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ENERGY CUMULATION UNDER CONSTANT – AMPLITUDE AND PROGRAMMED LOADINGS

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Abstract

In the paper there were presented test results of energy cumulation of plastic strain ΔW_{pl} in the specimen made of PA7 alluminium alloy under constant - amplitude and programmed loadings. During the tests it was found that the courses of changes of ΔW_{pl} energy on the same strain levels both under constant- amplitude and programmed loading were similar. It was shown that value of $\Sigma\Delta W_{pl}$ energy cumulated in the material till the fatigue failure is not constans and decreases with the increase of fatigue life.

Keywords*: cyclic properties, fatigue life*

1. Introduction

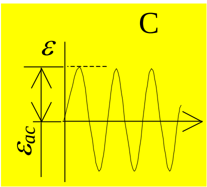
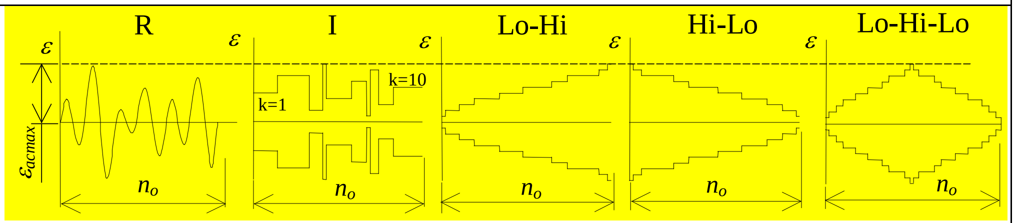
The base of fatigue life calculation effectiveness of construction elements is the insensitiveness of the standard value used during calculations to the changes of cyclic properties. It has been accepted that an energy description of the fatigue is less sensitive to changes of cyclic properties than stress or strain description. Among energy criteria of the fatigue process description various approaches can be distinguished. In one of them plastic strain energy $\Sigma\Delta W_{pl}$ [1] cumulated in the complete fatigue trial is accepted as the standard value. $\Sigma\Delta W_{pl}$ energy is the sum of plastic strain energies ΔW_{pl} from individual variable loading cycles till failure. The measure of the plastic strain energy ΔW_{pl} in the single loading cycle is the hysteresis loop area.

Available literature data concerning description of the course of energy cumulation mostly deal with constant - amplitude loadings or with simple (two step) programmed loadings. The course of operating loadings is most often very complex and is characterized by the variability of many of its parameters (amplitude, mean value, frequency, etc.). The research problem undertaken in this paper is the valuation of the influence of the kind and form of loading program and its parameters on the course of plastic strain energy cumulation during variable loading.

2. Tests description

Specimen for the tests were made of aluminium alloy PA7, which undergoes the strong hardening process during cyclic loading. Specimen were made according to the standard [2]. During the tests there were applied constant – amplitude (C), programmed (I) and random (R) loadings. In order to valuate the influence of the step sequence in the loading program on the course of energy cumulation there were applied diversified loading programs. Among them there were following loadings: gradually increasing *Lo-Hi*, gradually decreasing *Hi-Lo* and gradually increasing and then decreasing *Lo-Hi-Lo*. Common characteristics of these programs were the same values of maximum strain in the program ε_{acmax}, coefficient of spectrum density ζ and block capacity n_0. Loading programs consisted of the oscillatory cycles (R=-1), and as the steering value the amplitude of total strain was accepted. The schemes of the loading programs and their parameters are presented in Table 1.

Table 1 Loading programs parameters

Constant-amplitude	Programmed loading
ε_{ac}=0,35%; ε_{ac}=0,8% ε_{ac}=0,5%; ε_{ac}=1,0% ε_{ac}=0,65%	ζ=0,34, ζ=0,56, ζ=0,77, n_o=100, k=10 ε_{acmax}=0,35%; ε_{acmax}=0,5%; ε_{acmax}=0,8 %;
	

3. Tests results and their analysis

3.1. ΔW_{pl} energy changes and its cumulation during constant-amplitude loading

ΔW_{pl} energy values for individual cycles of constant – amplitude loading were determined with the use of momentary values of the loading force and specimen strains which were registered during the test. Momentary σ stress values were calculated dividing momentary value of the loading force by the initial cross - section area of specimen measurement point. The scheme illustrating process of ΔW_{pl} energy calculations and relation being used was presented in Fig. 2.

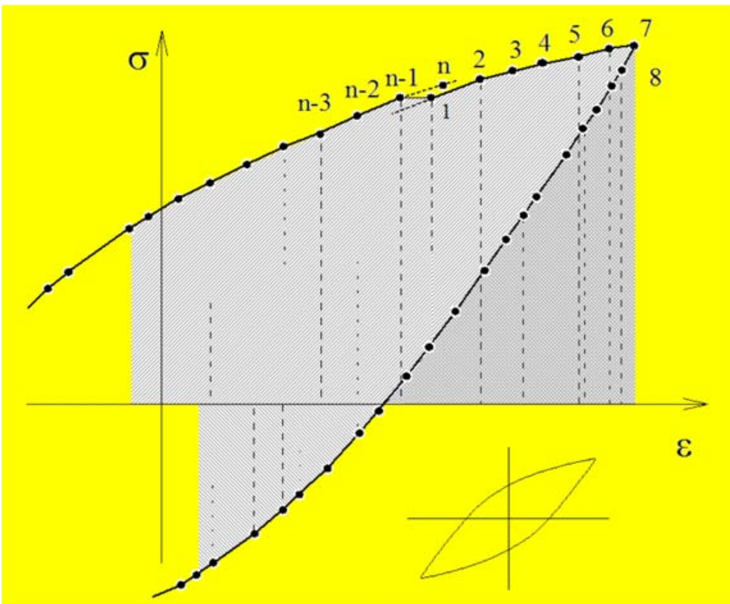


Fig. 2. Scheme for calculations of plastic strain energy ΔW_{pl}

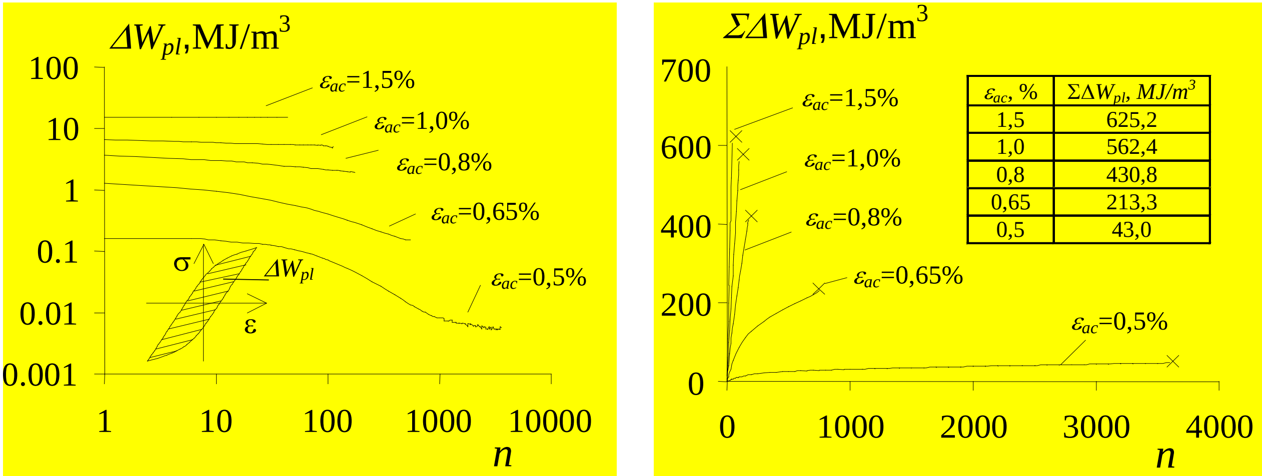
By using the specimen momentary values of stress σ_i and strain ε_i, the energy ΔW_{pl} for individual loading cycle was calculated from the relation (1):

$$\Delta W_{pl} = [\sum_{i=1}^{n-1} \frac{1}{2}(\sigma_i + \sigma_{i+1})(\varepsilon_{i+1} - \varepsilon_i)] + (\sigma_n + \sigma_1)(\varepsilon_1 - \varepsilon_n) \tag{1}$$

where: n – records number of momentary values of the force and strain taken during one loading cycle (n=200 points).

Cyclic hardening of the alloy, which appeared during constant – amplitude loading, was visible in ΔW_{pl} energy courses at five strain ε_{ac} levels. An example diagrams of ΔW_{pl} energy changes at these levels in double logarithmic coordinate system were shown in Fig. 3a.

a) b)



ε_{ac}, %	$\Sigma\Delta W_{pl}$, MJ/m^3
1,5	625,2
1,0	562,4
0,8	430,8
0,65	213,3
0,5	43,0

Fig. 3. ΔW_{pl} energy at five strain levels (a) and its cumulation (b)

Basing on the diagrams of energy ΔW_{pl} it can be stated that magnitude of the changes of cyclic properties (degree of the material hardening) increases with the decreasing of strain level ε_{ac}. Hardening of PA7 alloy is also visible in the energy cumulation diagrams (Fig. 3b). For the highest strain ε_{ac} levels energy $\Sigma\Delta W_{pl}$ cumulated in the material increases proportionally with the number of loading cycles, at the lowest levels, however, cumulation diagrams are characterized by distinct nonlinearity. The highest value of energy $\Sigma\Delta W_{pl}$ cumulated until specimen failure under constant – amplitude loadings was obtained for the strain level ε_{ac}=1,5% and the lowest one for the strain level ε_{ac} =0,5 %.

3.2. Energy changes during irregular loadings

For every sequence of programmed and random loadings there was observed similar quality of the energy ΔW_{pl} courses in the block of program. In order to visualize them, in Fig. 4 there were shown example diagrams of energy ΔW_{pl} in blocks of random and programmed loading registered in various periods of life. In the figures there were marked the numbers of the block repetitions for which the diagram was made. To make diagrams obtained under random loadings more legible (Fig. 4a) they were limited to the half volume in the block of the loading program (0,5n_o=50 cycles).

a) b)

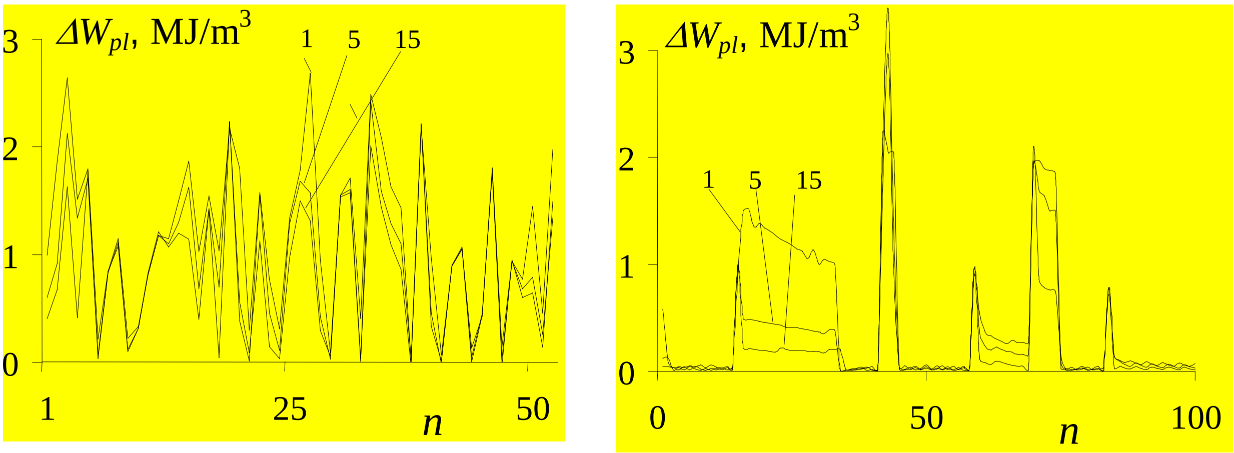


Fig. 4. Energy ΔW_{pl} in the block of loading program of various form a) „R", b) „I", (ε_{acmax}=0,8 %, ζ=0,56)

Basing on the analysis of ΔW_{pl} energy diagrams shown in Fig. 4 it can be stated that under random and programmed loading, similarly like during constant- amplitude loading, tested material is characterized by changes of cyclic properties and by absence of distinct period of stabilization. It is proved by diversified location of ΔW_{pl} energy diagrams for individual cycles (random loading) and steps of loading programs (programmed loading) which were realized in various periods of life. Succeeding ΔW_{pl} energy diagrams in the block are located below the diagrams obtained for the blocks of loading realized earlier. Such a location is the proof of cyclic hardening of the material which is also observed during constant – amplitude loading (Fig. 3a). In the paper there were analysed the courses of ΔW_{pl} energy changes for the cycles from individual steps of loading programs. In Fig. 5 there were shown the examples of energy changes on one step (ε_{ac}=0,65%) in succeeding repetitions of *Lo-Hi* and *I* programs. In diagrams there were marked the numbers of the block repetitions.

a) b)

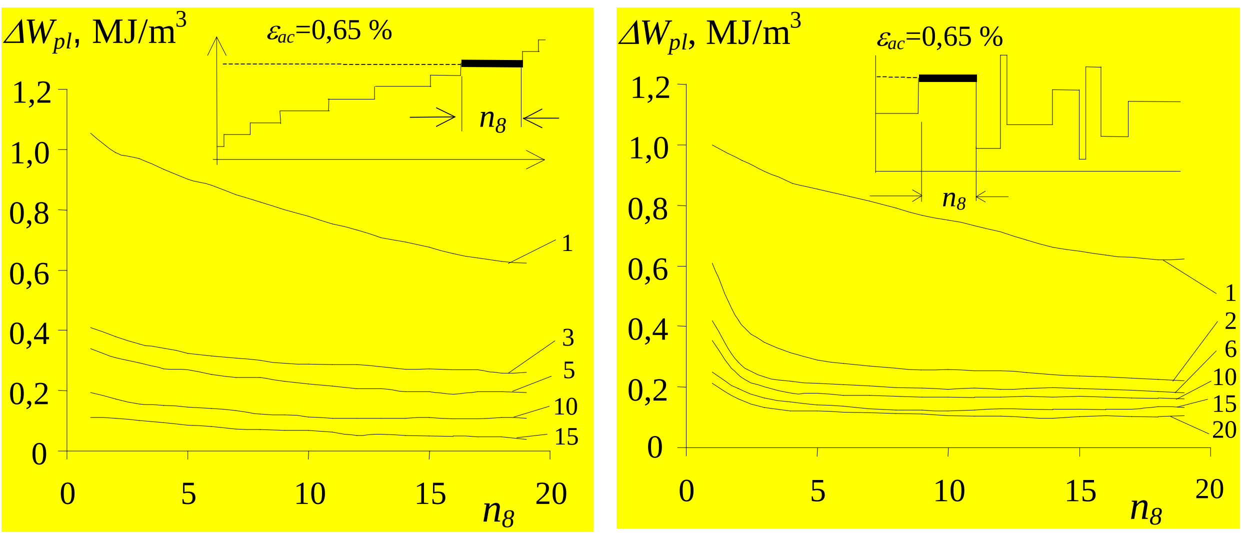


Fig. 5. Changes of ΔW_{pl} on the step with amplitude ε_{ac}=0,65 % carried out under loading with various sequence of steps in the block: a) Lo-Hi, b) I

Comparative analysis of ΔW_{pl} energy diagrams on the steps with the same strain amplitude realized for various program sequences allows to conclude that all loading programs result in the similar course of this energy. Its characteristic feature are lower and lower energy levels on the steps in the succeeding repetitions of the program block. Moreover the energy courses on the same levels of strain, which was realized under programmed loading, are qualitatively and quantitatively very similar to the value of ΔW_{pl} energy obtained under constant-amplitude loading. This was presented in Fig. 6 in the form of example ΔW_{pl} energy diagrams in the function of relative life *n/N* for constant-amplitude loading and selected sequences of random and programmed loading (ε_{ac}=0,65 %, ζ=0,56).

a) b)

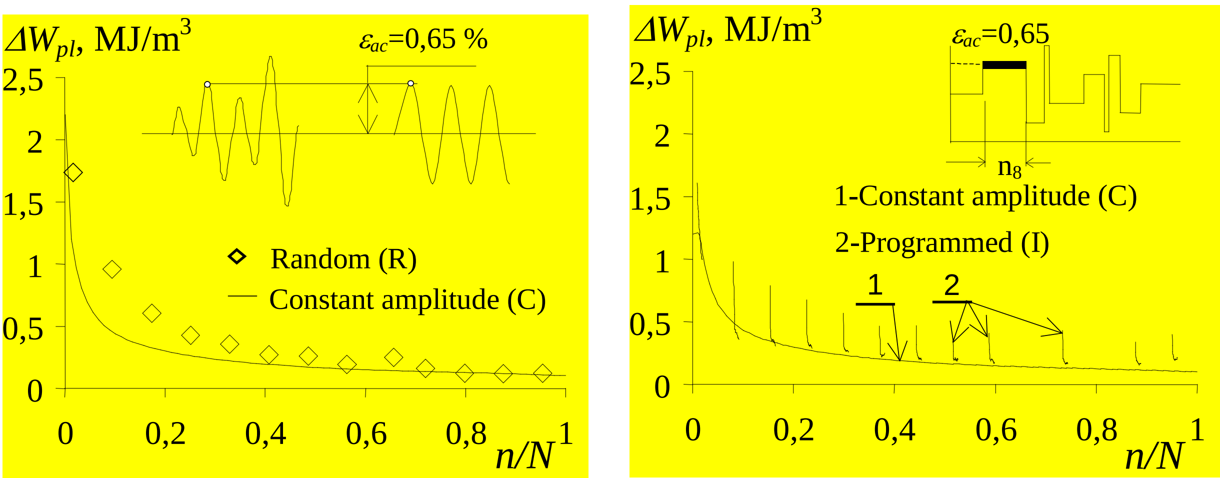


Fig. 6. Energy ΔW_{pl} under diversified loading with ε_{ac}=0,65 %: a) random and constant-amplitude, b) programmed and constant-amplitude

Basing on the mutual position of ΔW_{pl} energy diagrams in the succeeding repetitions of program block it can be concluded that despite the stabilization disturbances resulting from level changes of strain amplitude, material seems to “remember” the energy course which is typical for a given strain level. The tendency of these changes is clearly visible in ΔW_{pl} energy diagrams obtained under loadings with diversified steps sequence. Energy values obtained in the terminal cycles of the individual steps of programmed loading tend toward the level obtained during constant –amplitude loading.

In order to formulate some kind of genaral conclusions there was carried out the comparative analysis of ΔW_{pl} energy values for every strain level and every form of loading program. The comparison of ΔW_{pl} energy was carried out for the same periods of relative life *n/N*. In the case of programmed loadings with diversified sequence of steps (I, Lo-Hi, Hi-Lo, Lo-Hi-Lo) in the comparative analysis there were used ΔW_{pl} energy values defined for the last cycle of individual program steps. On every strain level there were obtained results supporting earlier observations which concerned the absence of the visible influence of the program form on the course of energy changes on the analysed strain level. An example results presenting these observations were shown in Fig. 7 on the diagrams of energy changes for two strain levels (ε_{ac}=0,65% and ε_{ac}=0,8%). To make these diagrams more legible there were ploted only the results obtained during one of the three trials realized for each sequence of loading program and the course of ΔW_{pl} energy obtained under constant-amplitude loading.

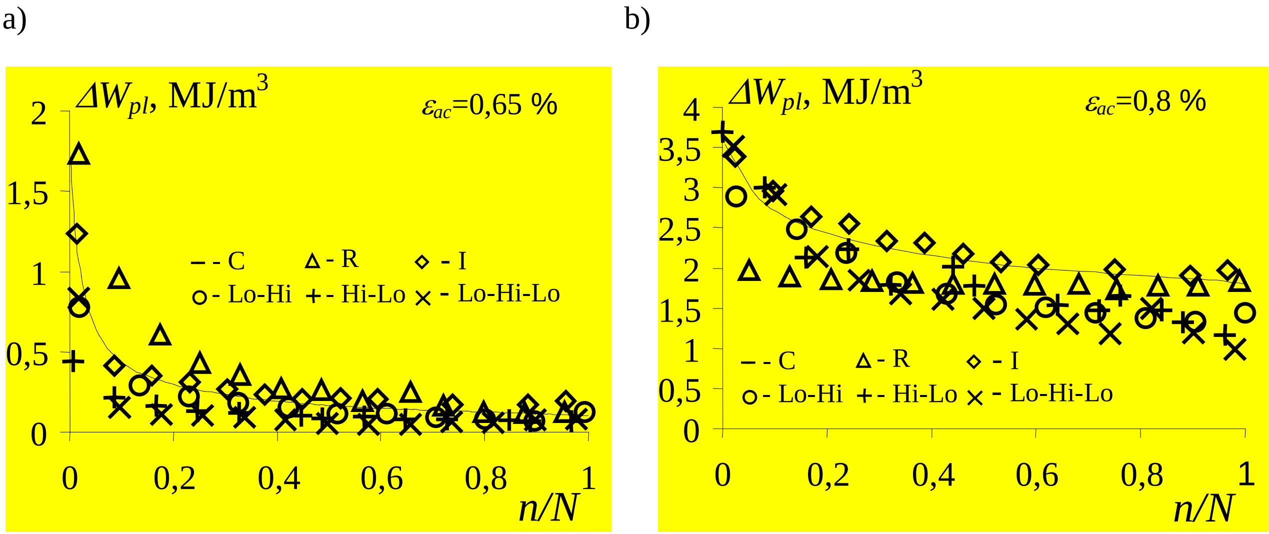


Fig. 7. Energy ΔW_{pl} under constant-amplitude and programmed loadings on two strain levels: a) ε_{ac}=0,65 %, b) ε_{ac}=0,8 %

Comparative analysis of ΔW_{pl} energy diagrams obtained for two strain levels realized in the programs of diversified form allows to conclude that they are locating themselves very closely to the diagrams presenting the courses of ΔW_{pl} energy changes obtained under constant-amplitude loading.

3.3. ENERGY CUMULATION UNDER IRREGULAR LOADING

As it was expected, steps sequence in the loading program influences the course of $\Delta W_{pl(i)}$ energy cumulation in one block of the program. In order to present the above observation, in Fig. 8a there were shown an example courses of ΔW_{pl} energy cumulation in the first block of random and programmed loading with diversified sequence of the steps in the program.

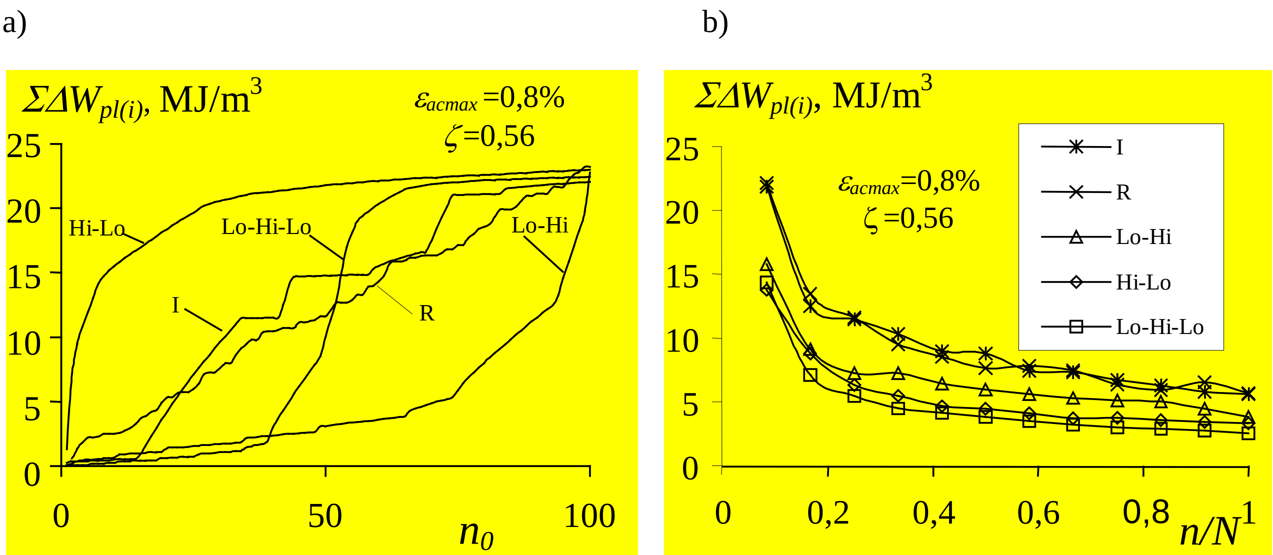


Fig. 8. The course of energy cumulation $\Sigma\Delta W_{pl(1)}$ in one block of the program with various sequence of steps (a), energy $\Sigma\Delta W_{pl(i)}$ in blocks from various periods of life (b)

Basing on the mutual position of energy cumulation diagrams in the first block, it can be concluded that the least differences are visible in the case of diagrams obtained during realization of the loading program with random succession of cycles (R) and irregular succession of steps (I). Despite the diversified course of $\Delta W_{pl(i)}$ energy cumulation in one block, the value of the energy cumulated $\Sigma\Delta W_{pl(i)}$ in the first and in succeeding blocks was very similar. In Fig. 8b there were shown an example calculation results of $\Sigma\Delta W_{pl(i)}$ energy cumulated in the blocks of loading realized in various periods of life. Dependence of the level of $\Sigma\Delta W_{pl(k)}$ energy cumulated in one block of loading from the period of life is the confirmation of ΔW_{pl} energy changes observed in Fig. 4 for individual cycles and steps of programmed loading.

Comparative analysis of $\Sigma\Delta W_{pl}$ energy cumulated in the entire fatigue trial for various sequences of loading program showed that its value is not influenced by the sequence of the steps in the loading program, too. In Fig. 9 there was shown schematically the course of energy cumulation during fatigue test for diversified step sequences of the loading program.
Despite the diversified course of energy cumulation in the block of loading program the value of energy cumulated in the entire fatigue test for various step sequences in the block is always very similar. The level of energy cumulated in the entire fatigue test is influenced significantly by the loading program parameters, i.e. ε_{acmax} and ζ (Fig. 9b). The highest values of $\Sigma\Delta W_{pl}$ energy cumulated until fatigue failure were obtained for the highest levels of maximal strain and coefficient of spectrum density ζ.

a) b)

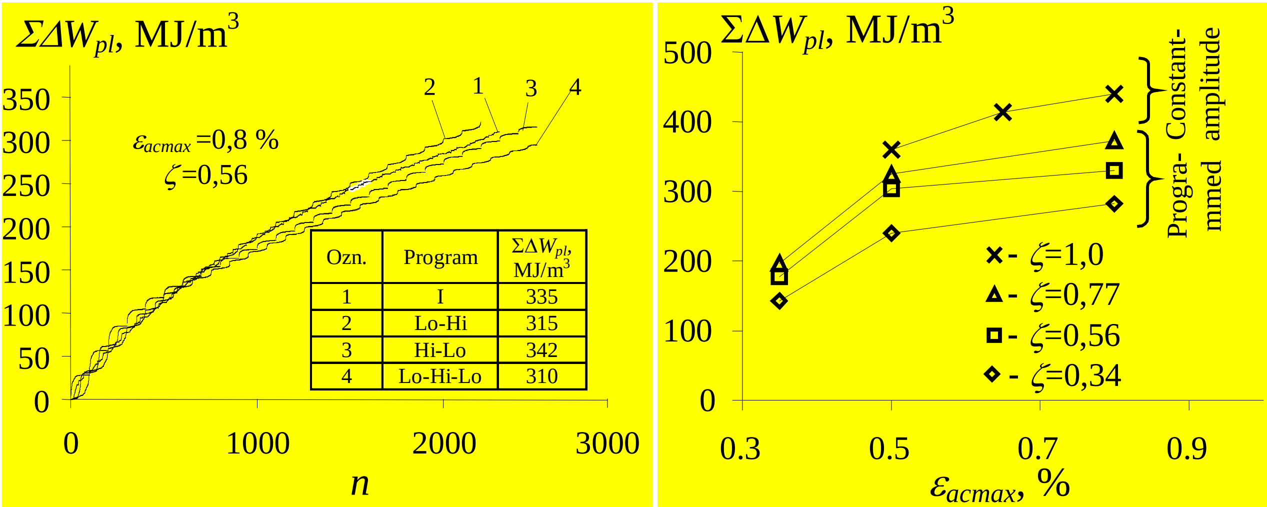


Fig. 9. The course of energy cumulation ΔW_{pl} under diversified loadings (a) and energy cumulated in relation to ε_{acmax} and ζ (b)

4. Summary

Plastic strain energy ΔW_{pl}, just like strain or stress, is sensitive to the changes of cyclic properties. The courses of ΔW_{pl} energy changes on the individual strain levels are very little influenced by the form and parameters of variable loading program. The quality of these courses, independently from the form and parameters of the loading program, is very similar, which is the proof of the cyclic hardening of the material. The volume of PA7 alloy hardening depends as in the case of the course of energy changes on the strain level.

For every form and sequence of loading program there was observed similar course of energy cumulation ΔW_{pl}. The characteristic feature of the tests results under random and programmed loading and also under constant- amplitude loading is that the energy cumulated in the material

$\Sigma\Delta W_{pl}$ until the fatigue failure is not constant value. Its level depend on the parameters of loading program (ε_{ac}, ε_{acmax}, ζ) and it decreases with the increase of fatigue life. Obtained test results do not support in this respect the energy criterion of fatigue failure appearance, in which it is accepted that failure is determined by the critical value of energy cumulated of plastic strain [3].

The use of plastic strain energy ΔW_{pl} as the criterion measure during fatigue life calculations of the construction elements made of cyclically unstable materials may lead to the discrepancy of calculations and tests results.

References

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