Journal of POLISH CIMAC
Faculty of Ocean Engineering & Ship Technology
GDAŃSK UNIVERSITY OF TECHNOLOGY

DOUBLE DYNAMIC VIBRATION ABSORBER

Henryk Holka

University of Technology and Life Sciences in Bydgoszcz
ul. Prof.S.Kaliskiego 7, 85-789 Bydgoszcz, Poland
tel.:+48 52 3408292
e-mail: holka@utp.edu.pl

Abstract

The paper presents the analysis of object vibration (a rigid beam) with asymmetrically located central point of mass supported on two elastic springs.
In order to eliminate the vibration of the main system the dynamic vibration absorber with two degrees of freedom has been joined.
Results of calculation for specific data have been demonstrated on the diagrams.

Keywords: *rigid beam, vibration damper, anti-vibration insulation*

1. Introduction

One of the known dynamics problems is an issue with insulating selected construction elements from foundation vibrations.

In Fig. 1 a carrying element (beam) on which sensitive to vibrations, mechanical-electric subsystems are placed, can be seen. Notice that in general cases point s does not coincide with the mass centre m.

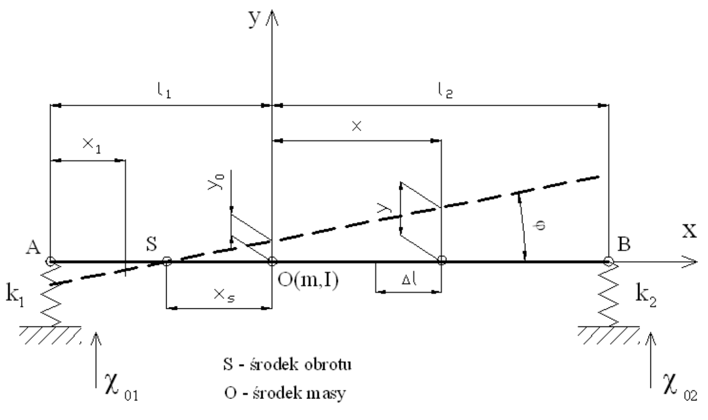


Fig.1. Beam supported on two springs

From the point of view of the designer a question arises considering the beam point's displacements and their values for specified excitation frequency.

The construction must meet the following condition: in certain beam segment the length Δl, vertical displacement y, velocity $\dot{y}$, and acceleration $\ddot{y}$ must be always smaller from the threshold value defined as admissible for the construction elements placed on the beam.

There are two possible options for restraining the beam's movement:

- changing the beam parameters,
- applying a dynamic vibration absorber.

In the first case we notice that the beam's movement is planar and the description of displacement y of beam's optional point x is possible, if we know position of rotation centre s and the value of the angle $\varphi = \varphi(t)$

$$x_{1s} = \frac{y_1(t)}{tg\varphi_1} = \frac{y_1}{\varphi_1}. \tag{1}$$

The values $y_1(t)$ and $\varphi_1(t)$ are determined from the beam's motion equations. The centre s depends from the system parameters and the excitation frequency. Defining of admissible beam vibrations is rather troublesome.

In the second case we receive good results by using a dynamic vibration absorber. Usually the vibration damper has one degree of freedom. In this particular problem a different solution has been applied – dynamic vibration damper with two degrees of freedom.

2. Motion equations

The following system is to be consided:

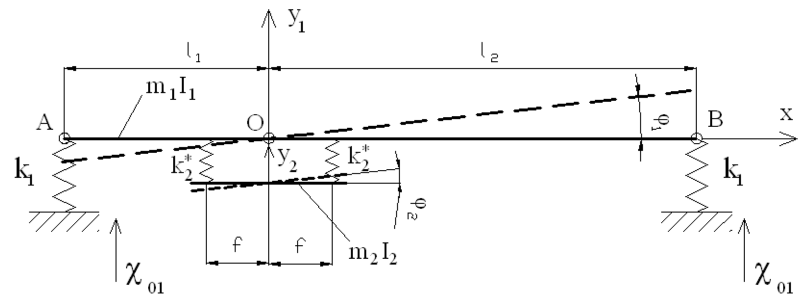


Fig. 2. Dynamic system with a double vibration absorber applied

As can be seen of the Fig. 2, an absorber with two degrees of freedom has been attached to the main beam. Hence the whole system has four degrees of freedom. In accordance to Fig. 2, the displacements of mass m_1 and m_2 are described by coordinates y_1, φ_1, y_2, and φ_2.

Equations of motion can be written in matrix form

$$[A]\left\{\ddot{x}\right\} + [B]\left\{\dot{x}\right\} + [C]\{x\} = \{Q(t)\}, \tag{2}$$

where: A – inertia matrix, B – damping matrix, C – rigidity matrix, Q – excitation matrix, $k_2^* = k_2 + sc$, $s = i\omega$.

Equation (2) is transformed to:

$$\begin{aligned} &Z(s)\cdot X(s)=Q(s)\\ &x(s)=col\{y_1(s),\varphi_1(s),y_2(s),\varphi_2(s)\}\\ &Q(s)=col\{F_1(s),F_2(s),0,0\}, \end{aligned} \tag{3}$$

where:

s – complex variable in Laplace transformation,

$Z(s)$ – rigidity matrix containing matrixes A,B and C,

$F_1(t)=2k_1\kappa_o\sin\omega t$, $F_2(t)=k_1(l_2-l_1)\kappa_o\sin\omega t$.

System response can be derived from Cramer's rule

$$x_r(i\omega)=\sum_1^n(-1)^{r+n}\cdot\frac{z_{rn}(i\omega)}{|Z(i\omega)|}F(i\omega)=\sum_1^n\alpha_{rn}(i\omega)\cdot F(i\omega), \tag{4}$$

where: z_m is minor of matrix Z,

$\alpha_{rn}(i\omega)$ – complex matrix of system receptance.

Matrix $|Z|$ has the form:

Z(iω)= (5)

$-m_1\omega_1+2k_1+2k_2^*$ a_{11}	$-k_1(l_1-l_2)$ a_{12}	$-2k_2^*$ a_{13}	0 a_{11}
$-k_1(l_1-l_2)$ a_{21}	$-I_1\omega^2+2k_2^*f^2+k_1(l_1^2+l_2^2)$ a_{22}	0 a_{23}	$-2k_2^*f^2$ a_{24}
$-2k_2$ a_{31}	0 a_{32}	$-m_2\omega^2+2k_2^*$ a_{33}	0 a_{34}
0 a_{41}	$-2k_2^*f^2$ a_{42}	0 a_{43}	$-I_2\omega^2+2k_2^*f^2$ a_{44}

Next, in accordance to (4), x_1 i φ_1 can be transformed to

$$\bar{x}_1(i\omega)=\frac{1}{|Z|}\left(\begin{vmatrix}a_{22}&0&a_{21}\\0&a_{33}&0\\a_{42}&0&a_{44}\end{vmatrix}\cdot F_1(t)-\begin{vmatrix}a_{21}&0&a_{24}\\a_{31}&a_{33}&0\\0&0&a_{44}\end{vmatrix}F_2(t)\right), \tag{6}$$

$$\bar{\varphi}_1(i\omega)=\frac{1}{|Z|}\left(\begin{vmatrix}a_{12}&a_{13}&0\\0&a_{33}&0\\a_{42}&0&z_{44}\end{vmatrix}F_1\cdot(t)-\begin{vmatrix}a_{11}&a_{13}&0\\a_{31}&a_{33}&0\\0&0&a_{44}\end{vmatrix}F_2(t)\right). \tag{7}$$

From the relation of (6) and (7) can be seen, that coordinates $x_1(t)$ and $\varphi_1(t)$ will equal zero, when elements a_{33} and a_{44} of matrix Z will also have zero value

$$x_1(t)=\varphi_1(t)=0 \quad \text{when} \qquad a_{33}(i\omega)=a_{44}(i\omega)=0. \tag{8}$$

Elements a_{33} i a_{44} have the form:

$$a_{33} = 2k_2 - m_2\omega^2 = 2k_2\left(1 - \frac{m_2\omega^2}{2k_o}\right), \tag{9}$$

$$a_{44} = 2k_2 f^2 - J_2\omega^2 = 2k_2 f^2\left(1 - \frac{J_2\omega^2}{2k_2 f^2}\right). \tag{10}$$

Notice that:

$$\omega_{x2}^2 = \frac{2k_2}{m_2} \text{ i } \omega_{\varphi 2}^2 = \frac{2k_2 f^2}{J_2}, \tag{11}$$

where ω_{x2} i $\omega_{\varphi 2}$ are free vibration of mass m_2.
We tune both values to the same frequency $\omega_{x2} = \omega_{\varphi 2}$

$$\frac{2k_2}{m_2} = \frac{2k_2 f^2}{J_2}, \tag{12}$$

hence

$$J_2 = m_2 f^2. \tag{13}$$

Eventually, by substituting (13) and (11) to (9) and (10), we get

$$a_{33} = 2k_2\left(1 - \frac{\omega^2}{\omega_{x2}^2}\right), \tag{14}$$

$$a_{44} = 2k_2 f^2\left(1 - \frac{\omega^2}{\omega_{x2}^2}\right). \tag{15}$$

From equations (14) and (15) can be seen, that

$$x_1(t) = \varphi_1(t) = 0 \qquad \text{when } \omega = \omega_{x2} = \omega_{\varphi 2}. \tag{16}$$

The determined rule is valid for all beam excitation frequencies.

3. Example

In order to verify the correctness of introduced dependencies, calculations on numerical data have been performed.

Following data has been used:

$m_1=25\ kg$, $I_1=50\ kgm^2$; $k_1=50000\ N/m$, $l_1=3m$, $l_2=1m$, $\kappa_o=0{,}001m$.

Free vibrations of the main beam, without absorber has been calculated from the formula:

$$\omega_{1,2}^2 = \frac{1}{2}\left[\frac{2k_1}{m_1} + \frac{k_1\left(l_1^2 + l_2^2\right)}{J_1}\right] = \sqrt{\left[\frac{2k_1}{m_1} + \frac{k_1\left(l_1^2 + l_2^2\right)}{I_1}\right]^2 - \frac{4k_1\left(l_1^2 + l_2^2\right)}{m_1 I_1}}.$$

After substituting data we receive:

$$\omega_1 = 53{,}6 \quad rad/s; \quad \omega_2 = 105{,}4\ rad/s.$$

Assuming, that the ratio $\frac{m_2}{m_1} = 0{,}2$ then $m_2=5kg$.

From the system tuning conditions I_2, k_2 can be determined,

$53{,}6 = \omega_{x2} = \omega_{\varphi 2}$. (the system operates near the place of first resonance)

hence $\sqrt{\dfrac{2k_2}{m_2} = 53{,}6} \quad i \quad k_2 = 7190\dfrac{N}{m.}$

Assuming *f=0,25m,* we get $J_2 = 0{,}3125\ kgm^2$.

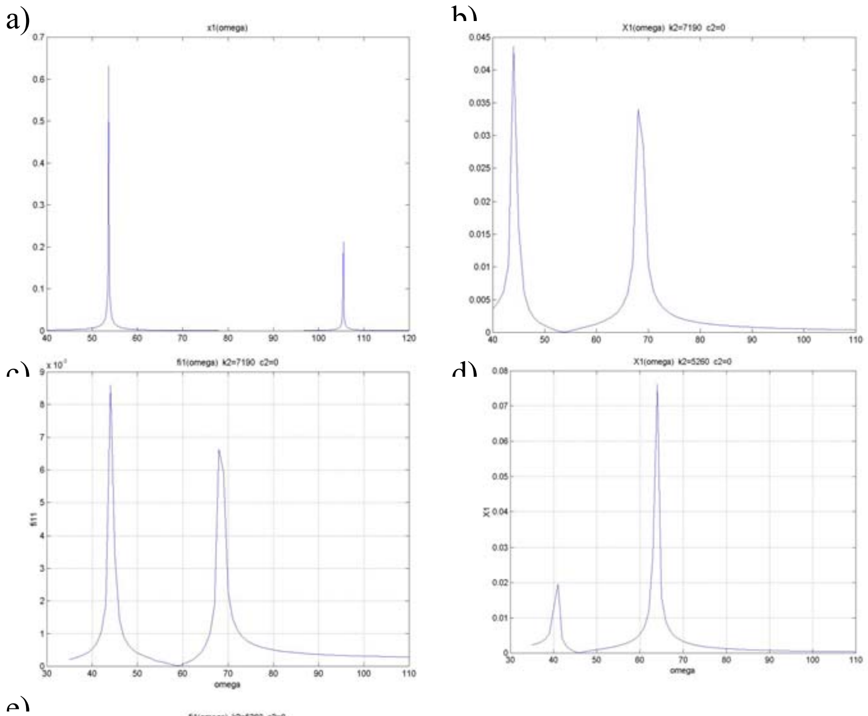
In the end we verify tuning conditions:

$$\omega = 53{,}6 = \omega_{x2} = \sqrt{\frac{2k_2}{m_2}} = \omega_{\varphi 2} = \sqrt{\frac{2k_2 f^2}{J_2}}\ .$$

Calculation results are shown on Fig.3.

The presented curves confirm calculations. Amplitude $x_1 = f(\omega))$ with absorber is equal to zero for the resonance frequency $\omega = 53.6$ and $\varphi_1 = f(\omega)$ is also equal to zero for $\omega = 58$.

The Fig. 3d shows the curves for $k_2 = 5260$ (optimum stiffness), $c_2 = 0$. In this case amplitudes for the resonance frequency are not zero , but the scope of small amplitudes is wider.



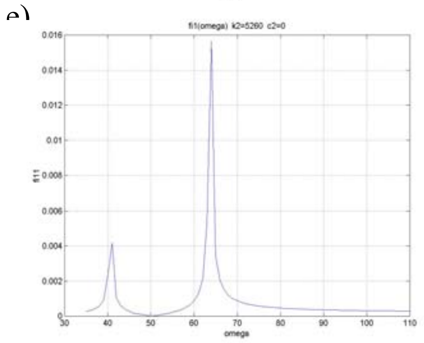


Fig.3. Results of calculation: a) vibration of the beam without vibration absorber; b,c,d,e) vibration of the beam with vibration absorber

4. Conclusions

1. The calculations confirm the analysis. Applying an absorber with two degrees of freedom eliminates main system vibrations.

2. If the foundation excitation frequency is changing, an absorber with damping should be used.
3. The presented solution may be applied in many designs: machine tools, damping pipeline vibrations, etc.

Reference

[1] Radziszewski, B., Różycki, A., *Układ o dwóch stopniach swobody jako „Dynamiczny izolator drgań"*, Mechanika Teoretyczna i Stosowana, PWN, Warszawa 1978.
[2] Holka, H.: *The dynamic vibration absorbe for the main system with two degrees of freedom.* International Symposium on Design and Synthesis, Tokyo, 1984.