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MODELLING OF THE CUTTING PROCESS BY THE DRUM CUTTING UNIT

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Abstract

This paper presents developed mathematical models of parameters that characterize the vegetable material cutting process using a drum cutting unit, including: elementary cutting resistance model, elementary cutting work referred to the cutting area and elementary cutting work model referred to the weight of the cut material. The models describe the relationships between key design features and parameters of the drum cutting unit as well as selected properties of the cut material.

Keywords: *cutting drum, modelling, vegetable material*

1. Introduction

The drum cutting unit is the main working unit of self-propelled, trailer type and stationary chaff-cutters [1, 2, 3, 4]. The drum cutting unit is designed to cut vegetable material (stalks or blades) into parts of defined length (chaff). Thanks to the use of such unit in chaff-cutters it is possible to obtain required degree of the material size reduction. Whereas to get desired feeding results, uniform length chaff is required. However the length depends on individual animal features and feeding method.

Cutting drums can be of open or closed design [1]. An open type drum consists of the shaft, where perforated disks are mounted. The disks are fitted with cutter holders. And the cutter holders hold cutters. Depending on the drum design, it is equipped with straight or helical cutters.

Moreover, cutters may be monolithic or sectional. The cutting drum is fixed with bearings mounted in side plates of the chaff-cutters.

While the shaft of the closed type cutting drum is fitted with the closed cylinder structure (instead of several disks) with brackets holding the cutters, mounted on cylinder side surface.

The cutting drum rotation causes the cutters to move as well. As the cutters move relative to fixed cutter, they press the vegetable material first and then cut it.

Vegetable material is fed between the cutter cutting edge and counter-cutting edge thanks to rotation of pulling-and-squeezing drums, which pre-shape and pre-compact the vegetable material.

The idea of the vegetable material feeding and cutting process using the cutting drum is shown in Fig. 1.

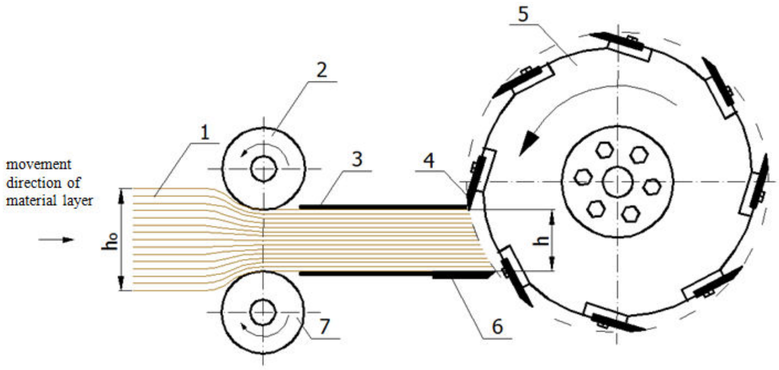

Fig. 1. The vegetable material feeding and cutting process using the cutting drum [1]:
1 – material layer, 2 – top pulling-and-squeezing drum, 3 - pressure plate, 4 – cutter, 5 – cutting drum,
6 – fixed cutter, 7 – bottom pulling-and-squeezing drum,
h_0 – height of the material layer before compaction, h – height of the material layer after compaction

The aim of this paper is to develop mathematical models that characterize the vegetable material cutting process using a drum cutting unit, including:

- unit cutting resistance model p_c,
- elementary cutting work model referred to the cutting area L_{jS},
- elementary cutting work referred to the weight of the cut material L_{jM}.

2. Mathematical models

Based on the analysis of the pressed vegetable material layer cutting process using the drum cutting unit, authors proposed mathematical model of the process.

Figure 2 shows the force system acting on the cutter during vegetable material layer cutting process.

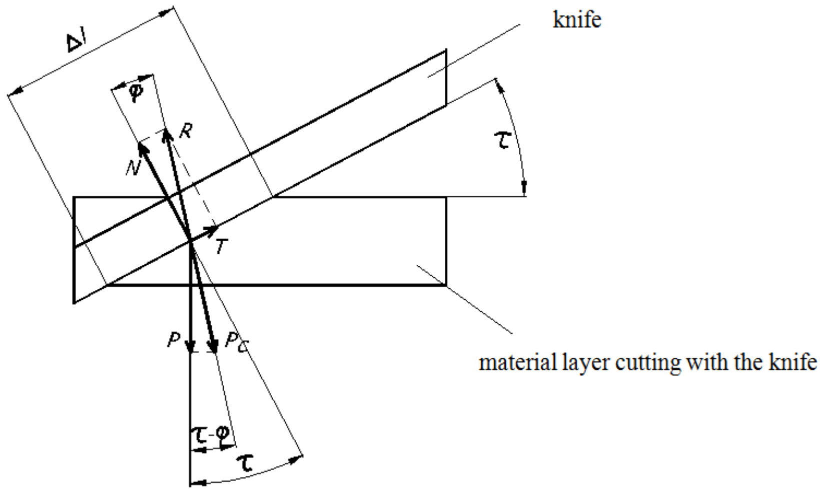

Fig. 2. The force system acting on the drum cutter during vegetable material layer cutting process

The objective of the circumferential force $\boldsymbol{P}$ shown in Fig. 2 is to overcome the resultant cutting resistance, consisting of: normal force $\boldsymbol{N}$ and friction force $\boldsymbol{T}$ generated as a result of the impact of the material layer on the cutter. The value of the normal force $\boldsymbol{N}$ depends on the unit cutting resistance $\boldsymbol{p}_c$ as well as effective length of the cutter $\Delta \boldsymbol{l}$:

$$N = p_c \Delta l \,. \tag{1}$$

The friction force $\boldsymbol{T}$ depends on the friction angle $\boldsymbol{\varphi}$ and it is expressed by the formula:

$$T = Ntg\varphi = p_c \Delta l tg\varphi \,. \tag{2}$$

The resultant cutting resistance $\boldsymbol{P_c}$ originating from the normal force and the friction force shall thus be expressed with the following formula:

$$P_c = \frac{p_c \Delta l}{cos\,\varphi} \,. \tag{3}$$

The circumferential force $\boldsymbol{P}$ is the vertical component of the cutting resistance force $\boldsymbol{P_c}$ and it is expressed by the formula:

$$P = P_c\, cos(\tau - \varphi) = \frac{p_c \Delta l}{cos\,\varphi} cos(\tau - \varphi) \,. \tag{4}$$

After conversions of the formula (4) and taking into account that: $tg\tau = \mu$, where μ is the friction factor between the cutter and the material layer, we get the formula of the circumferential force $\boldsymbol{P}$:

$$P = \frac{p_c \Delta l}{cos\,\varphi} cos\,\tau\, cos\,\varphi (1 + \mu tg\tau) = p_c \Delta l\, cos\,\tau (1 + \mu tg\tau) \,. \tag{5}$$

As a result of the analysis of rectangular vegetable material cutting, the authors divided the process into 3 stages, with the following assumptions:

a) the layer is cut only with one cutter each time,
b) the height of the material layer being cut is equal to the distance travelled by a given point of the cutter that passes through that layer.

STAGE I

At that stage the cutter is sinking into the material layer (the effective cutter length $\Delta \boldsymbol{l}$ is increasing).

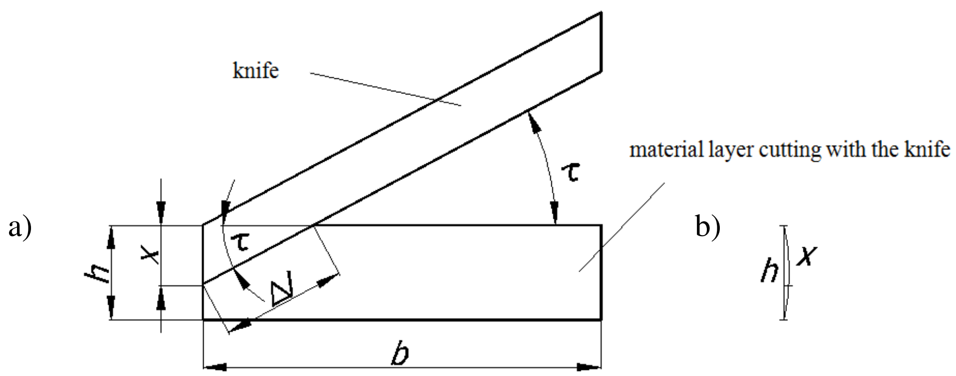

Fig. 3 Cutter sinking into the material layer (Stage I)
a) cross-section of the layer, b) comparison of the arc length and the layer height h

According to Fig. 3a, the difference between the length of the arc drawn by any point of the cutter from the top to the bottom edge of the material layer being cut is approximately equal the height of the layer h, which results from the fact that for small angles $h \approx h\frac{\Psi}{\sin\Psi}$, because $\sin\Psi \approx \Psi$. Using the trigonometric relationship $\frac{x}{\Delta l} = \sin\tau$, we get: $\Delta l = \frac{x}{\sin\tau}$. Whereas for $x = h$, we get: $\Delta l = \frac{h}{\sin\tau}$. Hence the cutting work L_{nI} at Stage I of the cutter movement can be expressed with the relationship:

$$L_{nI} = \frac{1}{r}\int_0^h M_c(x)dx, \tag{6}$$

where:
$M_c(x)$ – moment of cutting.

Taking into account that $M_c = C\frac{x}{\sin\tau}$, the cutting work within the range $0 \le x \le h$ can be expressed with the following relationship:

$$L_{nI} = \frac{C}{r}\int_0^h \frac{x}{\sin\tau}dx = \frac{C}{r}\left[\frac{x^2}{2\sin\tau}\right]_0^h = \frac{C}{r}\frac{h^2}{2\sin\tau}, \tag{7}$$

where:
$C = p_c\, r\cos\tau(1 + \mu\, tg\tau)$ – the constant occurring in the remaining relationships.

STAGE II

At that stage the cutter is cutting the layer of material (the effective cutter length Δl does not change).

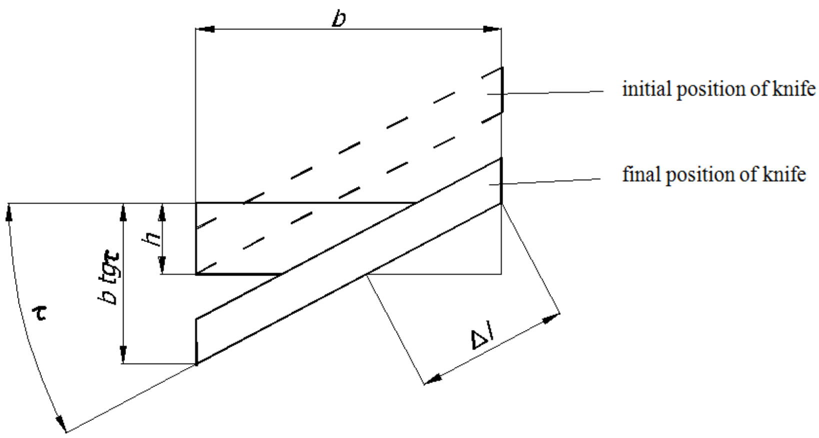

Fig. 4. Cutting of the material layer (Stage II)

According to Fig. 4, cutting process at stage II concerns the range $h \le x \le btg\tau$.
The value of Δl is constant within the range $h \le x \le btg\tau$, and it is expressed by the following relationship: $\Delta l = \frac{h}{\sin\tau}$.
Hence the cutting work is defined by the formula:

$$L_{nII} = \frac{C}{r}\int_{h}^{btg\tau} \frac{h}{\sin\tau} dx = \frac{C}{r}\frac{h}{\sin\tau}(btg\tau - h). \tag{8}$$

STAGE III

At that stage the cutter is coming out of the material layer (the effective cutter length Δl is decreasing).

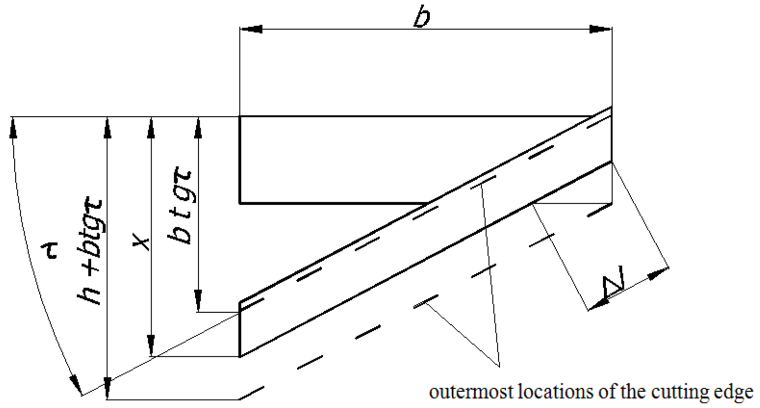

Fig. 5 Cutter coming out of the material layer (Stage III)

According to Fig.5, cutting process at stage III concerns the range $btg\tau \le x \le h + btg\tau$, i.e. it takes place over the same length as in stage I.

Within the range $btg\tau \le x \le h + btg\tau$, the value $\Delta l = \dfrac{x - btg\tau}{\sin\tau}$.

Hence the cutting work is denominated by the formula:

$$L_{nIII} = \frac{C}{r}\int_{btg\tau}^{h+btg\tau} \frac{(x - btg\tau)}{\sin\tau} dx = L_{nI} = \frac{C}{r}\frac{h^2}{2\sin\tau}.$$

As a result, the total work made by the cutter when passing through the vegetable material layer being cut shall be expressed with the formula:

$$L_n = L_{nI} + L_{II} + L_{nIII} = \frac{C}{r}\left(\frac{h^2}{2\sin\tau} + \frac{hbtg\tau}{\sin\tau} - \frac{h^2}{\sin\tau} + \frac{h^2}{2\sin\tau}\right) = \frac{C}{r}\frac{hb}{\cos\tau}. \tag{9}$$

Considering that:

$$(M_c)_{śr} = \frac{L_n z}{2\pi}, \tag{10}$$

where:

z - number of cutters taking part in the cutting process.

Hence, after conversions, we get:

$$\left(M_c\right)_{śr} = \frac{zhb\, p_c\left(1+\mu tg\,\tau\right)}{2\pi}. \tag{11}$$

Thus the unit cutting resistance is defined by the formula:

$$p_c = \frac{2\pi\left(M_c\right)_{śr}}{zhb\left(1+\mu tg\,\tau\right)} = \frac{2\pi J_b \frac{d\omega}{dt}}{zhb\left(1+\mu tg\,\tau\right)}, \tag{12}$$

where:
J_b - moment of inertia of the cutting drum,
$\frac{d\omega}{dt}$ - angular acceleration of the cutting drum.

While the elementary cutting work referred to the cutting area can be calculated based on the following relationship:

$$L_{jS} = \frac{L_n}{hb} = p_c\left(1+\mu tg\,\tau\right), \tag{13}$$

and the elementary cutting work referred to the weight m_j of the cut material can be determined based on the formula:

$$L_{jM} = \frac{L_n}{m_j} = \frac{z\omega hb\, p_c\left(1+\mu tg\,\tau\right)}{2\pi W}, \tag{14}$$

where:
$m_j = W\frac{2\pi}{z\omega}$; W - the cutting unit throughput.

3. Summary

The developed mathematical models are of significant scientific importance as they explain the essence of the vegetable material cutting process using a drum cutting unit. No studies on that subject can be found in the professional literature. Moreover, after positive experimental verification, they can be used for simulations at the design stage of new drum cutting units. It is very important as agricultural works are of seasonal nature. Due to that nature it was impossible to create "data base" sufficient to facilitate quick design of working units of that type, despite of sometimes long-years experiments.

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