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MODYFICATION OF SELECTED METHODS OF RAPID DETERMINATION OF FATIGUE CHARAKTERISTICS IN THE RANGE OF LIMITED FATIGUE LIFE

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Abstract

In a number of design situations acquiring complete experimental data on fatigue properties used in material design solutions is impossible. It is justifiable then to use one of the common approximate (analytical) methods, preferably a method of verified accuracy. The problem of accuracy of analytical methods was earlier addressed in a publication [12]. The fatigue life assessment errors shown there for the range of limited fatigue life must be considered unacceptably high. This paper quotes selected, known from literature, applicable proposals and own modifications suggested by the Authors. The modifications were verified based on experimental data from a number of papers. A significant decrease in the fatigue life assessment error was reported.

Keywords: *high-cycle fatigue strength , fatigue plots, analytical methods of estimating Wőhler characteristics*

1. Wöhler characteristics and their importance

In design calculations it is necessary to know fatigue properties of the construction materials applied. As for the construction elements exposed to fatigue loads, the properties (for the calculations in the range of a high cycle number) are determined based on the S-N curve. The curve is applied, in practise, in the area of unlimited fatigue life and limited fatigue life. Performing design calculations in the first of the areas has a much more profound tradition and it has been applied by designers for a few dozen years already. The methods and relationships used in that way have been well verified and recognised. Design element dimensioning due to the limited life has been used only for the recent years and the state of knowledge and the accumulated sources of numerical quantities describing that area are definitely less satisfying. With that in mind, the attention of the Authors of this paper focuses on the second area.

In the range of limited fatigue life the Wöhler curve is also described with equation (1), for the determination of quantities, see Table 1. Considering the knowledge of the value of fatigue limit Z_G and the related limit number of cycles N_0, one can write the equation (2) or such notation to determine the value of slope m describing the curve in the range we are interested in. The scope of experimental studies necessary to determine value m applying experimental methods is given symbolically in Fig. 1a. The result of value m is accurate, however, unfortunately, due to a high number of samples which would need to be brought to fatigue damage, it is time-consuming. For example norm [9] recommends performing tests at a minimum of 5 load levels and at least 3 samples for each, which gives us at least 15 samples to be damaged. Besides the result is conservative in a sense of being related to the conditions of the test.

$$\sigma_a^m N = const., \tag{1}$$

$$\sigma_a^m N = Z_G^m N_0, \tag{2}$$

$$m = \frac{log\left(\frac{N_o}{N}\right)}{log\left(\frac{\sigma_a}{Z_G}\right)}, \tag{3}$$

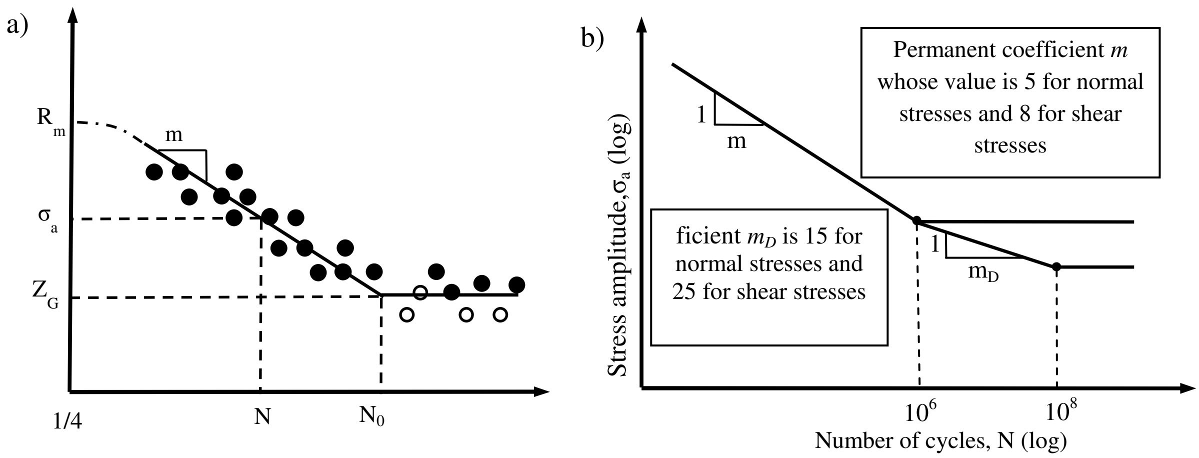

Fig. 1. Method of plotting the S-N curve according to a) experimental and b) analytic approach according to the FITNET method

Tab. 1. Breakdown of sample relationships from calculation methods in the high-cycle fatigue area

Method characteristics		Determined quantities	Model applied	Required input quantities
High-cycle range calculations	Real fatigue factor	δ – real fatigue factor in the range of limited fatigue life	$\delta = \frac{Z_N}{\sigma_{n\,a}}$ $Z_N = Z_G \sqrt[m]{\frac{N}{N_0}}$	Z_G, σ_a, σ_m, ε, β, N, N_0, m
Hypotheses of totalling of fatigue damage in the aspect of stress	Palmgren-Miner hypothesis	D_n – admissible sum of accumulated damage	$D_{PM} = \sum_{i=1}^{q} \frac{n_i}{N_i}$ $N_i = \left(\frac{Z_G}{\sigma_a}\right)^m N_o$	σ_{ai}, n_i, N_0, Z_G, m
Legend: Z_G – fatigue limit, σ_a – stress amplitude, σ_m – mean stress, ε – object size coefficient, β – notch effect coefficient, N – executed number of cycles, N_0 – limit number of cycles, N_i – executed number of cycles at a given load level, m – exponent.				

The importance of knowing exponent *m,* shown with selected examples, is given in Table 1. Table 1 presents sample notations connected with performing calculations of the values of the real fatigue factor and the values of fatigue damage determined according to the hypothesis by Palmgren - Miner. More details on the importance of that quantity in fatigue calculations are provided by the authors of monographs, e.g. [4].

2. Selected methods of acquiring fatigue characteristics and developing the paper hypothesis

Acquiring data on the fatigue characteristics (mostly equation exponent) can be made experimentally (treating the method as accurate) or with the analytic method, treating the method to be approximate.

The experimental method is described in detail in a number of norms, e.g. [9]. Sample analytic methods, namely the FITNET method and the method from the publication [5], were described in article [13]. The methods were selected for verification since they refer to fatigue life in the range of limited fatigue life. Those are the methods which have been proposed relatively recently, to address practical applications by experts. Besides determining the fatigue curve, according to the authors of those methods, can be supported by performing a simple experiment which involves a static test of determining material tensile strength. The verification of the accuracy of calculations of those methods is addressed in paper [12], which demonstrates considerable errors in the fatigue life assessment. For selected conditions the errors range from 14% to 573 %.

The authors of this paper have formulated a working hypothesis of the justifiability of a modified, analytically-experimental, approach which would involve the use of common analytical methods and a partial support from a simple fatigue experiment, own or provided in literature. Applying common methods of rapid determining of fatigue characteristics, offered in literature, accompanied by experimental support, it is possible to obtain satisfactory approximation accuracy result of the estimation for accurate characteristics. The aim of the paper is to verify, in a selected field, the above hypothesis.

3. Analytic-experimental method – own proposal

The own proposal has been based on the notations of FITNET procedure. Since, as demonstrated, determining characteristics according to FITNET methodology in the range of limited fatigue life is insufficiently accurate, the procedure of plotting the S-N curve in that range has been modified. The modification involves the variation of coefficient *m* in a wider range than in in the

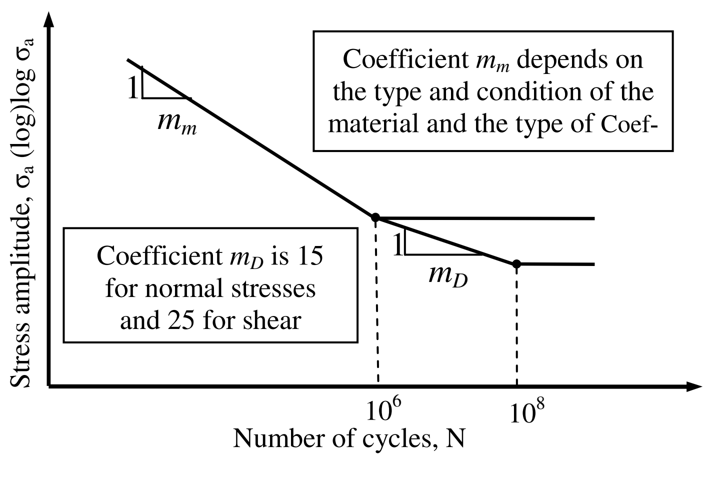

Fig. 2. Proposed modified curve

original methodology. Coefficient m depended on the type of material and the type of load. The numerical values of the slope used to implement that idea will be proposed based on the literature data and experiments. Fig. 2 presents, in a form of a diagram, the way the S-N curve is plotted according to the modified FITNET method. Plotting the curve in the range of unlimited fatigue life is made following the original method.

4. Verification of the hypothesis

4.1. Conditions of verification

Setting-up of the values of exponent m_m (symbols, see Fig. 2) for the reporting verification range in this paper is given in Table 2. The data are taken from papers [2,4,5,8] in which the ranges of variation of coefficient *m* are quoted.

Tab. 2. Values of the slope m_m

Type material	State	Data source	Value of coefficient m_m	
			Normal stress	Shear stress
Steel	Smooth specimen	[2],[4],[8]	12	12
Steel	Element with notch	[4],[5]	7	7
Steel	Heat treatment	[4]	18	18
Iron	Smooth specimen	Own proposal	11	
Aluminium alloy	Smooth specimen	Own proposal	8	8

At attempt at generalising the notation of value *m* involves formulating a general equation describing the dependence of slope *m* on the value of the ratio of the yield limit (R_e) to tensile strength (R_m). The notation has been presented as equations (4) and (5). To differentiate from the values in Table 2, symbol m_e has been introduced. In those equations there was assumed limiting the range of limited fatigue life in advance, by value 0.9 R_e.

$$m_e = \frac{\log\left(\frac{10^6}{N_{Re}}\right)}{log\left(\frac{0,9R_e}{Z_G}\right)}, \quad (4)$$

$$N_{Re} = 400\left(\frac{R_e}{R_m}\right)^{-10}, \quad (5)$$

Fig. 3a presents the regression curve described with equation (5). The curve was plotted based on the experimental data reported in literature (quoted in Table 3). For the curve plotted in that way the coefficient of correlation is 0.831, while the graphic interpretation of the formula (4) is presented in Fig. 3b.

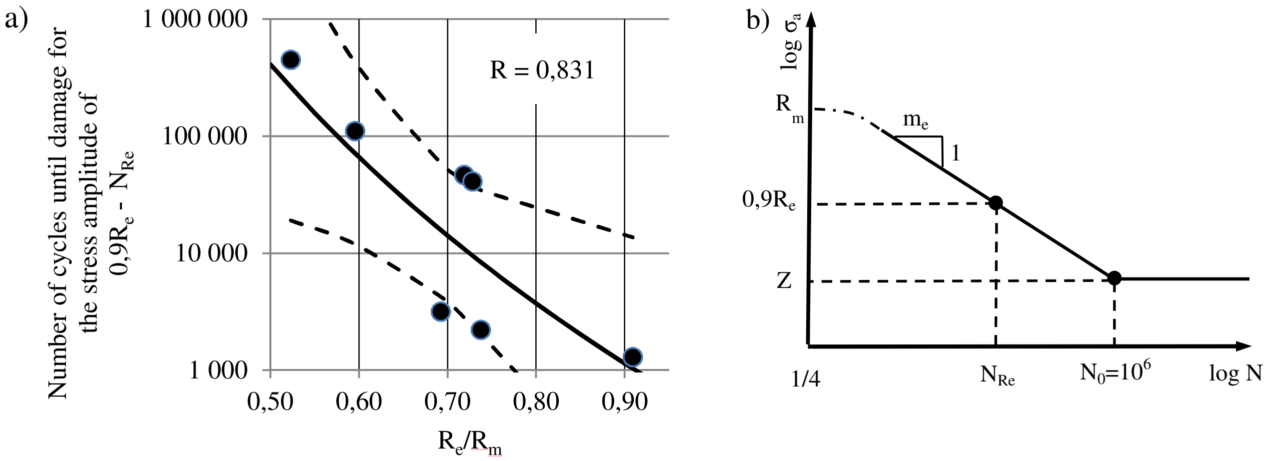

Fig. 2. a) Relationship between the number of cycles until damage and the ratio of R_e/R_m, b) graphic representation of symbols from formula 4

4.2. Results of the verification

The verification of the proposed analytical-experimental approach was made based on the calculations for the stress amplitude corresponding to the level of 10^5 cycles on the experimental curve. A comparison was made between the fatigue life determined from the experimental curve (namely 10^5 cycles) with the fatigue life determined for the same stress amplitude from the estimated curve. The results of that comparison are shown in Table 4. The results of the fatigue life assessment error range from 12% to 286%. A graphic representation of selected results of curve calculations estimated according to the original FITNET method and the modified method are given in Figs. 4a, 4b and 4c.

Tab. 3. Comparison of the recorded fatigue life according to the proposal conditioning 0.9 value of the yield limit with the number of cycles until damage and the experimental data.

Material	Value of slope *m* according to the experiment		R_e/R_m	Number of cycles for crossing of 0.9 of the yield limit with the experimental curve	Calculated number of cycles N_{Re}	Determined value of slope *m*
S355J0	12,3	[12]	0.74	2.21×10^3	8.41×10^3	12.3
C40	12.8	[1]	0.60	1.10×10^5	7.11×10^4	15.1
C45	11.1	[6]	0.52	4.46×10^5	2.61×10^5	29.8
34CrMo4	8.4	[1]	0.72	4.63×10^4	1.09×10^4	12.5
42CrMo4	23.2	[1]	0.73	4.09×10^4	9.54×10^3	12.4
42CrMoS4	14	[10]	0.91	1.30×10^3	1.03×10^3	11.5
D38MSV5S	17.7	[4]	0.69	3.16×10^3	1.58×10^4	12.7

An attempt at generalising the notation of value *m* was verified for the materials specified in column 1 Table 3. The table makes it possible to compare exponent *m* taken directly from the experiment (column 2) with the value estimated based on Fig. 3a (the last column).

Tab. 4. Comparison of the recorded fatigue life according to the modified FITNET method (constant m for a group of materials) and the experimental data

Material	Value of slope *m* according to the experiment		Value of slope *m*	Number of cycles until damage	Difference between the experimentally determined number of cycles and the number of cycles recorded	Fatigue life assessment error [%]
Normal stresses						
S235JR	11.5	[9]	12	9.4×10^3	9.1×10^4	91
S355J0	12.3	[13]	12	3.9×10^5	-2.9×10^5	-286
E355	18.8	[3]	12	3.1×10^5	-2.1×10^5	-214
C40	12.8	[2]	12	1.12×10^5	-1.2×10^4	-12
C45	11.1	[7]	12	1.16×10^5	-1.6×10^4	-16
15Cr2	20	[9]	18	3.6×10^3	9.6×10^4	96
14CrMoV69	16.1	[2]	18	4.1×10^2	9.96×10^4	99,6
30CrNiMo8	11.7	[12]	18	1.3×10^5	-3×10^4	-30
34CrMo4	8.4	[2]	12	3.8×10^4	6.2×10^4	62
42CrMo4	23.2	[2]	12	1.7×10^4	8.3×10^4	82
42CrMoS4	14	[11]	18	3.2×10^4	9.4×10^4	94
SAE 8630	11.8	[2]	18	2.4×10^5	-1.4×10^5	-139
D38MSV5S	19.5	[5]	12	2.1×10^5	-1.1×10^5	-110
Shear stresses						
C45	10,1	[6]	12	1.4×10^5	-4×10^4	-40
30CrNiMo8	19,3	[12]	18	4×10^3	9.6×10^4	96
D38MSV5S	10,9	[4]	12	2.1×10^4	7.9×10^4	79

Tab. 5. Comparison of the fatigue life recorded according to the modified FITNET method (m dependent on the ratio R_e to R_m) and the experimental data

Material	Value of slope *m* according to the experiment		Determined value of slope *m*	Number of cycles until damage	Difference between the experimentally determined number of cycles and the number of cycles recorded	Fatigue life assessment error [%]
S355J0	12.3	[12]	12.3	3.77×10^5	-2.77×10^5	-277
C40	12.8	[1]	15.1	6.35×10^4	3.65×10^4	36.5
C45	11.1	[6]	29.8	4.77×10^3	9.52×10^4	95.2
34CrMo4	8.4	[1]	12.5	3.39×10^4	6.61×10^4	66.1
42CrMo4	23.2	[1]	12.4	1.54×10^4	8.46×10^4	84.6
42CrMoS4	14	[10]	11.5	1.55×10^4	8.45×10^4	84.5
D38MSV5S	17.7	[4]	12.7	1.91×10^5	-9.06×10^4	-90.6

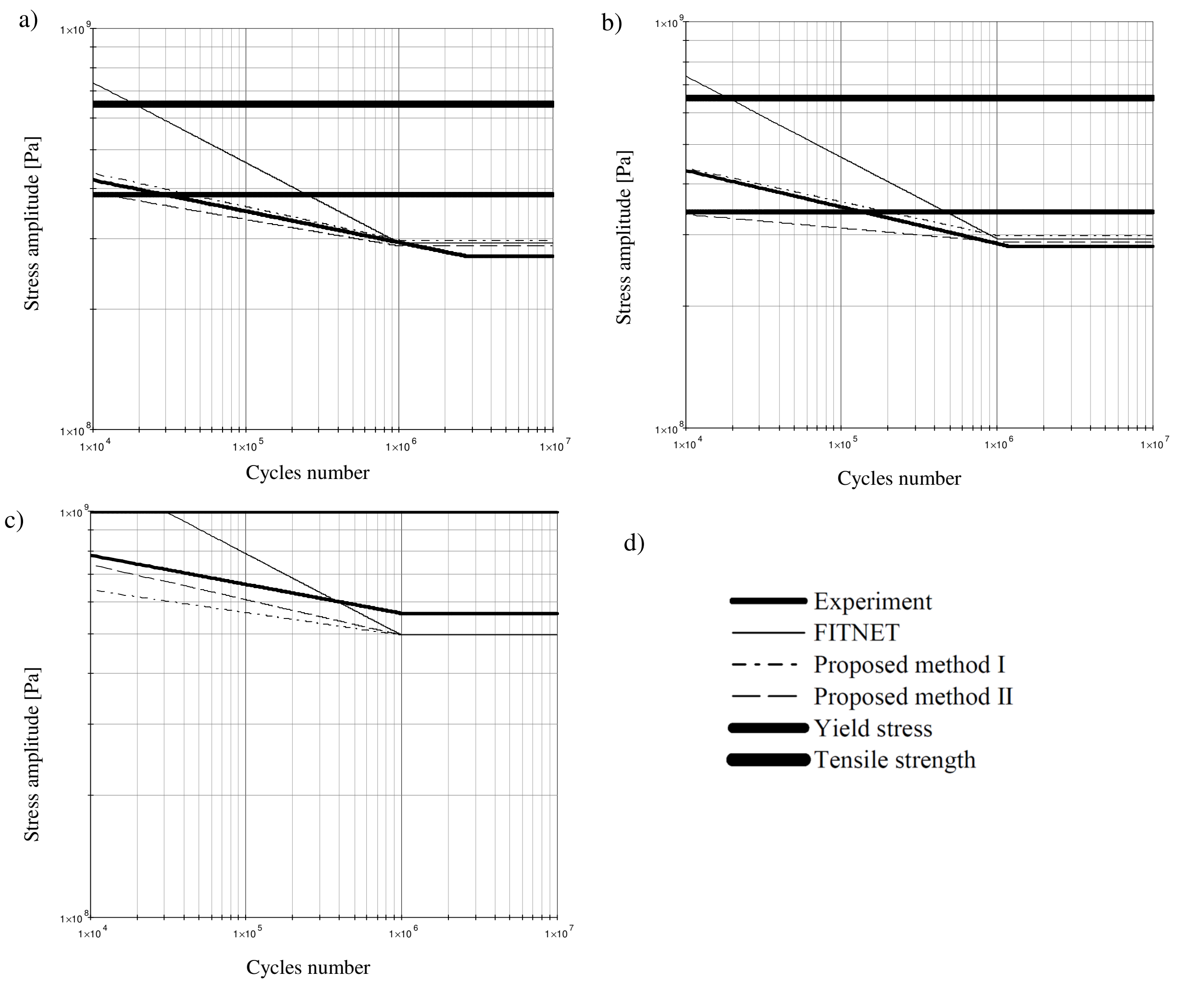

Fig. 3. Diagrams presenting a comparison of methods of evaluating the high-cycle fatigue life for: a) steel C40 in normalized state, b) steel C45 in normalized state, c) steel 42CrMoS4 in thermally improved state, and d) legend

6. Summary and conclusions

Based on the above verification results the following conclusions can be formulated:

– Using only analytical methods can lead to considerable fatigue life assessment errors (as much as 573%).
– The application of experimental methods results in considerable financial inputs and leads to the results conservative in use.
– As pointed in this paper, it is possible to combine the existing knowledge on analytical methods with purposeful support with data from similar experiments; such approach has been referred to as the analytical-experimental approach or it can be referred to in short as the hybrid approach.
– An essential quality of the hybrid approach is the dependence of exponent m from the type and state of the construction element material the approximate value of which can be determined based on the literature data or experimental data of similar materials.
– Two analytical methods have been recognised as are worth considering for engineering applications; the method with a proposed value of exponent m for a specific group of materials and the method with the slope exponent dependent on the ratio of Re to Rm.
– The application of the hybrid approach leads to decreasing the fatigue life assessment error from 573% (as shown in paper [12]) to 286%, as demonstrated in Tables 4 and 5 (for the assumed area of analysis).

Interestingly, the presented breakdown of data, the analysis and its conclusions concern a limited group of materials. The verification points to specific tendencies and finds it justifiable to present it for a wider group of materials.

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