and intensity two times larger as that shown in Fig. 6 taken 82 rpm.

After analysis performed it frequencies were found and compared. For those vibrations induced at low engine speed dominant frequency was approximately 50 Hz when frequency of injection was 0.9 Hz. And for those forced vibrations at high engine speed main frequency was 50 when frequency of injection was 1.4 Hz. In order to understand the reason for such difference in frequencies and vibrational characteristics analytical model has to be analyzed.

Only we are missing now the data for internal pressure amplitude and frequency, which are required in Eq. 18. Those will allow answering the question regarding stability of the vibrating system, which is studied here.

Let us write down the parameters from Eq. (18a):

$$\lambda = \frac{4}{\omega^2} p_k^2 (1 - q_k^2), \quad h^2 = \frac{b_k^2}{2(1 - q_k^2)}.$$

Numerical values of those $\lambda^2$ and $h^2$ for two cases outlined in the experimental part can be
found by using data from the factory test records. Wartsila has reported test results for its Sulzer RT-flex 82C fuel injection control unit (ICU) performed in 2009 [6]. As it can be seen from the diagram of the fuel pressure before fuel injectors shown in Fig. 7, when injection is done, oscillation process takes in a fuel line. Frequency of those oscillations can be estimated as 200 Hz.

![Fig. 5. Bending vibration of high pressure fuel pipe when engine load is 55 rpm](image)

![Fig. 6. Bending vibration of high pressure fuel pipe when engine load is 82 rpm](image)

Obviously, frequencies of the forced vibration for the partial load and nominal load cases should be different. It follows from the vibrational theory. If time between fuel injection is long enough, bending vibration in a fuel pipe to be induced by the post injection oscillations.

Otherwise, frequency of injection should be the frequency of the forced vibration, since post injection oscillations are dumped at once.
Thus, looking at the diagram in Fig. 7 we can define the necessary numerical values for the final study. For the partial load we take $P_a = 20$ MPa, $P_0 = 5$ MPa and $\omega = 1256$ rad/s. And for the engine speed of 82 rpm values will be $P_a = 60$ MPa, $P_0 = 5$ MPa and $\omega = 8.6$ rad/s. Results for the parameters $\lambda$ and $h^2$ calculated for the two interested cases are presented in Table 1. There were following numerical values used for the pipe geometry: $L = 2.85$ m, $r_i = 0.004$ m. The Young’s modulus was taken as $E = 210$ MPa. Moment of inertia is calculated with the next formula:

$$J = \frac{\pi D_o^4}{64} \left(1 - \beta^4\right), \quad \beta = \frac{d_i}{D_o},$$

where $d_i$ is the internal diameter, $D_o$ is the outer diameter, $D_o = 0.0325$ m.

![Fig. 7. Injection test record for ICU](image)

By using the data from Table 1 and the stability chart in Fig. 3, we can observe that at the partial load parametric resonance can take place on the third vibrating mode; however we do not see it on the experimental oscillograms. Probably amplitude of the third mode is too small. In the same time first vibrating mode is very close to the area of instability, and this should be the cause of amplifying vibration with frequency of 25 Hz, which is very close to the first natural frequency of the pipe.

Table 1. Dependence of the vibrating fuel pipe stability on the engine load

<table>
<thead>
<tr>
<th>Engine load</th>
<th>Stability parameters</th>
<th>Value $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda$</td>
<td>1</td>
</tr>
<tr>
<td>Partial</td>
<td></td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>$h^2$</td>
<td>5.674e-4</td>
</tr>
</tbody>
</table>
Natural frequency of the pipe $p_1$, rad/s

<table>
<thead>
<tr>
<th>Nominal</th>
<th>$\lambda$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>748.235</td>
<td>5.803e3</td>
</tr>
<tr>
<td></td>
<td>36.309</td>
<td>100.857</td>
</tr>
<tr>
<td></td>
<td>6.104e4</td>
<td>326.777</td>
</tr>
</tbody>
</table>

In its turn parameters for the nominal engine load look fine. They all lie far in the stability zone. Experiment has shown availability of vibration with frequency of 50 Hz. This leads us to the second vibrating mode of the pipe. Obviously, first vibrating mode is being dumped by the forced vibration.

6. Conclusions

The reliability connected to bending vibrations of the high pressure fuel pipes of marine low speed diesel engines has been analyzed. Equation of motion has been studied for stability for two different loads of the diesel engine with use of Mathieu-Hill method.

Basing on the results of the study, there are following conclusions can be made:

- High pressure fuel system reliability depends on the mode of diesel engine operation;
- Although high pressure fuel systems of modern diesel engines are built according to the SOLAS Convention requirements, they cannot be considered reliable and safe;
- When designing fuel system of low speed diesel engine, every maker needs to perform reliability analysis with an approach presented in this paper or similar;
- Geometrical properties of high pressure fuel pipes can be adjusted to avoid the negative influence of fuel pressure oscillations on the parametric vibrations of those.

Literature