Journal of Polish CIMAC
Gdansk University of Technology
The Faculty of Ocean Engineering and Ship Technology

THE ASPECTS OF DIMENSIONING OF THE CONSTRUCTIONS WITH USE OF THE FRACTURE MECHANICS

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Abstract

In the first part of this work the classic strength effort hypothesis and their usefulness to dimensioning of the materials and constructions were characterised. There was given their small usefulness when the defects such as cracks are presented. In this situation very useful to dimensioning of the materials and constructions may be the fracture mechanics with their parameters and criterions. The further part of this work was devoted to characterising of the basic parameters and criterions of the fracture mechanics. Such parameters as stress intensity factor K_n , strain energy release rate G, crack tip opening displacement δ (CTOD) and integral Rice – Čerepanov J and their criterions are presented. Last part is devoted to practical aspect of use of above parameters and criterions to dimensioning of the constructions.

Key words: *rules of dimensioning, fracture mechanics, parameters and criterions, analytical examples*

1. Introduction

The problem of optimal design of the welded structures is very complex and various attempts have been made to obtain effective methods which might be used in engineering, with requires appropriated dimensioning of the materials and constructions. The application of the classical strength effort hypothesis is inconsiderable when defects, such as cracks, occur. The classical effort hypothesis, such as Huber – Mises equivalent of stress and their modifications, does not assess the effort of the construction as no real conditions of material effort are considered. For example [1], [2]:

- the “scale effect” and the geometric feature of the constructions has not taken into consideration,
- the state of the microstructure of the materials, their heterogeneity, internal discontinuity and other defects which are formed under manufacturing process and exploitation leaves out of account,
- a question of the run of the material damage does not consider,
- the constitutive physical rules are created by use of the continuum model of material.

Various approximate methods of calculation are often used in designing, e.g. reducing internal forces, yet the best approach is based on fracture mechanics and their parameters and criterions. The application of fracture mechanics parameters to materials and constructions dimensioning is in a great step towards effort process modelling compliant with the modelling rules, which justifies the aim of the mechanics and its applicability to welded structure designing and dimensioning [2].

2. Characteristic of the fracture mechanics parameters and criterions

Stress intensity factor K_n

The stress intensity factor is a parameter which determines the level of the stress or of the strain, energy density, elastic singularities near the tip of an ideal crack in a stressed linear elastic solid. The asymptotic stress and displacement components may be expressed as follows [1],[2]:

$$\sigma_{ij}(r,\theta)=\frac{1}{\sqrt{2\pi r}}\left[K_I f_{ij}^{I}(\theta)+K_{II} f_{ij}^{II}(\theta)+K_{III} f_{ij}^{III}(\theta)\right], \tag{1}$$

$$u_i(r,\theta)=\frac{1}{2\mu}\sqrt{\frac{r}{2\pi}}\left[K_I g_{ij}^{I}(\theta)+K_{II} g_{ij}^{II}(\theta)+K_{III} g_{ij}^{III}(\theta)\right], \tag{2}$$

where:

r	-	distance from the crack tip,
θ	-	angle, second polar coordinate,
$f_i^n(\theta), g_i^n(\theta)$	-	are dimensionaless functions of the angle θ,
K_n	-	stress intensity factor (n = I, II, III distinguish the models: I –opening mode, II–sliding mode, III–tearing or antiplane mode),
i, j	-	the index refer to either the cartesian coordinates (x, y, z) or the cylindrical co-ordinates (r, θ, z).

Compliant with above, the stress intensity factor K_n is a fundamental quantity that governs the level of stress field in the vicinity of the crack tip or of the strain. Furthermore, it affects the strain energy density, elastic singularities near the tip of an ideal crack in a stressed linear elastic solid at r = 0 ($r \to 0$, $\sigma_{ij} \to \infty$) – Fig. 1a. In reality some inelasticity in the neighbourhood of the crack tip is always present in the form of plasticity Irwin presents a simplified model for determination of the plastic zone attending the crack tip under small–scale yielding. The observation led Irvin to suggest that the effect of plasticity makes the plate behave as if it had a crack longer than the actual crack size, Fig.1b: $a_{eff} = a + r_p$.

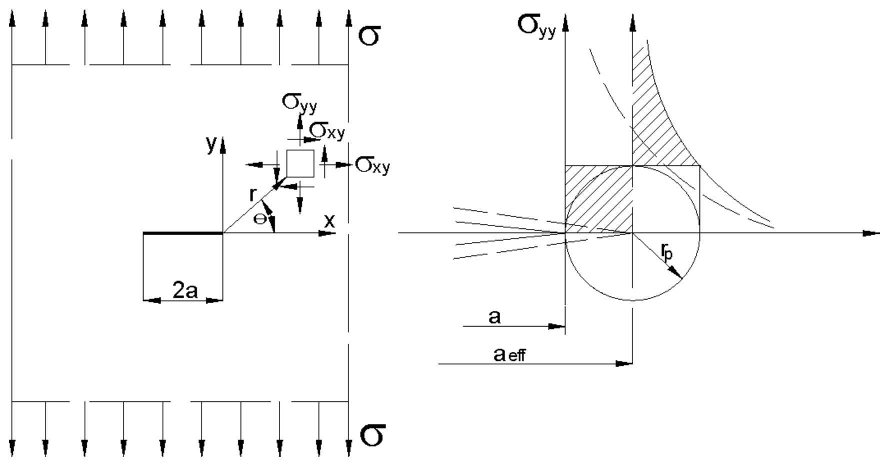

Fig.1. Model of the plate with the crack 2a and the state of stress at the crack tip:
a. in elastic state of material with ideal crack under tension,
b. with extended crack length by plastic deformation a_{eff} – fictitious crack length.

A study of the local stress fields for the three models (I, II, III) of loading showed their general applicability and can be generally given by the values of three stress intensity factors [4]:

$$K_I = \lim_{x \to a} \sqrt{2\pi(x-a)}\sigma_{yy}(x,0,0) = \lim_{x \to a} \sqrt{2\pi r}\sigma_{yy}(x,0,0), \tag{3}$$

$$K_{II} = \lim_{x \to a} \sqrt{2\pi(x-a)}\sigma_{xy}(x,0,0) = \lim_{x \to a} \sqrt{2\pi r}\sigma_{xy}(x,0,0), \tag{4}$$

$$K_{III} = \lim_{x \to a} \sqrt{2\pi(x-a)}\sigma_{yz}(x,0,0) = \lim_{x \to a} \sqrt{2\pi r}\sigma_{yz}(x,0,0), \tag{5}$$

For structural elements different then infinity plate and unit thickness with crack 2a the stress intensity factor is a function of the loading of the body, including applied loads and displacements, of the crack size, of the geometry of the body and of the crack. For example, at the plate with finite dimension 2H x W and thickness B with the crack 2a is affected by the action from tension stress σ then the stress intensity factor is eq. [2, 3]:

$$K_I = \sigma\sqrt{\pi a}F_I(\alpha,B), \tag{6}$$

where: $\alpha = 2a/W,\ B = 2H/W.$

The diagram of the function $F_1(\alpha, B)$ is presented in Fig. 2.

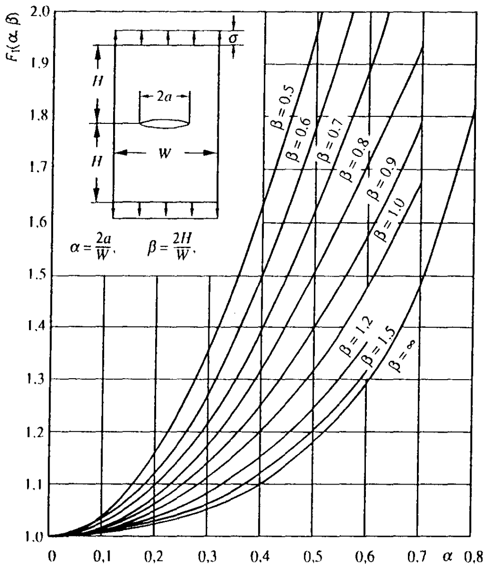

Fig.2. Diagram of the function $F_I(\alpha, B)$ [5].

The above solutions are obtained by making the assumption that the radius of curvature at the crack tip $\rho_e \to 0$. When $\rho_e \neq 0$, K_I can be assessed as follows:

- plane strain

$$K_I = \sqrt{2\pi}(1-\nu)\mu\sqrt{\rho e}\,, \tag{7}$$

- plane stress $$K_I = \sqrt{4\pi}\,\mu\sqrt{\rho e}\,, \tag{8}$$

where:

ν - Poisson ratio,
μ - Shear modulus.

In dynamic problems the stress intensity factor is the functions of time $K_n(t)$ for models n = I, II, III.

Strain energy release rate G

The parameter G is related to the stress intensity factors as follows:

- in plane stress $$G_I = \frac{K_I^2}{E}, \tag{9}$$

$$G_{II} = \frac{K_{II}^2}{E}, \tag{10}$$

$$G_{III} = (1+\nu)\frac{K_{III}^2}{E} = \frac{K_{III}^2}{2\mu}, \tag{11}$$

- in plane strain $$G_I = \left(1+\nu^2\right)\frac{K_I^2}{E}, \tag{12}$$

$$G_{II} = \left(1+\nu^2\right)\frac{K_{II}^2}{E}, \tag{13}$$

$$G_{III} = \left(1+\nu^2\right)\frac{K_{III}^2}{E} = \frac{K_{III}^2}{2\mu}, \tag{14}$$

At the composed state of loading (n = I, II, III) the total value of G as:

$$G = G_I + G_{II} + G_{III}, \tag{15}$$

After inserting the equations (9)÷(11) and (12)÷(14) into (15) we received:

- in plane stress $$G = \frac{\left(K_I^2 + K_{II}^2\right)}{E} + \frac{K_{III}^2}{2\mu}, \tag{16}$$
- in plane strain $$G = \frac{\left(K_I^2 + K_{II}^2\right)\left(1-\nu^2\right)}{E} + \frac{K_{III}^2}{2\mu}, \tag{17}$$

G is also related to the variation of the compliance C of the element with as follows:

$$G = \frac{1}{2} F^2 \frac{\partial C}{\partial A}, \tag{18}$$

where:

C - compliance $\left(u = C F \rightarrow C = u F^{-1}\right) N^{-1} m$,

F - force, N.

Crack tip opening displacement CTOD - δ

According to [4] the opening of the effective crack at the tip of the crack is given by:

$$\delta = \frac{8\sigma_y a}{\pi E} ln\left(sec \frac{\pi\sigma}{2\sigma_y} \right), \tag{19}$$

By expanding equation (19) in Maclaurin series and retaining only the first term for small value of σ / σ_y we get:

$$\delta = \frac{K_I^2}{E\sigma_y}, \tag{20}$$

If we base on the Irwing's solution the distance δ of the faces of the fictitious crack at the tip of the initial crack at the tip of the initial crack of length a is given by:

$$\delta = \frac{4}{\pi E} \frac{K_I^2}{\sigma_y}, \tag{21}$$

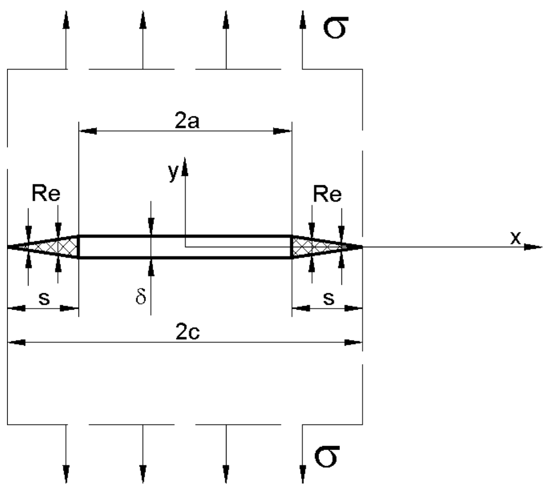

Fig. 3. Dougdale model for the mode I crack of length 2a situated in very thin and an infinite plate subjected to uniaxial uniform stress σ at infinity perpendicular to the crack plane.

By comparing eq. (20) and (21) we can deduce that the Irwin model over-estimates δ as compared to the Dougdale model by 27 per cent.

The eq. (20), (21) combined with eq. (9) yield, respectively:

$$G = \sigma_y \delta ,\tag{22}$$

$$G = \frac{\pi}{4} \sigma_y \delta ,\tag{23}$$

Experimental investigations to indicate that above relations are appropriate when $\sigma \le \sigma_y$.

Rice – Čerepanov J integral

The mathematical formulation of the conservation laws applicable in elasto-statics in the form of path independent integrals of some functionals of the elastic field over the bounding surface of a closed region lastly was proposed by J.R.Rice and independently Čerepanow [4]. The J-integral applied to notch problems was introduced by them.

The increment of the difference between the stored elastic energy W_e and work of the external loads U per unit increment of an ideal crack surface area A in nonlinear elasticity enable to establish the J as [1, 4]:

$$J = -\frac{\Delta P}{\Delta A} ,\tag{24}$$

or

$$J = -\frac{\partial P}{\partial a} ,\tag{25}$$

where: P – potential energy.

In elasticity the strain energy release rate J is equal to the Rice–Čerepanov J integral defined as:

$$J = \int_{F} \left(W dy - \sigma_{ij} n_j \frac{\partial u_i}{\partial x} ds \right) ,\tag{26}$$

where:

W	-	the strain energy density,
Γ	-	the contour around the tip joining one point on a face of the crack to another point on the opposite face, Fig.4,
ds	-	is a differential element of the contour,
u_i	-	is a displacement vector,
n	-	the unit outward normal to Γ,
$\sigma_{ij} n_j$	-	the normal component of the stress tensor to the contour Γ at ds, called as a traction vector,
σ_{ij}	-	stress tensor.
$\sigma_{ij} n_j \frac{\partial u_i}{\partial x} ds$	-	the unit energy density release from stress field σ_{ij}
x, y	-	is a cartesian coordinate system.

In linear elasticity J = G. Furthermore, the J integral is independent of the contour in elasticity. This property is also extended in plasticity in monotonic radial loading. Under the same conditions the strain energy release rate can be assumed to be equal to the J integral, as in elasticity.

The J integral characterises the crack tip singularities of stress, strain and strain energy density in non-linear elasticity and in plasticity.

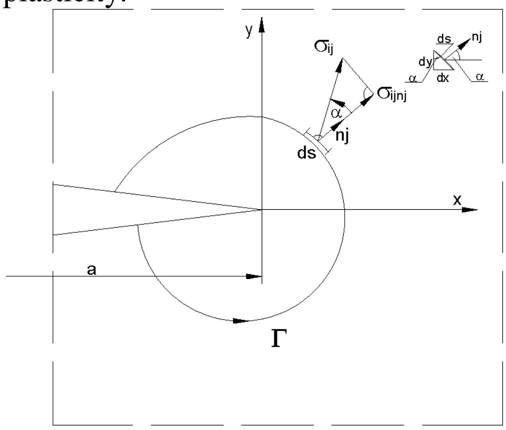

Fig. 4. A two-dimensional cracked body with a path Γ starting from lower and ending to the upper face of a notch with traction vector.

In the framework of deformation plasticity, it can be shown that J control the stress and strain near-tip fields as K does in linear elasticity. The proof was given by Hutchinson, Rice and Rosengreen [1, 3] and are designated as (HRR) fields. Let (r, φ) be polar coordinates with origin in the crack tip. Then for r→0 the stress and deformation as a function of J integral we assess as:

$$\sigma_{ij} = \sigma \left[\frac{J}{I_n \sigma_o \varepsilon_o A r} \right]^{\frac{1}{n+1}} \cdot \tilde{\sigma}_{ij}(\varphi, n), \tag{27}$$

$$\varepsilon_{ij} = \sigma \left[\frac{J}{I_n \sigma_o \varepsilon_o A r} \right]^{\frac{1}{n+1}} \cdot \tilde{\varepsilon}_{ij}(\varphi, n), \tag{28}$$

where:

A, n, ε_o, σ_o	-	are the constants of the Ramberg–Osgood law [2],
I_n	-	is the dimensionless factor depends on the hardening exponent n and on the crack mode (I, II, III) and is tabulated, e.g. in [4],
$\tilde{\sigma}_{ij}(\varphi, n)$ $\tilde{\varepsilon}_{ij}(\varphi, n)$	-	are the dimensionless function determined in [1, 3].

Near the crack tip, the strain energy density, which varies as r^{-1}, is proportional to J.

Fracture criterions

In agreement with [2, 4] the basic fracture criterions can be established as follows:

$$K_n = K_{nC}, \tag{29}$$

$$G_n = G_{nC}, \tag{30}$$

$$\delta = \delta_C, \tag{31}$$

$$J_n = J_{nC}, \tag{32}$$

where: n = I, II, III modes.

The left side-hand of criterions (31)÷(32) are characterised previously as above. The right-hand side of this equation are characterised the fracture toughness which described a material resistance to crack extension. The criterions (33) and (34) are mainly used for brittle fracture in the range of linear elastic fracture mechanics. The residual criterions (35), (36) are used in elasto-plasticity.

For example at I mode condition the criterions be express as follow:

$$K_I = K_{IC}, \tag{33}$$

$$G_I = G_{IC}, \tag{34}$$

$$\delta = \delta_C, \tag{35}$$

$$J_I = J_{IC}, \tag{36}$$

The fracture resistance at unstable crack propagation prior to 0,2 mm of crack growth or to a pop-in [2, 4].

Under plane strain condition the critical value of J, J_{IC} is related to the plane strain fracture toughness K_{IC} by equation:

$$J_I = \frac{1-\nu^2}{E} K_{IC}, \tag{37}$$

3. Conclusion

Fracture mechanics design methodology is based on more realistic parameters and criterions than the classic continuous mechanics. With the above procedures there were determined the maximum allowable loads applied for a specified crack size, or the maximum permissible crack size for specified load applied. Some further effort is needed on the effects of constraint on deformation behaviour during the fracture process, specially in weldments.

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