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PROBABILISTIC APPROACH FOR ANALITICALLY-DETERMINED FATIGUE CURVE

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Abstract

When a new machine element is to be designed, the dimension of the element must be defined by calculating fatigue strength or life. To calculate it, fatigue characteristic of this part must be known. To acquire the information on such properties, the analytical method can be applied. Fatigue curve plotted that way is corresponding to 50% probability of the element failure. However, for engineering calculations there must be used a characteristic curve of 95% level of reliability or higher. The paper presents the method of the translocation of the fatigue curve, applying the analytical method to the technical requirements of probability. To verify this approach, an experiment was carried out to get fatigue characteristics of material C45+C and 42CrMo4 and comparisons were made for that estimated curve with the characteristics provided by the analytical method to require reliability. The fatigue test was made using smooth specimen and loaded by rotating bending. The comparison performed indicates sufficient analytical curve as compared with the characteristics estimated from the experiment. One shall note that the reliability coefficient can be used to substitute the safety coefficient which can assume a large value as there is no applicable knowledge available.

Keywords: *high-cycle fatigue strength, fatigue plots, analytical methods of estimating Wöhler characteristics.*

1. Introduction

In designing of new machine elements their dimensioning is performed. To do so, calculations are carried out to evaluate fatigue strength or life. For make that calculation, fatigue characteristics of the material or construction element must be known. To get such data, long and costly fatigue tests are required. Frequently at the beginning of the design project, it is not possible to realize such test and so analytical methods are used, e.g. the one presented in [2,3,6]. The characteristics provided using any of the methods correspond to 50% of the probability of the element failure. From this perspective, real construction reliability is required at 95% level or higher. Further on the paper presents the approach which can be feasible for such characteristics to be received. When an engineer does not have any data on the probability of failure of the design element, the safety coefficient is used. The value of the coefficient ranges from 1.3 to 2.5 according to [1], where the authors noted that the persons who do not have experience in designing elements loaded by variable force should apply the values from 2.5 to 4. However publication [4] suggests the range from 1.1 to 6.12 if there is no credible data available to calculate the value.

2. Presentation of selected analytical method

The method presented in paper [7] is based on FITNET method where the definition of coefficient m has changed. The first step of this approach determines the fatigue limit using the following equations (after[3]):

$$\sigma_W = f_{W,\sigma} \cdot R_m \text{ lub} \quad (1)$$

$$\tau_W = f_{W,\tau} \cdot \sigma_W, \quad (2)$$

where:
R_m – ultimate strength,
$f_{W,\sigma}, f_{W,\tau}$ –coefficient depending from type of material (after[3]).

In the method proposed, the value of slope coefficient is calculated with the following equations [7]:

$$m_e = \frac{\log\left(\frac{10^6}{N_{Re}}\right)}{log\left(\frac{0{,}9R_e}{Z_G}\right)}, \quad (3)$$

$$N_{Re} = 400\left(\frac{R_e}{R_m}\right)^{-10}, \quad (4)$$

where:
R_e – yield strength of material.

Graphic representation of the approach is shown in fig. 1.

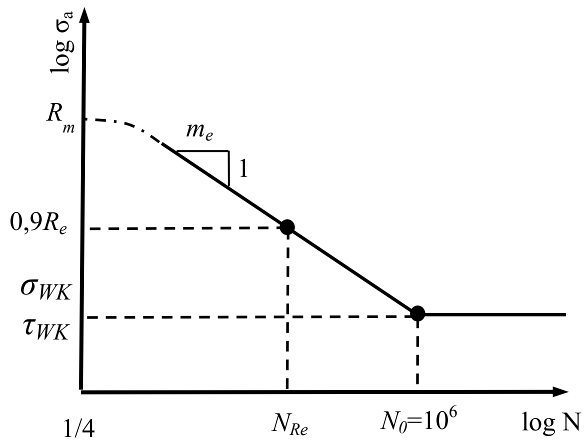

Fig. 1. Fatigue curie according analytical method from [7]

3. Probabilistic approach to determined analytical fatigue limit

It is commonly known that the distribution of fatigue limit is similar to the normal distribution [1,2]. With such assumption it is possible to define the probability of the strength value of unlimited fatigue life [2]. Using the equation for standardizing normal distribution, the following can be noted:

$$Z = \frac{Z_{GR} - \mu}{s}, \quad (5)$$

where:
Z – value of random variable from normal distribution,
Z_{GR} – random variable of fatigue limit,
μ – value of mean of fatigue limit (it can assumption Z_G),

s – standard deviation.

Assuming that probability of failure of sample loaded by defined stress amplitude is given by following equation:

$$P(Z < \mu) = \Phi(Z) = 1 - R, \tag{6}$$

$$Z = \Phi^{-1}(1 - \mathrm{R}), \tag{7}$$

where:
$\Phi(Z)$ – standard normal density function,
R – level of reliability.

Coefficient of variation define as:

$$V = \frac{s}{\mu}, \tag{8}$$

and to equation (7) substituting Z with equation (5) and replacing quantity s by the quantity obtained from equation (8) we will get the following:

$$Z_{GR} = \mu + \Phi^{-1}(1 - R) \cdot \mu \cdot V. \tag{9}$$

By introducing the coefficient of reliability which reduces the value of fatigue limit to the value adequate to assuming the probability of failure, the following equation is obtained:

$$Z_{GR} = C_R \cdot \mu, \tag{10}$$

where:
C_R – coefficient of reliability.

By substituting C_R to equation (9), we will obtain the following equation:

$$C_R = 1 + \Phi^{-1}(1 - R) \cdot V. \tag{11}$$

Assuming that the coefficient of variation of fatigue limit assumes the value 0.08 (according [2,4]), it is possible to determine the value of the coefficient of reliability. The value calculated has been presented in Table 1.

Tab. 1 Value of coefficient of reliability for diferend levels of reliability

R	C_R
0,9	0,897
0,95	0,868
0,98	0,836
0,99	0,814
0,999	0,753
0,9999	0,702

Finally the dependence to calculate the fatigue limit for the required level of reliability Z_{GP} can be given as follows:

$$Z_{GP} = Z_G \cdot C_R. \tag{12}$$

4. Comparison of characteristics obtained by analytical and experimental method

The characteristics estimated experimentally for material C45+C and 42CrMo4 are presented in Figures 4 and 5. Static properties are given in Tab. 2. The first material was tested in delivery state in form of cold-drawn bars 10 mm in diameter. The second material (42CrMo4) has been quenched and tempered to harden the surface up to 30 HRC. The dimensions of the specimen are given in Fig. 2. The samples were loaded by rotary bending at the frequency of 28 Hz. The test equipment of own design; the device verification is presented in paper [6].

Tab. 2 Static properties of material C45+C and 42CrMo4

Material	*Yield limit /proof stress [MPa]*	*Tensile strength [MPa]*
C45+C	-/647	826
42CrMo4	1095/-	1172

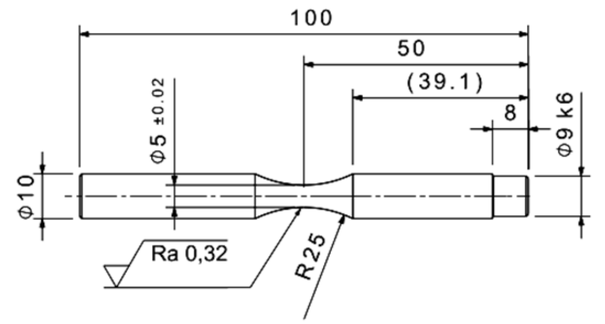

Fig. 2. Sample used for tests

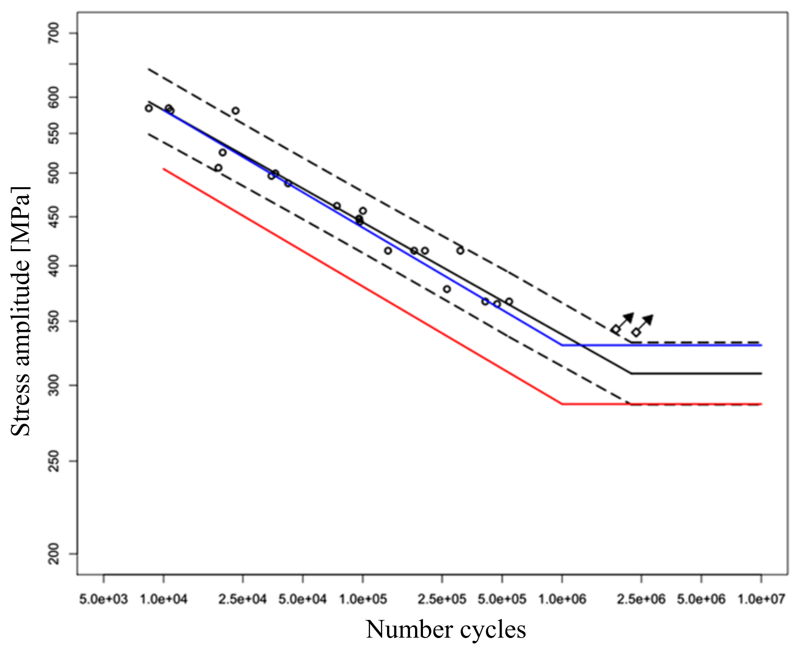

Fig. 3. Fatigue curves for C45+C steel where black line - estimated curve for experimental data for 50% probability, dotted line – fatigue curve for probability of failure 95% and 5%, the curve obtained using the analytical method according to [7], red line – the curve obtained applying the analytical method according to [7] with the fatigue limit corrected with equation (12) for the reliability of 95%

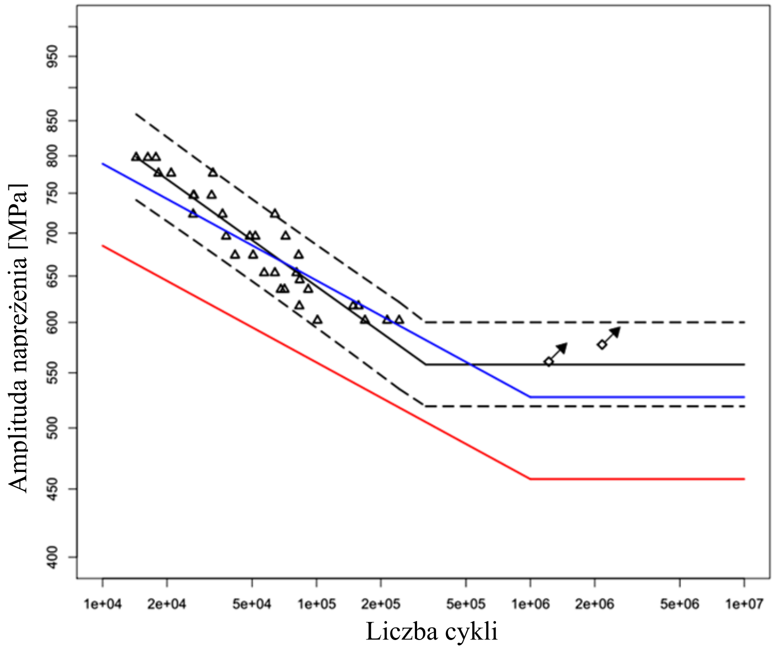

Fig. 4. Fatigue curves for 42CrMo4 steel where black line – the estimated curve for experimental data for 50% probability, dotted line – fatigue curve for probability of failure 95% and 5%, blue line- curve obtained using the analytical method according to [7], red line – the curve plotted applying the analytical method according to [7] with the fatigue limited corrected with equation (12) for the reliability of 95%

Based on the figures plotted, there were determined the strength for 10^5 cycles according to the characteristics of 50% and 95% probability. The results for each material and method are given in Table 3.

Tab. 3 Value of fatigue strength for 10^5 cycles according analytical and experimental method

Method of determine fatigue curve	Material	Fatigue strength for 10^5 cycles for 50% probability [MPa]	Fatigue strength for 10^5 cycles for 95% probability [MPa]	Ratio of strength for 50% and 95% probability characteristics
Analytical method	C45+C	421,1	365,5	1,15
	42CrMo4	645,1	560	1,15
Experimental method	C45+C	443,8	411,9	1,08
	42CrMo4	631,8	594,4	1,06

5. Summary and conclusions

Applying equation (12), it is possible to define the fatigue limit, determined using the analytical method, of the required level of probability of the forecast correctness. Based on that assumption one can receive fatigue characteristics at the reliability level required. The figures show the characteristics are on ‘a safe’ side (underestimated fatigue life/strength estimated).

Additionally, the application of probabilistic approach gives the designer a possibility of giving up conservative values of the safety coefficient. With such approach the engineer can decide himself what safety level is adequate for the machine being designed. In the present example of determining the fatigue characteristics with the analytical method the coefficient of reliability was used at the level of 95%. In that case there was obtained the coefficient of safety of 1.15. It is the value which is relatively low as compared with those offered in literature. One shall note that if required by safety requirements, the value can be increased applying the coefficient of reliability at a higher level.

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Literatura

[1] Kocańda S., Szala J., *Podstawy obliczeń zmęczeniowych*, PWN, Warszawa, 1997.

[2] Lee Yung-Li, Pan Jwo, Hathaway R. B., Barkey M. E., *Fatigue testing and analysis*, University of Alabama, Elsevier 2005.

[3] Neimitz A., Dzioba I., Graba M., Okrajni J., *Ocena wytrzymałości, trwałości i bezpieczeństwa pracy elementów konstrukcyjnych zawierających defekty*, Politechnika Świętokrzyska, Kielce 2008.

[4] Niezgodziński M.E., Niezgodziński T., Wzory wykresy i tablice wytrzymałościowe, wyd. 9, Warszawa WNT, 2004.

[5] Pook L., Metal fatigue what it is. Why it matters, London Springer 2009*,*

[6] Strzelecki P., Sempruch J., *Experimental Verification of the Analytical Method for Estimated S-N Curve in Limited Fatigue Life*. Materials Science Forum, Vol. 726, 2012, pp. 11-16.

[7] Strzelecki P., Sempruch J., *Modification of selected methods of rapid determination of fatigue characteristics in the range of limited fatigue life,* Journal of Polish Cimac Selected problems of designing and operating technical systems Vol. 6 No 3; 2011, pp. 289-296.