



## THE PROCESS OF MODELING CURVE OF AN INEQUALITY GRINDING RICE GRAINS

Andrzej Tomporowski

University of Technology and Life Sciences  
ul. Prof. S. Kaliskiego 7, 85-789 Bydgoszcz  
Tel./fax: +48 52 3408255  
[a.tomporowski@utp.edu.pl](mailto:a.tomporowski@utp.edu.pl)

### Abstract

*There has been the modelling of regularity of weights of multiedge biological material mincers conducted at work. The record of irregularity, registered in the time interval, is called the execution of stochastic irregularity of the grinding process. The construction features of the grinding tool, parameters of the process and individual features of ground and biological material of anisotropic structure influence their shape. For adopted assumptions, there has been proposed the psychical model to be published in trigonometric and exponential function. The suggested methodology to mathematically model the curve of the machine run non-uniformity and the grinding process itself fulfils the expectations concerning the development aimed at: high process effectiveness and product usable quality.*

*With the conformity of the above model for real course, the evaluated function ( $Q=||y-y^*||$ ) was determined at the level of 6-12% depending on rice properties.*

*The necessary condition to increase uniformity and efficiency as well as further development of grinding biological and fibrous materials is to elaborate an effective method to model and describe the grinding non-uniformity curve.*

**Keywords:** non-uniformity, grinding, biological materials, multi-edge grinders

### 1. Introduction

The process of angular velocity of the torque and grinding power is of impulse character. It is associated with the periodical operation of the grinding edges and determined operation of biological material, its function of biological materials cannot be definitely predicted, it is a stochastic phenomenon. The registered parameter in the function, for example time, of which values all the time change at random, is called the record of stochastic starting signal. However, one can depict the shape, structure and operating elements of the machine. The course of the grinding irregularity and state the range  $(t_1, t_2)$  defined by time or angle of rotation  $(\alpha_n; \alpha_k)$  and approximated moment value of measured parameter. The very important feature of stochastic register of irregularity of grinders' operation is the independence of the registered parameter (power or angular velocity) of the property, from the freely chosen time intervals, in which one makes the analysis of the objective course of the starting signal.

In the real course of grinding materials of the farm and food industry, one finds it difficult to predict, occurring one after the other, states of changes and stresses. On these grounds we deal with, from the point of view of work regularity, the stochastic process. The basic difference that

distinguishes this type of processes from the determined processes is that: the stochastic process is an accumulation of different executions, and the executions last relatively long.

The research problem was formulated in the form of a question: What are the formalization possibilities of real processes of dynamic (M) irregularity and kinematic one ( $\omega$ ), with the use of parameters and properties of random function?

## 2. Assumptions

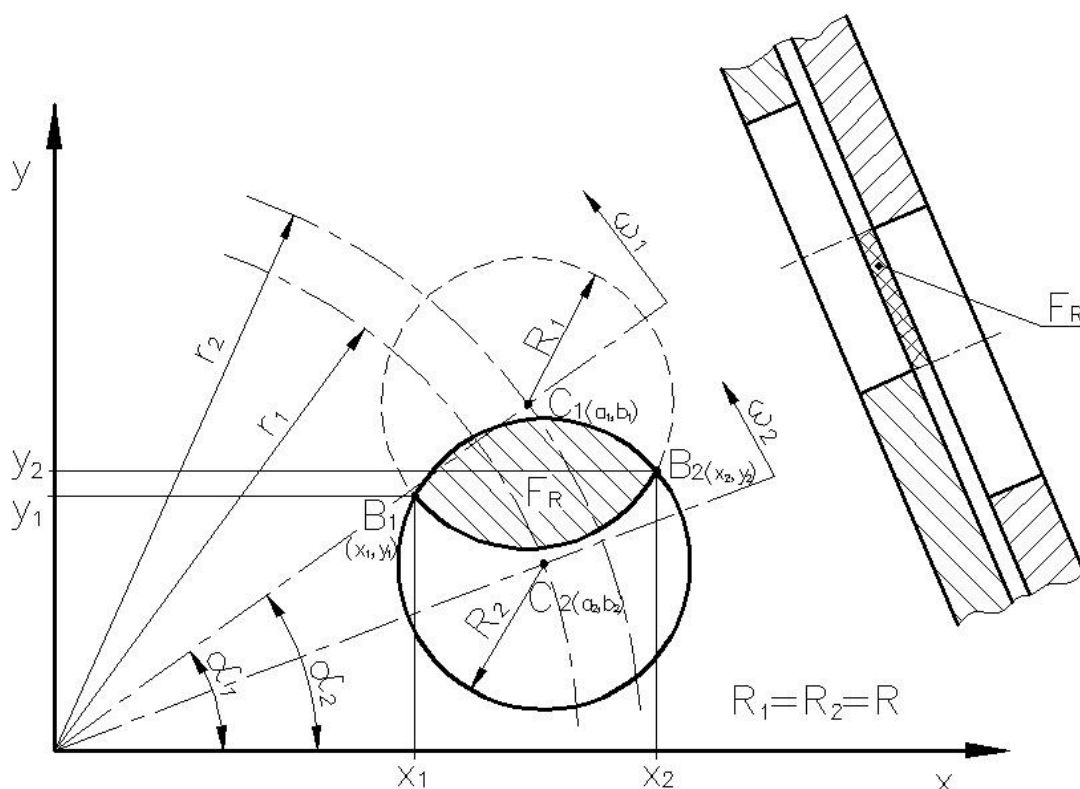


Fig.1. Sectional area and effective grinding surface for two grinding discs between edges of the grinding holes [1,2]

During the time a single blade goes through the material being ground (Fig. 1), the grinder shaft torque increases from zero (start of cutting) up to the maximum value, and then it gradually goes down to zero (at the moment when the cutting operation is finished). The torque value inside a multi-hole grinder depends on the design parameters of the quasi-shearing assembly

In turn great changeability of torque makes alternate variations of the angular velocity. The changes in the angular velocity are of a pulsing nature. The measure of uniformity of work of the grinders is inaccuracy of realization of the preset motion function, it means non-uniformity of run, loads, and even efficiency [1, 2].

The power requirement to drive a blade is very unstable (Fig. 2), since there is a period of the machine idle run between cutting the consecutive pieces of material. Therefore significant changes in power consumption may occur.

The changing difference of the active torque and the torque of driving forces is a reason for changeability of the angular velocity of the grinder shaft. The active torque  $M_{cz}$  is considered to be the torque which occurs when cutting. While the torque of driving forces  $M_{sn}$  is used to denote the torque taken from an electric motor. If the shaft makes  $n$  revolutions, then the average angular velocity being equal to the nominal velocity is

$$\omega_{ir} = \omega_{nom} \approx \frac{\omega_{max} + \omega_{min}}{2}. \quad (1)$$

The variations of the shaft angular velocity during the steady motion take place between  $\omega_{\max}$  and  $\omega_{\min}$ . The aforementioned changes are included in the coefficient of the non-uniformity of the machine run [6], which is determined by

$$\delta_k = \frac{\omega_{\max} - \omega_{\min}}{\omega_{sr}} \quad (2)$$

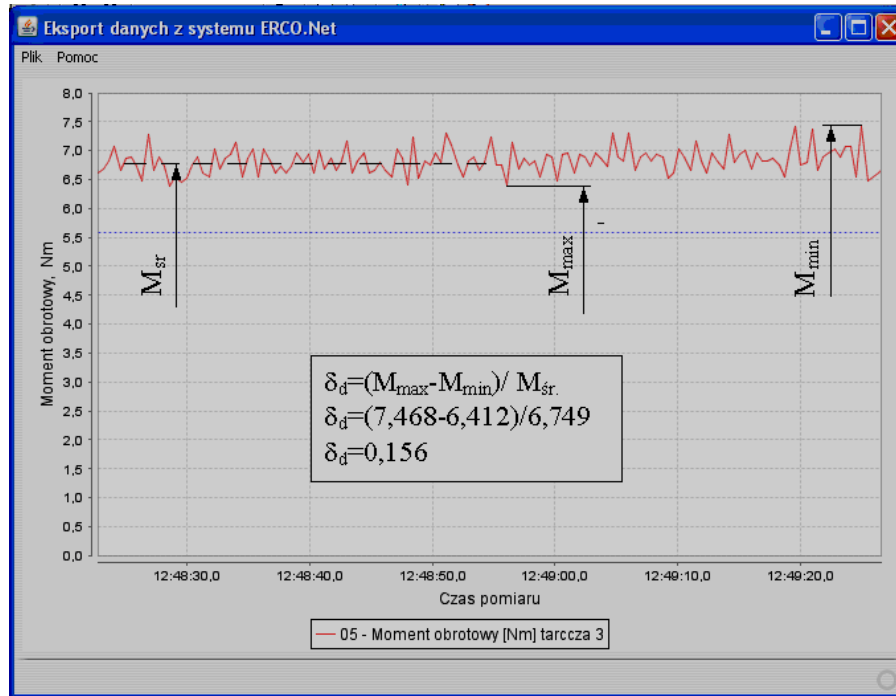


Fig. 2. Fragment of torque sequence and evaluation of non-uniformity of loads of the five-shielded grinder RWT-5KZ, while grinding a rice grain

## 2. Approximation of the grinding non-uniformity curve to obtain its mathematical form

The recorded courses of kinematic and dynamic non-uniformity (Fig. 4) of the grinder work may be described in a mathematical form by applying essential elementary function systems:

- exponential,

$$M(t) = M_1 e^{jn\omega t} + M_2 e^{jn\omega t} + \dots + M_n e^{jn\omega t}, \quad (3)$$

- trigonometric,

$$M(t) = a_0 + c_1 \cos(n\omega t + \phi_1) + c_2 \cos(n\omega t + \phi_2) + \dots + c_n \cos(n\omega t + \phi_n). \quad (4)$$

For the needs of transformation of the measurement curve (e.g. stochastic one) to obtain its mathematical form, it is advantageous to select the relationships which shall assure representation of a real course at the minimum level of the acceptable deviation error [4]. This is a characteristic feature of trigonometric and polynomial functions. A periodic function describing the investigated non-uniformity courses, which meets the necessary conditions may be set as the Fourier series, made of the sum of the individual trigonometric functions, with various amplitudes, and a constant component. The physical form of the period course of grinding non-uniformity is a sufficient condition for existence of a real mathematical function.

The periodic function of non-uniformity function developed to the Fourier series has many theoretical and strictly practical applications. It facilitates presentation of the course of non-uniformity of grinding operation when presenting frequency analysis for sinusoidally changing

courses. Each component of a pulse, an increase or a decrease in load may be investigated separately for the linear representations.

Presentation of the course of non-uniformity of the grinding machine course as the Fourier series is equivalent to decomposition of the essential function into its components; a constant component and individual components (respective answers to loads). The investigated non-uniformity curve may be approximated by means of a finite number of harmonic courses so that after adding next sequential harmonic components an image showing more and more precise approximation of the real course can be obtained.

The essential, graphic mathematical models describing the courses of non-uniformity of the grinding machine work are affected by some error – a deviation from the real values. They may be presented as a regular wave with a typical geometrical configuration by means of an infinite trigonometric series, with the functions having the form of the dominant course:

- with an isosceles triangle outline

$$M(t) = 8 \frac{A}{\pi^2} \left( \frac{\sin \omega t}{1} - \frac{\sin 3\omega t}{9} + \frac{\sin 5\omega t}{25} - \dots \right), \quad (5)$$

- with a right-angled triangle outline;

$$M(t) = 2 \frac{A}{\pi} \left( \frac{\sin \omega t}{1} - \frac{\sin 2\omega t}{2} + \frac{\sin 3\omega t}{3} - \dots \right), \quad (6)$$

- with a trapezoidal outline;

$$M(t) = 4 \frac{A}{\alpha \pi^2} \left( \frac{\sin \pi \alpha \sin \omega t}{1} + \frac{\sin 3\pi \alpha \sin 3\omega t}{9} + \frac{\sin 5\pi \alpha \sin 5\omega t}{25} + \dots \right). \quad (7)$$

In the analysed courses of non-uniformity of work of the multi-edge grinders the amplitudes of individual components are inversely proportional to frequency, therefore the components with smaller frequency have greater amplitude than the components with higher frequency.

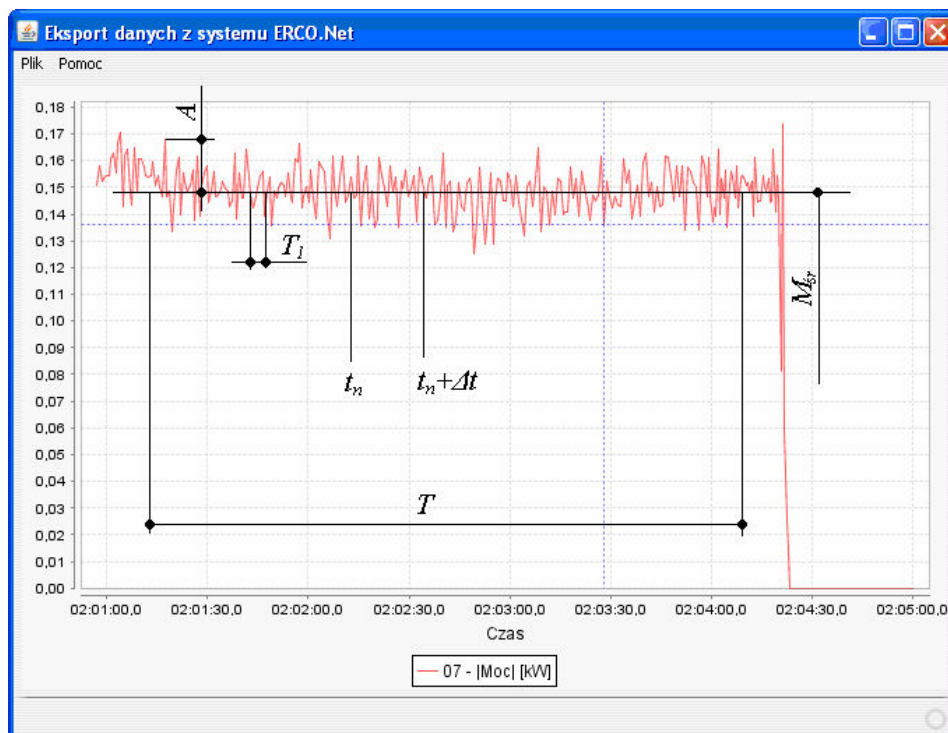


Fig. 3. The examination power of the stochastic process values of one-shield grinder during grinding grinder RWT-5KZ, while grinding a rice grain

Approximation of a real course to obtain the corresponding model curve is simplified or it becomes more general after applying the exponential Fourier series.

$$M(t) = A_0 + A_1 e^{j\omega t} + A_2 e^{j2\omega t} + A_3 e^{j3\omega t} + \dots + A_n e^{jn\omega t}, \quad (8)$$

where:

$M(t)$  — approximated function - torque,

$n = 1, 2, 3, \dots$ ,

$\omega = 2\pi/T$ ,

$T$  - period.

The coefficient  $A_n$  facilitates to determine a set of harmonic components which when summed up make the course of the function  $M(t)$ .

Exemplary presentation of the sequence of the rectangular pulses (velocity or torque) by means of the trigonometric series is formulated as:

$$M(t) = \frac{\alpha A}{T} \left[ 1 + 2 \left( \sin \frac{1\alpha\pi}{T} \cos 1\omega t + \sin \frac{2\alpha\pi}{T} \cos 2\omega t + \dots \right) \right]. \quad (9)$$

With the conformity of the above model for real course, the evaluated function ( $Q = \|y - y^*\|$ ) was determined at the level of 6-12% depending on the rice properties.

### 3. The statistic characteristics of irregularities processes

Making average of the obtained values of the torque or rotational speed of the grinder's shaft, as a random value in relation to the time, one can make the average of one of the execution in relation to the time parameter or rotational angle of the operational shaft.

It is essential, in the analysis of the stochastic process, to check whether in the tested process of irregularity of the grinding machine operation do not occur the statistic regularities which indicated the process at least partially determined. It may be performed using the non-complicated examination procedure. One needs to register repeatedly the process of irregularity of the machine process. The achieved results one needs to compare with one another in a few, and even several, similar time intervals from  $t_1$  to  $t_n$  (or whether depending on the kept register, rotational angle of the operational shaft from  $\alpha_1$  to  $\alpha_n$ ). Compare and make the analysis, read in the chosen ranges the values of the registered parameter: of torque for the dynamic register and angular velocity for the kinematic one, from  $M_1(t_1)$  to  $M_1(t_n)$ , and in turn all ones, till to the value from  $M_n(t_n)$ . In order to exclude the dependence of the achieved function on the accepted time values, the checking procedure should be repeated for the moved times. As a consequence one should make the diagrams of cumulative frequency of the random variable  $E(M, \omega)$ , in the form of:

- for the dynamic irregularity

$$E_t(M) = P [M(t) < x] \quad \text{and} \quad E_{t+\Delta t}(M) = P [M(t+\Delta t) < x], \quad (10)$$

- for the kinematic irregularity

$$E_t(\omega) = P [\omega(t) < \omega] \quad \text{and} \quad E_{t+\Delta t}(\omega) = P [\omega(t+\Delta t) < \omega], \quad (11)$$

where:

$M$  – the torque on the grinder's shaft,

$\omega$  - the angular velocity of the grinder's shaft.

The process is stationary, if the achieved cumulative frequencies are equal, which means

$$E_t(M, \omega) = E_{t=\Delta t}(M, \omega). \quad (12)$$

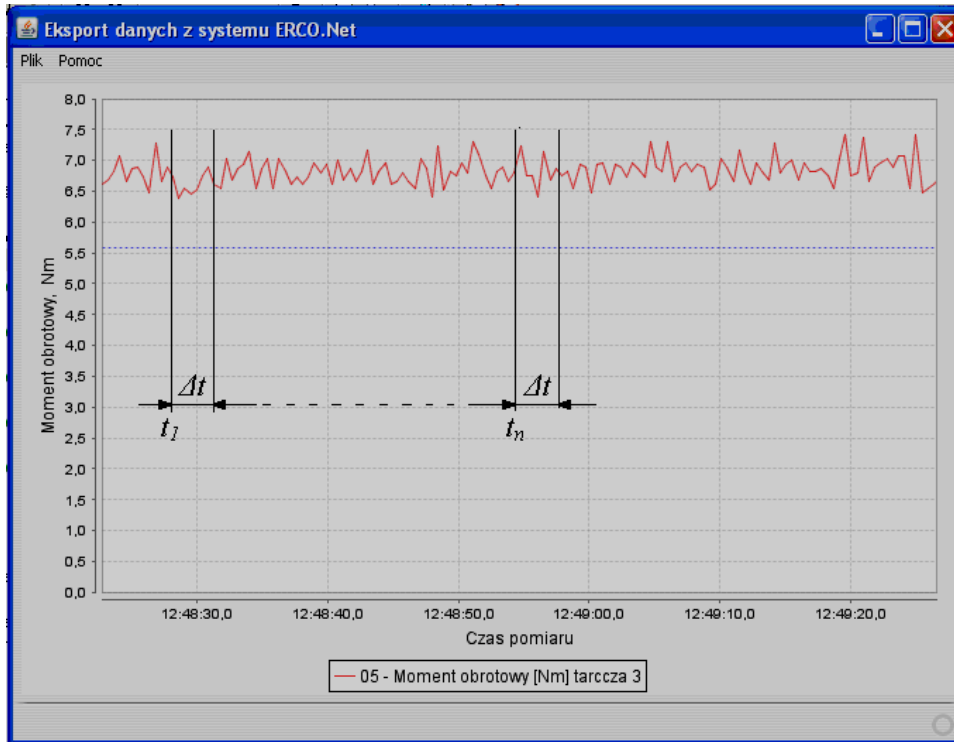


Fig.4. The examination of the stochastic process values of one-shield grinder during grinding grinder RWT-5KZ, while grinding a rice grain

One cannot predict the course of the irregularities of grinding the biological materials, it is completely stochastic phenomenon. Correctly designed process of multi-edged grinding technology let us predict some properties of statistical registered record of irregularities.

The record of the process of grinder's work irregularity may be read in double ways:

- reading of the record data – statistic feature of the model, record,
- reading of the changes speed, registered starting point – describes statistically the variability of the objective record.

The analysis of irregularities as of the stochastic phenomenon defines in the statistic sense the process of multi-edged grinding. Practically, for a correct description of the grinding process one should register the irregularities of the stochastic record in a very long time interval [5].

#### 4. Static characteristics of irregularities course

The average value (Fig. 3) of the random form of irregularity of the grinding machine process  $M(t)$  and  $\omega(t)$  defines the static component, one which is not accidental. As a result of its subtracting from the irregularity value, we receive the value including only the random variable component.

The random mean value may be presented in the form:

- for the dynamic irregularity:

$$M_{sr}(t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T M(t) dt, \quad (13)$$

– for the kinematic irregularity:

$$\omega_{sr}(t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \omega(t) dt . \quad (14)$$

For the description of the statistical properties of the irregularity register of the grinding machine course, one may use its mean-square value, as a measure characterizing the irregularity course, both the dynamic and kinetic one, of the course of the biological materials of grinders;

$$M_{sr}^2(t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T M^2(t) dt , \quad (15)$$

where:

T – time interval.

The likelihood that the temporary value of the irregularity M(t) shall be included in the range (M<sub>A</sub>, M<sub>B</sub>) is expressed by the formula;

$$P[M(t) \in \langle M_A ; M_B \rangle] = \lim_{T \rightarrow \infty} \frac{1}{T} (t_1 + t_2 + \dots + t_k) , \quad (16)$$

where:

t<sub>k</sub> - times of which the temporary values of the signal M(t).

The density of likelihood of the temporary values of the multi-openings grinder's irregularity course may be written in the form of the formula;

$$p(M) = \lim_{\Delta x \rightarrow \infty} \frac{P[M(t) \in \langle M_A ; M_B \rangle]}{t_1 + t_2 + t_3 + \dots} . \quad (17)$$

The arrangement of the likelihood of the continuous random variable M (moment) or ω (the angular speed) in the time is the dependence of the likelihood density p(x) from the independent variable x.

$$p(M) = const \Leftrightarrow M \in \langle M_{min} ; M_{max} \rangle ,$$

and

$$p(M) = 0 \Leftrightarrow M \notin \langle M_{min} ; M_{max} \rangle . \quad (18)$$

The density of the likelihood of the irregularity (dynamic) function of the grinder's course of the model approximated to the form of described with the formula;

$$M(t) = 2M_i \frac{1 - 2\left(\frac{\cos \omega t}{3} + \frac{\cos 2\omega t}{15} + \frac{\cos 3\omega t}{35} + \dots\right)}{\pi} , \quad (19)$$

is:

$$P(M) = \pi \sqrt{M_{max}^2 - M_i^2} , \quad (20)$$

where:

M<sub>i</sub> – temporary values.

For the description variation of the irregularities course of the biological materials grinding, one may use the function of autocorrelation as a measure of the dependence of the function values M(t) or ω(t) distant to each other of the difference value of their measurement. It may be depicted with the formula:

$$S(\Delta t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T M(t_i)M(t_i + \Delta t)dt. \quad (21)$$

## 5. Summary

The necessary condition to increase uniformity and efficiency as well as further development of grinding biological and fibrous materials is to elaborate an effective method to model and describe the grinding non-uniformity curve [6]. At each stage such a design solution of the tool and grinding unit is sought which shall guarantee, during the preset time, extreme function of: effectiveness, quality and uniformity.

The up to present examinations of the irregularity course [1, 2, 3], directed at the experimental cognition of the course values and characteristics, prove the formalization need. The formalization of the grinding irregularities courses carried out on the example of biomaterials, for the chosen random function parameters prove the soundness of the accepted assumptions. Such a proceedings should cover the detailed verification of the ergodicity assumptions and the stationarity of the characteristics of the torque, angular speed, and the power on the grinder's shaft.

## References

- [1] Flizikowski, J., *Konstrukcja rozdrabniaczy żywności*, Wydawnictwo Uczelniane Akademii Techniczno – Rolniczej w Bydgoszczy, Bydgoszcz 2005.
- [2] Flizikowski, J., Bieliński, M., *Rozdrabniacz wielotarczowy zwłaszcza do materiałów ziarnistych*, Patent RP.
- [3] Flizikowski, J., Tomporowski, A., *Mechanical processing of elastic – plastics corns*, 12-th International Conference on Developments in Machinery Design and Control, Nowogród 2008.
- [4] Mroziński, A., *Modelling of waste-paper stock treatment process in disc refiners*, Journal of POLISH CIMAC nr 3/2010, Vol. 5, str. 113-119.
- [5] Tomporowski, A., *Testing thy power of multi - disc shrebing grains of rice*, Ekologia i Technika nr 3, s. 161-164.
- [6] Tomporowski, A., *Nierównomierność obciążeń przy rozdrabnianiu drewna*, Recykulacja, Bydgoszcz 2008.
- [7] Tomporowski, A., *Rozdrabniacz do wilgotnych ziaren zbóż*, Inżynieria maszyn 22/2004, Bydgoszcz 2004.