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GRAPHICAL ANALYSIS OF BUS LIFETIME

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Abstract

The lifetime distribution is very important in reliability studies. The shape of lifetime distribution can vary considerably. It frequently cannot be approximated by simple distribution functions. The purpose of this paper is to introduce and describe graphical tools for lifetime data. This article is connected with problem of finding of lifetime distribution for a heterogeneous population of lifetime data. A heterogeneous population can be represented by a two component mixture. The numerical examples are given to illustrate two lifetime model. The parameter estimation is based on the maximum likelihood method. The methodology is illustrated by two real data set. The graphical illustration on lifetime distribution is presented.

Keywords*: lifetime, mixture of distribution, Weibull distribution, exponential distribution, TTT-plots, reliability function, IFR, DFR.*

1. Introduction

The modeling and analysis of lifetimes is an important aspect of statistical work in a wide variety of scientific and technological fields. An important topic in the field of lifetime data analysis is to select the most appropriate lifetime distribution. This distribution describes the time to failure of a component, subsystem or system. The probability distribution of the lifetime of a technical object can be characterized by the failure rate function. The failure rate function is a basic concept in reliability theory and reliability practice. If lifetime distribution is absolutely continuous what very often can be assumed, the failure rate function uniquely determines the lifetime distributions. An important class of the lifetime distribution arises when the failure rate function is non-monotonic.

Occurrence of instantaneous or early failures in lifetime testing is observed in sets of failures of machines. This occurrence may be due to faulty constructions or inferior quality. Somme failures result from natural damages of the machine while the other failures may be caused by inefficient repairs of previous failures resulting from incorrect organization of the repairs.

In the papers [8] and [14] the set of failures of a machine is divided into two subsets, namely into set of primary failures and the set secondary failures. This suggests that the population of lifetime is heterogeneous. The population of the time before failure can be described by using the statistical concept of mixture. This mixture, in particular case, has the unimodal failure rate function [9]. The purpose of this note is to bring attention to the use of use graphical solution methods based on the total time on test TTT transform for detecting early failures. TTT-transform was introduced by Barlow and Campo [3] and further extended by Bergman and Klefsjo [4]. An application of TTT-plots in reliability theory is presented in paper [9]. Total time test (TTT) transformation plots are useful for analyzing non-negative data. The plots help choosing a mathematical model for the reliability data and provide the information about the failure rate function. In this study, graphical methods based on TTT-transform will be used to illustrate the variety of the failure rate shapes.

2. Basic definitions and ageing properties

Let F(t) be a lifetime distribution with finite mean ET and F(t) = 0 for t < 0.T is random variable (lifetime) with distribution function F(t), reliability function R(t) = 1− F(t) and the failure rate function $\lambda(t) = f(t) / R(t)$.

2.1. Mixture

We consider a mixture of two lifetimes T_1, T_2 with densities $f_1(t)$, $f_2(t)$, reliability functions $R_1(t)$, $R_2(t)$,failure rate function $r_1(t)$, $r_2(t)$ and weights p and $q = 1 – p$, where $0 < p < 1$. The mixed density is then written as

$$f(t) = f_1(t) + (1 – p) f_2(t)$$

and mixed reliability functions is

$$R(t) = p R_1(t) + (1 – p) R_2(t).$$

The failure rate function of the mixture can be written as the mixture [8]

$$r(t) = \omega(t) r_1(t) + (1 – \omega(t)) r_2(t),$$

where $\omega(t) = pR_1(t) / R(t)$.

2.2. TTT transformation

During another year, the fundamental concept was defined, studied and proven to be a useful tool. In the lifetime data analysis the total time test (TTT) is very useful rate function. The function:

$$H^{-1}(t) = \int_0^{F^{-1}(t)} R(u)du \quad \text{for } 0 \le t \le 1$$

is TTT transformation of F(t). The mean of F(t) is given by

$$\mu = H^{-1}(1) = m(0).$$

The scale invariant transformation

$$H^{-1}(t)/\,\mu = \frac{1}{\mu}\int_0^{F^{-1}(t)} R(u)du$$

Different concepts are used not only for parametric modeling but also to define various nonparametric classes of lifetime distribution. The most well known of these are : IFR – increasing failure rate function and DFR – decreasing failure rate function.

2.3. Early and instantaneous failures

We consider a family of continuous distribution functions F(x; Δ), where Δ is a set of parameters, F(0, Δ) = 0. To accommodate a real life situation, where instantaneous failures are observed at the origin, the model F(x; Δ) is modified to model G(x; Δ, p) by using a mixture in the proportion 1– p and p respectively of the singular random variable Z at zero and with random variable T with the distribution function F(x; Δ).Thus, the modified distribution function of lifetime is given as:

$$G(x;\Theta,p) = \begin{cases} 1-p & \text{for} \quad x = 0 \\ 1-p+pF(x;\Theta) & \text{for} \quad x > 0 \end{cases}$$

and the corresponding probability density function as:

$$f(x;\Theta,p) = \begin{cases} 1-p & \text{for} \quad x = 0 \\ 1-p+pf(x;\Theta) & \text{for} \quad x > 0 \end{cases}$$

The problem of statistical inference about (Δ, p) has received considerable attention particularly when T is exponential. Some of the early references are: Aitchison [2], Kleyle and Dahiya [7], Jayade and Parasad [5], Muralidharan [10], [11],[12] Kale and Muralidharan [6] and the references contained therein. Muralidharan and Kale [6] considered the case where F is a two parameters gamma distribution with shape parameter β and scale parameter α, and they obtained confidence interval for δ = pαβ assuming α as being known and unknown.

3. The lifetime model for bus engine

The analysis of n =190 data sets of lifetimes suggests that as the distribution time between two successive failures we can accept the distribution of mixture of two point distribution with exponential distribution. In our experiment the time between two successive failures was registered one time per day. Analysis of lifetime data shows that the number of recorded failures for t = 0 and t = 1 is considerably greater than for other time values. Firstly, in order to identify the shape of the life time distribution, we shall consider graphical methods based on Total Time on Test (TTT). Hence, we conclude that the statistical population is heterogeneous. This fact suggests that the distribution of lifetime may be the mixture of the two point distribution:

$$P\{X = 0) = a,\ P\{X = 1) = b, \text{ where } a + b = 1,\ a \geq 0,\ b \geq 0 \qquad (1)$$

and the exponential distribution with the distribution function:

$$F_2(t) = 1-\exp(-\lambda t) \quad \text{dla } x > 0 \tag{2}$$

This fact suggests that the distribution of lifetime may be the mixture of the two point distribution and the exponential distribution. The distribution function of the mixture has a form:

$$F(t) = p\, F_1(t) + (1-p)\, F_2(t) \tag{3}$$

The distribution function depends on parameters (a, b, λ, p).These parameters are estimated by numerical maximum likelihood methods, thus obtaining: a = 0.792, b = 0.208, λ = 0.051, p = 0.336.

The goodness fit test shows high consistence of both distributions. The value of the statistics λ-Kolmogorow's λ = 0.37it gives p-value=0.75, whereas, the Person's χ^2 = 23.13 with p-value = 0.84.

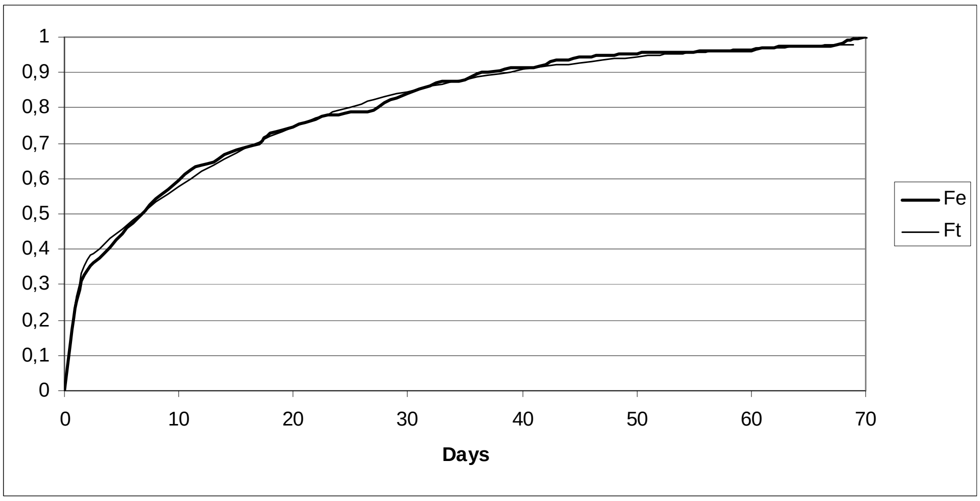

Fig.1. Empirical distribution and distribution of mixture

Fig.1 shows the charts of the empirical distribution functions and the mixture. Fig.2 demonstrates TTT-plots for the empirical distribution and the distribution after separation of the two-point distribution. The distribution after separation shows the good cosistency with an exponential distribution.

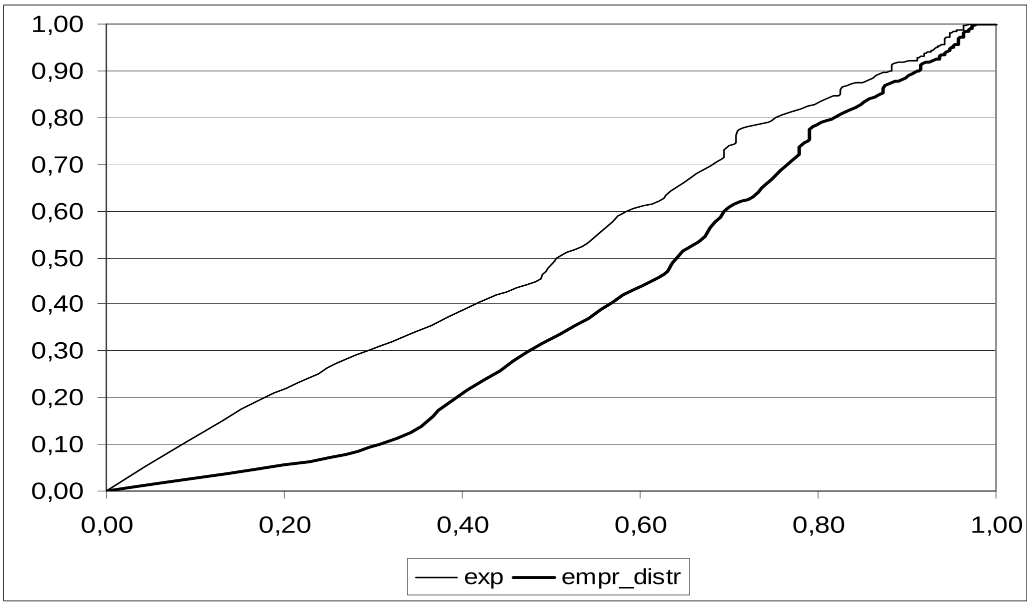

Fig. 2. TTT-plots for empirical distribution and the distribution after separation

4. The model of time between failures of the bus electrical system

The analysis of n = 1576 the lifetime between successive failures shows that the distribution of times until the failure is a mixture of an exponential distribution and Weibull distribution with the reliability function in the form:

$$R_2(t) = \exp(-\, c\, x^d), \text{ where } x>0,\ c > 0,\ d > 0.$$

can be the model for this lifetime data.
The reliability function for distribution (4) has the form:

$$\lambda(t) = c\, d\, t^{d-1} \text{ dla } x>0.$$

In this case the reliability function of mixture

$$R(t) = p \exp(-\lambda t) + (1-p) \exp(-\, c\, t^d) \text{ dla } x > 0$$

depends on four parameters (c, d, λ, p).
These parameters are estimated by numerical maximum likelihood methods thus obtaining:
c = 0.785, d = 0.487, λ = 0.0625, p = 0.627. Goodness of fit test of the empirical distribution with the distribution of the mixture gives for statistics λ − Kołmogorow's $\lambda_k = 0.649$, for p − value =0.83.Goodness of fit test Pearson's $\chi^2 = 74.45$ for p − value = 0.82. Both tests confirm good consistence of the empirical distribution and mixture distribution.

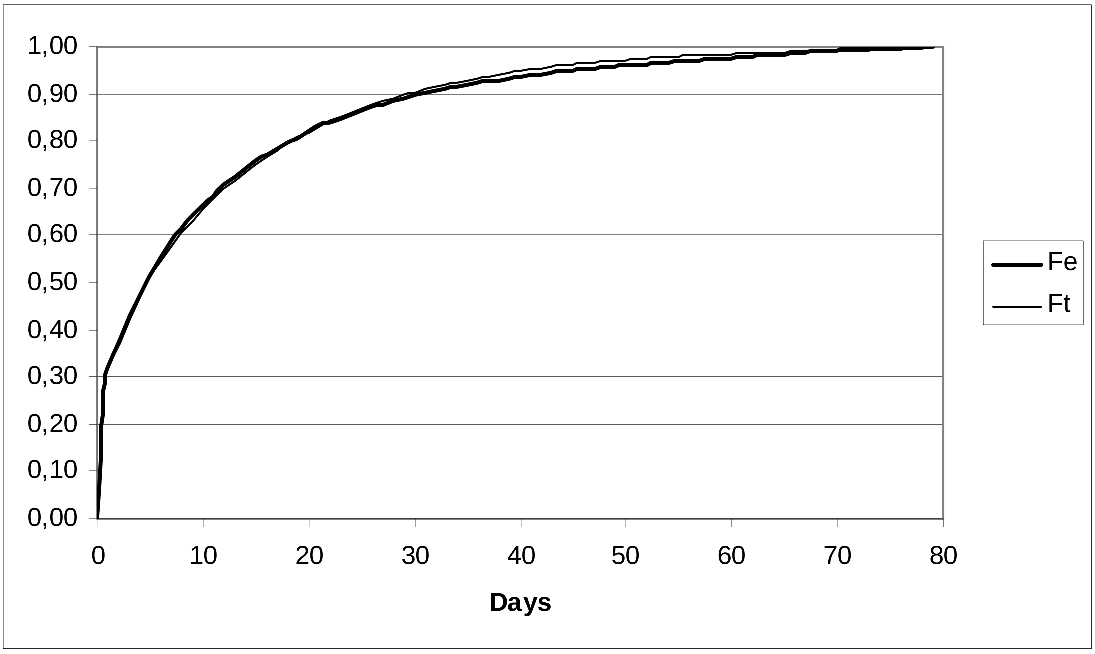

Fig..3 The empirical distribution and the distribution of mixture

Fig. 3 shows the plots of the empirical and mixture distributions and fig. 4 shows TTT-plots the for both components of the mixture.

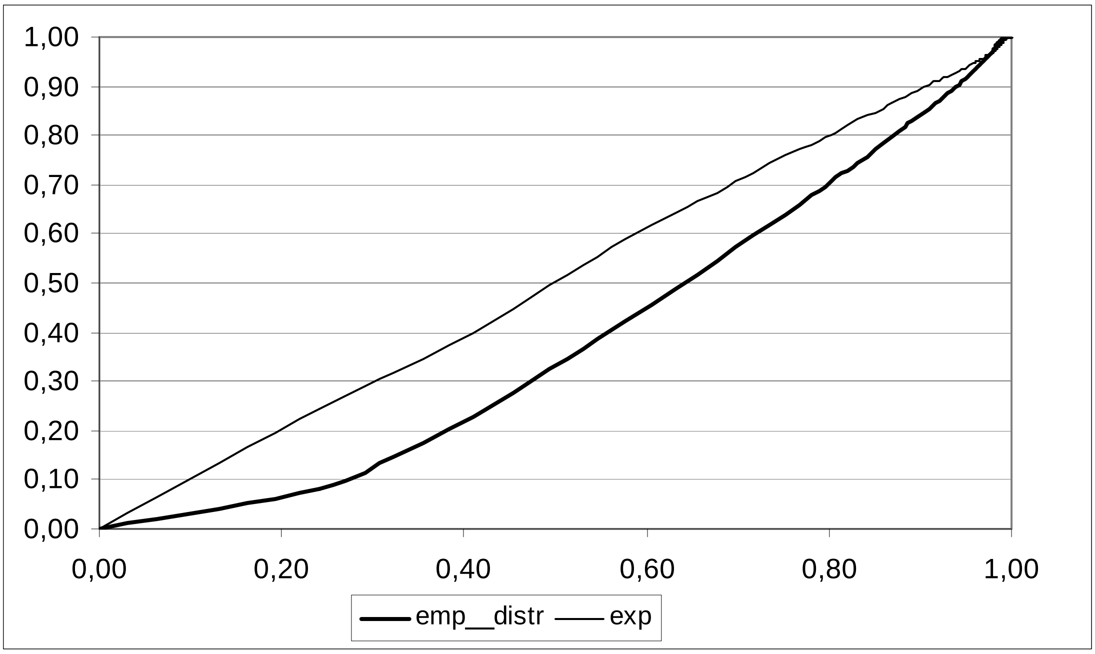

Fig. 4. TTT-plots of the empirical components of the mixture

5. Concluding remarks

In this paper, the lifetime distribution as a mixture of two known distributions, has been provided and discussed. It can be seen that the proposed model is a complete model for describing lifetime data of the bus subsystem. In the first example, a mixture of two points and exponential distribution as a lifetime of engine is considered. However, in the second example, Weibull mixture and an exponential distribution have been discussed as the electrical system's lifetime. It is proved that an empirical distribution and a mixture distribution fit each other.

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